

ADVANCED COORDINATE GEOMETRY: CIRCLES

Complete Master Problem Bank & Practice Assignment

Comprehensive Transcribed Collection: All 46 Pages (Parts 1, 2, 3 & 4)

Analytical, Proof-Based, Locus, Tangency, Orthogonal Systems, Matrix Match & Objective Problems

Master Assignment Structure & Scope:

- **Part 1 (Pages 1–12):** Foundations, tangents, orthocentre/centroid loci, coaxal systems, multi-concept problems.
- **Part 2 (Pages 13–24):** Multi-conics, common tangents, geometric progression packings, and similitude circles.
- **Part 3 (Pages 25–36):** Radical axes, touching circle families, chord bisection, and linked comprehensions.
- **Part 4 (Pages 37–46):** Conjugate points, circumcircle properties, matrix matchings, and extremal problems.

PART 1: ADVANCED PROBLEM SET

Pages 1 to 12 • Foundations, Tangents, Orthogonality, Loci & Multi-Concept Problems

§ Page 1

1. The line $lx + my + n = 0$ intersects the curve $ax^2 + 2hxy + by^2 = 1$ at the points P and Q . The circle on PQ as diameter passes through the origin. Prove that:

$$n^2(a + b) = l^2 + m^2$$

2. $A(-a, 0)$ and $B(a, 0)$ are fixed points. C is a point which divides AB in a constant ratio $\tan \alpha$. If AC and CB subtend equal angles at P , prove that the equation of the locus of P is:

$$x^2 + y^2 + 2ax \sec 2\alpha + a^2 = 0$$

3. A circle is drawn with its centre on the line $x + y = 2$ to touch the line $4x - 3y + 4 = 0$ and pass through the point $(0, 1)$. Find its equation.

4. A point moving around the circle $(x + 4)^2 + (y + 2)^2 = 25$ with centre C broke away from it either at point A or point B on the circle and moved along a tangent to the circle passing through the point $D(3, 3)$. Find the following:

- Coordinates of the points A and B .
- Equation of the tangents at A and B .
- Angle ADB and the maximum and minimum distances of the point D from the circle.
- Area of quadrilateral $ADBC$ and $\triangle DAB$.
- Equation of the circle circumscribing $\triangle DAB$ and also the intercepts made by this circle on the coordinate axes.

5. Find the locus of the midpoint of the chord of a circle $x^2 + y^2 = 4$ such that the segment intercepted by the chord on the curve $x^2 - 2x - 2y = 0$ subtends a right angle at the origin.

6. Find the equation of a line with gradient 1 such that the two circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 10x - 14y + 65 = 0$ intercept equal lengths on it.

7. Consider a circle S with centre at the origin and radius 4. Four circles A, B, C , and D , each with radius unity and centres $(-3, 0)$, $(-1, 0)$, $(1, 0)$, and $(3, 0)$ respectively, are drawn. A chord PQ of the circle S touches circle B and passes through the centre of circle C . If the length of this chord can be expressed as $\sqrt{x}$, find x .

8. Consider a curve $ax^2 + 2hxy + by^2 = 1$ and a point P not on the curve. A line is drawn from the point P intersecting the curve at points Q and R . If the product $PQ \cdot PR$ is independent of the slope of the line, show that the curve is a circle.

9. The line $2x - 3y + 1 = 0$ is tangent to a circle $S = 0$ at $(1, 1)$. If the radius of the circle is $\sqrt{13}$, find the equation of the circle S .

10. Find the equation of the circle which passes through the point $(1, 1)$ and touches the circle $x^2 + y^2 + 4x - 6y - 3 = 0$ at the point $(2, 3)$ on it.

11. Let a circle be given by $2x(x - a) + y(2y - b) = 0$ ($a \neq 0, b \neq 0$). Find the condition on a and b if two chords, each bisected by the x -axis, can be drawn to the circle from the point $(a, b/2)$.

12. Show that the equation of a straight line meeting the circle $x^2 + y^2 = a^2$ in two points at equal distances d from a

point (x_1, y_1) on its circumference is:

$$xx_1 + yy_1 - a^2 + \frac{d^2}{2} = 0$$

§ Page 2

1. The radical axis of the circles $x^2 + y^2 + 2gx + 2fy + c = 0$ and $2x^2 + 2y^2 + 3x + 8y + 2c = 0$ touches the circle $x^2 + y^2 + 2x - 2y + 1 = 0$. Show that either $g = 3/4$ or $f = 2$.
2. Find the equation of the circle through the points of intersection of circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 4y - 12 = 0$, cutting the circle $x^2 + y^2 - 2x - 4 = 0$ orthogonally.
3. The centre of the circle $S = 0$ lies on the line $2x - 2y + 9 = 0$ and $S = 0$ cuts orthogonally the circle $x^2 + y^2 = 4$. Show that circle $S = 0$ passes through two fixed points and find their coordinates.
4. Find the equation of the circle whose radius is 3 and which touches the circle $x^2 + y^2 - 4x - 6y - 12 = 0$ internally at the point $(-1, -1)$.
5. Show that the locus of the centres of a circle which cuts two given circles orthogonally is a straight line, and hence deduce the locus of the centres of the circles which cut the circles $x^2 + y^2 + 4x - 6y + 9 = 0$ and $x^2 + y^2 - 5x + 4y + 2 = 0$ orthogonally. Interpret the locus.
6. A variable circle passes through the point $A(a, b)$ and touches the x -axis; show that the locus of the other end of the diameter through A is $(x - a)^2 = 4by$.
7. Consider a family of circles passing through two fixed points $A(3, 7)$ and $B(6, 5)$. Show that the chords in which the circle $x^2 + y^2 - 4x - 6y - 3 = 0$ cuts the members of the family are concurrent at a point. Find the coordinates of this point.
8. Find the equation of the circle passing through $(1, 1)$ belonging to the system of co-axial circles that are tangent at $(2, 2)$ to the locus of the point of intersection of mutually perpendicular tangents to the circle $x^2 + y^2 = 4$.
9. Find the locus of the midpoint of all chords of the circle $x^2 + y^2 - 2x - 2y = 0$ such that the pair of lines joining $(0, 0)$ and the point of intersection of the chords with the circle make equal angles with the x -axis.
10. The circle $C : x^2 + y^2 + kx + (1 + k)y - (k + 1) = 0$ passes through the same two points for every real number k . Find:
 - (i) The coordinates of these two points.
 - (ii) The minimum value of the radius of a circle C .

11. Show that the locus of the point from which the tangents to the circle $x^2 + y^2 - a^2 = 0$ include a constant angle α is:

$$(x^2 + y^2 - a^2)^2 \tan^2 \alpha = 4a^2(x^2 + y^2 - a^2)$$

12. A circle with centre in the first quadrant is tangent to $y = x + 10$, $y = x - 6$, and the y -axis. Let (h, k) be the centre of the circle. If $h + k = a + b\sqrt{a}$ where $\sqrt{a}$ is a surd, find the value of $a + b$.
13. A circle is described to pass through the origin and to touch the lines $x = 1$, $x + y = 2$. Prove that the radius of the circle is a root of the equation:

$$(3 - 2\sqrt{2})t^2 - 2\sqrt{2}t + 2 = 0$$

14. Find the condition such that the four points in which the circles $x^2 + y^2 + ax + by + c = 0$ and $x^2 + y^2 + a'x + b'y + c' = 0$ are intercepted by the straight lines $Ax + By + C = 0$ and $A'x + B'y + C' = 0$ respectively, lie on another circle.

§ Page 3

1. A circle C is tangent to the x and y axes in the first quadrant at the points P and Q respectively. BC and AD are parallel tangents to the circle with slope 1. If the points A and B are on the y -axis while C and D are on the x -axis, and the area of the figure $ABCD$ is $\sqrt{2}$ sq units, find the radius of the circle.
2. The circle $x^2 + y^2 - 4x - 4y + 4 = 0$ is inscribed in a triangle which has two of its sides along the coordinate axes. The locus of the circumcentre of the triangle is $x + y - xy + K\sqrt{x^2 + y^2} = 0$. Find K .
3. An isosceles right-angled triangle whose sides are 1, 1, $\sqrt{2}$ lies entirely in the first quadrant with the ends of the hypotenuse on the coordinate axes. If it slides, prove that the locus of its centroid is:

$$(3x - y)^2 + (x - 3y)^2 = 32$$

4. Find the equation of a circle which touches the lines $7x^2 - 18xy + 7y^2 = 0$ and the circle $x^2 + y^2 - 8x - 8y = 0$, and is contained inside the given circle.
5. Let W_1 and W_2 denote the circles $x^2 + y^2 + 10x - 24y - 87 = 0$ and $x^2 + y^2 - 10x - 24y + 153 = 0$ respectively. Let m be the smallest possible value of a for which the line $y = ax$ contains the centre of a circle that is externally tangent to W_2 and internally tangent to W_1 . Given that $m^2 = p/q$ where p and q are relatively prime integers, find $p + q$.
6. Find the equation of the circle which passes through the origin, meets the x -axis orthogonally, and cuts the circle $x^2 + y^2 = a^2$ at an angle of 45° .
7. The ends A, B of a fixed straight line of length a and ends A', B' of another fixed straight line of length b slide upon the axis of x and the axis of y (one end on axis of x and the other on axis of y). Find the locus of the centre of the circle passing through A, B, A' , and B' .
8. The angle between a pair of tangents drawn from a point P to the circle $x^2 + y^2 + 4x - 6y + 9\sin^2\alpha + 13\cos^2\alpha = 0$ is 2α . The equation of the locus of the point P is:
- (a) $x^2 + y^2 + 4x - 6y + 4 = 0$ (b) $x^2 + y^2 + 4x - 6y - 9 = 0$
 (c) $x^2 + y^2 + 4x - 6y - 4 = 0$ (d) $x^2 + y^2 + 4x - 6y + 9 = 0$
9. Find the intervals of values of a for which the line $y + x = 0$ bisects two chords drawn from a point $\left(\frac{1+\sqrt{2}a}{2}, \frac{1-\sqrt{2}a}{2}\right)$ to the circle $2x^2 + 2y^2 - (1 + \sqrt{2}a)x - (1 - \sqrt{2}a)y = 0$.
- 10.
- (a) The chords of contact of the pair of tangents drawn from each point on the line $2x + y = 4$ to the circle $x^2 + y^2 = 1$ pass through a fixed point. Find its coordinates.
 (b) Let C be any circle with centre $(0, \sqrt{2})$. Prove that at most two rational points can lie on C . (A rational point is a point both of whose coordinates are rational numbers).
11. The number of common tangents to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 6x - 8y = 24$ is:
- (A) 0 (B) 1 (C) 3 (D) 4
12. C_1 and C_2 are two concentric circles, the radius of C_2 being twice that of C_1 . From a point P on C_2 , tangents PA and PB are drawn to C_1 . Prove that the centroid of the triangle PAB lies on C_1 .

§ Page 4

1. If two distinct chords, drawn from the point (p, q) on the circle $x^2 + y^2 = px + qy$ (where $pq \neq 0$) are bisected by the x -axis, then:
- (A) $p^2 = q^2$ (B) $p^2 = 8q^2$ (C) $p^2 < 8q^2$ (D) $p^2 > 8q^2$
2. Let T_1, T_2 be two tangents drawn from $(-2, 0)$ onto the circle $C : x^2 + y^2 = 1$. Determine the circles touching C and having T_1, T_2 as their pair of tangents. Further, find the equations of all possible common tangents to these circles, when taken two at a time.
3. The triangle PQR is inscribed in the circle $x^2 + y^2 = 25$. If Q and R have coordinates $(3, 4)$ and $(-4, 3)$ respectively, then $\angle QPR$ is equal to:
- (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$
4. Find the equation to the circle which is such that the lengths of the tangents to it from the points $(1, 0)$, $(2, 0)$, and $(3, 2)$ are $1, \sqrt{7}$, and $\sqrt{2}$ respectively.
5. Find the point on the straight line $y = 2x + 11$ which is nearest to the circle $16(x^2 + y^2) + 32x - 8y - 50 = 0$.
6. Let PQ and RS be tangents at the extremities of the diameter PR of a circle of radius r . If PS and RQ intersect at a point X on the circumference of the circle, then $2r$ equals:
- (a) $\sqrt{PQ \cdot RS}$ (b) $\frac{PQ+RS}{2}$
 (c) $\frac{2PQ \cdot RS}{PQ+RS}$ (d) $\sqrt{\frac{PQ^2+RS^2}{2}}$
7. A circle of radius 2 units rolls on the outer side of the circle $x^2 + y^2 + 4x = 0$, touching it externally. Find the locus of the centre of this outer circle. Also find the equations of the common tangents of the two circles when the line

joining the centres of the two circles makes an angle of 60° with the x -axis.

8. Let $2x^2 + y^2 - 3xy = 0$ be the equation of a pair of tangents drawn from the origin O to a circle of radius 3 with centre in the first quadrant. If A is one of the points of contact, find the length of OA .

9. Tangents TP and TQ are drawn from a point T to the circle $x^2 + y^2 = a^2$. If the point T lies on the line $px + qy = r$, find the locus of the centre of the circumcircle of triangle TPQ .

10. If the tangent at the point P on the circle $x^2 + y^2 + 6x + 6y = 2$ meets the straight line $5x - 2y + 6 = 0$ at a point Q on the y -axis, then the length of PQ is:

- (A) 4 (B) $2\sqrt{5}$ (C) 5 (D) $3\sqrt{5}$

11. If $a > 2b > 0$, then the positive value of m for which $y = mx - b\sqrt{1+m^2}$ is a common tangent to $x^2 + y^2 = b^2$ and $(x-a)^2 + y^2 = b^2$ is:

- (a) $\frac{2b}{\sqrt{a^2-4b^2}}$ (b) $\frac{\sqrt{a^2-4b^2}}{2b}$
(c) $\frac{2b}{a-2b}$ (d) $\frac{b}{a-2b}$

12. The radius of the circle, having centre at $(2, 1)$, whose one chord is a diameter of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is:

- (A) 1 (B) 2 (C) 3 (D) $\sqrt{3}$

§ Page 5

1. A circle is given by $x^2 + (y-1)^2 = 1$; another circle C touches it externally and also touches the x -axis, then the locus of its centre is:

- (a) $\{(x, y) : x^2 = 4y\} \cup \{(x, y) : y \leq 0\}$ (b) $\{(x, y) : x^2 + (y-1)^2 = 4\} \cup \{(x, y) : y \leq 0\}$
(c) $\{(x, y) : x^2 = y\} \cup \{(0, y) : y \leq 0\}$ (d) $\{(x, y) : x^2 = 4y\} \cup \{(0, y) : y \leq 0\}$

2. The length of the tangent drawn from any point on the circle $x^2 + y^2 + 2gx + 2fy + p = 0$ to the circle $x^2 + y^2 + 2gx + 2fy + q = 0$ is:

- (A) $q - p$ (B) $\sqrt{p - q}$ (C) $\sqrt{q - p}$ (D) None of these

3. Equation of the circle cutting orthogonally the three circles $x^2 + y^2 - 2x + 3y - 7 = 0$, $x^2 + y^2 + 5x - 5y + 9 = 0$, and $x^2 + y^2 + 7x - 9y + 29 = 0$ is:

- (a) $x^2 + y^2 - 16x - 18y - 4 = 0$ (b) $x^2 + y^2 - 7x + 11y + 6 = 0$
(c) $x^2 + y^2 + 2x - 8y + 9 = 0$ (d) None of these

4. The circumference of the circle $x^2 + y^2 - 2x + 8y - q = 0$ is bisected by the circle $x^2 + y^2 + 4x + 12y + p = 0$, then $p + q$ is equal to:

- (A) 25 (B) 100 (C) 10 (D) 48

5. The chords of contact of the pair of tangents drawn from each point on the line $2x + y = 4$ to the circle $x^2 + y^2 = 1$ pass through the point:

- (A) $(1, 2)$ (B) $(1/2, 1/4)$ (C) $(2, 4)$ (D) None

6. Find the equation to the circle which touches the axis of x at a distance 3 from the origin and intercepts a distance 6 on the axis of y .

7. Tangents are drawn from the point (h, k) to the circle $x^2 + y^2 = a^2$; prove that the area of the triangle formed by them and the straight line joining their points of contact is:

$$\frac{a(h^2 + k^2 - a^2)^{3/2}}{h^2 + k^2}$$

8. Find the equations to the common tangents of the circles $x^2 + y^2 - 2x - 6y + 9 = 0$ and $x^2 + y^2 + 6x - 2y + 1 = 0$.

9. If $(a, 1/a)$, $(b, 1/b)$, $(c, 1/c)$, and $(d, 1/d)$ are four distinct points on a circle of radius 4 units, then $abcd$ is equal to:

- (A) 4 (B) 16 (C) 1 (D) None

10. From the point $A(0,3)$ on the circle $x^2 + 4x + (y - 3)^2 = 0$, a chord AB is drawn and extended to a point M such that $AM = 2AB$. The equation of the locus of M is:

- (a) $x^2 + 8x + y^2 = 0$ (b) $x^2 + 8x + (y - 3)^2 = 0$
 (c) $(x - 3)^2 + 8x + y^2 = 0$ (d) $x^2 + 8x + 8y^2 = 0$

§ Page 6

1. Two thin rods AB and CD of lengths $2a$ and $2b$ move along OX and OY respectively, where O is the origin. The equation of the locus of the centre of the circle passing through the extremities of the two rods is:

- (a) $x^2 + y^2 = a^2 + b^2$ (b) $x^2 - y^2 = a^2 - b^2$
 (c) $x^2 + y^2 = a^2 - b^2$ (d) $x^2 - y^2 = a^2 + b^2$

2. Let x and y be the real numbers satisfying the equation $x^2 - 4x + y^2 + 3 = 0$. If the maximum and minimum values of $x^2 + y^2$ are M and m respectively, then the numerical value of $M - m$ is:

- (A) 2 (B) 8 (C) 15 (D) None of these

3. The area of the triangle formed by the tangents from the point $(4,3)$ to the circle $x^2 + y^2 = 9$ and the line joining their points of contact is:

- (A) 192/25 (B) 192 (C) 25 (D) 250

4. A point $A(2,1)$ is outside the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ and AP, AQ are tangents to the circle. The equation of the circle circumscribing the triangle APQ is:

- (a) $(x + g)(x - 2) + (y + f)(y - 1) = 0$ (b) $(x + g)(x - 2) - (y + f)(y - 1) = 0$
 (c) $(x - g)(x + 2) + (y - f)(y + 1) = 0$ (d) None

5. The locus of the midpoints of the chords of the circle $x^2 + y^2 + 4x - 6y - 12 = 0$ which subtend an angle of $\pi/3$ radians at its circumference is:

- (a) $(x - 2)^2 + (y + 3)^2 = 6.25$ (b) $(x + 2)^2 + (y - 3)^2 = 6.25$
 (c) $(x + 2)^2 + (y - 3)^2 = 18.75$ (d) $(x + 2)^2 + (y + 3)^2 = 18.75$

6. If the circle $C_1 : x^2 + y^2 = 16$ intersects another circle C_2 of radius 5 in such a manner that the common chord is of maximum length and has a slope equal to $3/4$, then the coordinates of the centre of C_2 are:

- (a) $(\pm 9/5, \pm 12/5)$ (b) $(\pm 9/5, \mp 12/5)$
 (c) $(\pm 12/5, \pm 9/5)$ (d) $(\pm 12/5, \mp 9/5)$

7. If from any point P on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, tangents are drawn to the circle $x^2 + y^2 + 2gx + 2fy + c \sin^2 \alpha + (g^2 + f^2) \cos^2 \alpha = 0$, then the angle between the tangents is:

- (A) α (B) 2α (C) $\alpha/2$ (D) None

8. If the length of a common internal tangent to two circles is 7, and that of a common external tangent is 11, then the product of the radii of the two circles is:

- (A) 36 (B) 9 (C) 18 (D) 4

9. Through a fixed point (h, k) , secants are drawn to the circle $x^2 + y^2 = r^2$. Show that the locus of the midpoints of the portions of the secants intercepted by the circle is $x^2 + y^2 = hx + ky$.

10. Find the equation of a circle passing through the origin if the line pair $6y^2 - 3xy + 2y - 6 = 0$ is orthogonal to it. If this circle is orthogonal to the circle $x^2 + y^2 - kx + 2ky - 8 = 0$, then find the value of k .

11. Find the equation of the circle which cuts each of the circles $x^2 + y^2 = 4$, $x^2 + y^2 - 6x - 8y + 10 = 0$, and $x^2 + y^2 + 2x - 4y - 2 = 0$ at the extremities of a diameter.

12. Find the values of a for which the point $(2a, a + 1)$ is an interior point of the larger segment of the circle $x^2 + y^2 - 2x - 2y - 8 = 0$ made by the chord whose equation is $x - y + 1 = 0$.

§ Page 7

1. Match Column-I with Column-II (Geometric Properties):

Column I	Column II
(A) Number of values of a for which the common chord of the circles $x^2 + y^2 = 8$ and $(x - a)^2 + y^2 = 8$ subtends a right angle at the origin is	(p) 4
(B) A chord of the circle $(x - 1)^2 + y^2 = 4$ lies along the line $y = 2\sqrt{3}(x - 1)$. The length of the chord is equal to	(q) 2
(C) The number of circles touching all the three lines $3x + 7y = 2$, $21x + 49y = 5$, and $9x + 21y = 0$ is	(r) 0
(D) If radii of the smallest and largest circle passing through the point $(\sqrt{3}, \sqrt{2})$ and touching the circle $x^2 + y^2 - 2\sqrt{2}y - 2 = 0$ are r_1 and r_2 respectively, then the mean of r_1 and r_2 is	(s) 1

2. Match Column-I with Column-II (Common Tangents):

Column I	Column II
(A) Number of common tangents of the circles $x^2 + y^2 - 2x = 0$ and $x^2 + y^2 + 6x - 6y + 2 = 0$ is	(p) 1
(B) Number of indirect common tangents of the circles $x^2 + y^2 - 4x - 10y + 4 = 0$ and $x^2 + y^2 - 6x - 12y - 55 = 0$ is	(q) 2
(C) Number of common tangents of the circles $x^2 + y^2 - 2x - 4y = 0$ and $x^2 + y^2 - 8y - 4 = 0$ is	(r) 3
(D) Number of direct common tangents of the circles $x^2 + y^2 + 2x - 8y + 13 = 0$ and $x^2 + y^2 - 6x - 2y + 6 = 0$ is	(s) 0

3. Statement-Reason Analysis 1:

Statement-1: If three circles are such that their centres are non-collinear, then exactly one circle exists which cuts the three circles orthogonally.

Statement-2: Radical axis for two intersecting circles is the common chord.

- (A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
 (B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
 (C) Statement-1 is True, Statement-2 is False.
 (D) Statement-1 is False, Statement-2 is True.

4. Statement-Reason Analysis 2:

Statement-1: If a line $L = 0$ is tangent to the circle $S = 0$, then it will also be a tangent to the circle $S + \lambda L = 0$.

Statement-2: If a line touches a circle, then the perpendicular distance of the line from the centre of the circle is equal to the radius of the circle.

- (A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
 (B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
 (C) Statement-1 is True, Statement-2 is False.
 (D) Statement-1 is False, Statement-2 is True.

§ Page 8

1. For each natural number k , let C_k denote the circle with radius k cm and centre at the origin. On the circle C_k , a particle moves k cm in the counterclockwise direction. After completing its motion on C_k , the particle moves to C_{k+1} in the radial direction. The motion of the particle continues in this manner. The particle starts at $(1, 0)$. If the particle crosses the positive direction of the x -axis for the first time on the circle C_n , then $n =$

2. A circle passes through three points A, B , and C with the line segment AC as its diameter. A line passing through A intersects the chord BC at a point D inside the circle. If angles DAB and CAB are α and β respectively, and the distance between the point A and the midpoint of the line segment DC is d , prove that the area of the circle is:

$$\frac{\pi d^2 \cos^2 \alpha}{\cos^2 \alpha + \cos^2 \beta + 2 \cos \alpha \cos \beta \cos(\beta - \alpha)}$$

3. A circle having centre at C is made to pass through the point $P(1, 2)$, touching the straight lines $7x - y = 5$ and $x + y + 13 = 0$ at A and B respectively, then:

- (A) Area of quadrilateral $ACBP$ is 100 sq units
(B) Radius of smaller circle is $\sqrt{50}$
(C) Area of quadrilateral $ACBP$ is 200 sq units
(D) Radius of smaller circle is 10

4. Given pair of lines $2x^2 + 5xy + 2y^2 + 4x + 5y + a = 0$ and line $L \equiv bx + y + 5 = 0$. Match the conditions on b in Column-I with values in Column-II:

Column-I	Column-II
(A) If there exist 4 circles which touch the pair of lines and the line L simultaneously, then the value of b can be	(p) $1/2$
(B) If there exist 2 circles which touch the pair of lines and the line L simultaneously, then the value of b can be	(q) 2
(C) If there exist no circle which touches the pair of lines and the line L simultaneously, then the value of b can be	(r) 5
(D) If there exist infinite circles which touch the pair of lines and the line L simultaneously, then the value of b cannot be	(s) 4
	(t) 1

§ Page 9

Comprehension Passage / Instructions:

Linked Comprehension:

Tangents are drawn to the parabola $y^2 = 4x$ from the point $P(6, 5)$ to touch the parabola at Q and R . C_1 is a circle which touches the parabola at Q and C_2 is a circle which touches the parabola at R . Both the circles C_1 and C_2 pass through the focus of the parabola.

1. Area of the ΔPQR equals:

- (A) $1/2$ (B) 1 (C) 2 (D) $1/4$

2. Radius of the circle C_2 is:

- (A) $5\sqrt{5}$ (B) $5\sqrt{10}$ (C) $10\sqrt{2}$ (D) $\sqrt{210}$

3. The common chord of the circles C_1 and C_2 passes through the:

- (A) Incentre of ΔPQR (B) Circumcentre of ΔPQR
(C) Centroid of ΔPQR (D) Orthocentre of ΔPQR

§ Page 10

1. 8 spheres of radius 1 unit are kept on a table with their centres at the vertices of a regular octagon and each sphere touching its two neighbours. If a sphere is placed in the centre on the table touching all of the 8 spheres, then its radius is:

- (A) $\sqrt{2} - 1$ (B) $1 - \frac{1}{\sqrt{2}}$
(C) $\sqrt{2} + 1$ (D) $1 + \frac{1}{\sqrt{2}}$

2. Let ellipse $4x^2 + 16y^2 = 64$ and circle $x^2 + y^2 = r^2$ have a common tangent touching at A and B respectively. The maximum length of AB can be:

- (A) 4 (B) 3 (C) 2 (D) 5

3. Let $S_1 \equiv x^2 - 10x + y^2 + 16 = 0$, S_2 be the image of S_1 w.r.t. the line $x - y = 0$, and S_3 be the image of S_2 w.r.t. $y = 0$. Let S' be the circle which cuts all three circles orthogonally and S'' be the circle of minimum radius which contains all three circles, then:

- (A) Equation of circle S' is $x^2 + y^2 = 16$ (B) Equation of circle S'' is $x^2 + y^2 = 64$
(C) The radius of S' is 4 (D) The radius of S'' is 8

§ Page 11

1. A pair of perpendicular lines passing through $P(1, 4)$ intersect the x -axis at Q and R . Then the locus of the incentre

of ΔPQR is:

- (a) $x^2 + y^2 - 2x - 8y + 17 = 0$ (b) $x^2 + y^2 + 2x - 8y + 17 = 0$
 (c) $x^2 - y^2 - 2x - 8y + 17 = 0$ (d) $x^2 + y^2 - 2x + 8y + 17 = 0$
2. Triangle formed by the lines $x + y = 0$, $x - y = 0$ and $lx + my = 1$. If l and m vary subject to the condition $l^2 + m^2 = 1$, then the locus of its circumcentre is:
- (a) $(x^2 - y^2) = x^2 + y^2$ (b) $(x^2 - y^2)^2 = x^2 + y^2$
 (c) $(x^2 + y^2)^2 = 4x^2y^2$ (d) $(x^2 - y^2)^2 = (x^2 + y^2)^2$

§ Page 12

1. The line $lx + my + n = 0$ intersects the curve $ax^2 + 2hxy + by^2 = 1$ at the points P and Q . The circle on PQ as diameter passes through the origin, then:
- (A) $n^2(a + b) = l^2 + m^2$ (B) $l^2(a + b) = n^2 + m^2$
 (C) $m^2(a + b) = l^2 + n^2$ (D) None of these

PART 2: ADVANCED PROBLEM SET

Pages 13 to 24 • Tangency, Multi-Conics, Orthogonal Families, Inscribed Packings & Extremal Geometry

§ Page 13

1. A circle C is tangent to the x and y -axis in the first quadrant at the points P and Q respectively. BC and AD are parallel tangents to the circle with slope -1 . If the points A and B are on the y -axis while C and D are on the x -axis and the area of figure $ABCD$ is $900\sqrt{2}$ sq. units, and the radius of the circle is k , then $\frac{k}{3}$ is equal to:
2. If p_1, p_2, p_3 are the altitudes of a triangle which circumscribes a circle of diameter $\frac{4}{3}$ units, then the least value of $p_1 + p_2 + p_3$ is equal to:
3. The centres of two circles C_1 and C_2 each of unit radius are at a distance of 6 units from each other. Let P be the midpoint of the line segment joining the centres of C_1 and C_2 , and C be a circle touching C_1 and C_2 externally. If a common tangent to C_1 and C passes through P and is also a common tangent to C_2 and C_1 , then the radius of circle C is:
4. Match the following List-I with List-II:

List-I	List-II
(A) Diameter of the circle touching the line $(x - 1)\cos\theta + (y - 1)\sin\theta = 1$ for all values of θ	(p) 2
(B) Radius of smallest circle which touches the circle $x^2 + y^2 = 1$ and $x^2 + y^2 - 6x - 8y + 21 = 0$	(q) 1
(C) Number of values of m for which the line $(y - 2) = m(x - 1)$ cuts the circle $x^2 + y^2 = 5$ at two real points	(r) 4
(D) Number of circles touching both the axes and the line $x + y = 4$	(s) Infinite
	(t) 3

5. A circle S passes through the point $(0, 1)$ and is orthogonal to the circles $(x - 1)^2 + y^2 = 16$ and $x^2 + y^2 = 1$. Then:
- (A) Radius of S is 8 (B) Radius of S is 7
 (C) Centre of S is $(-7, 1)$ (D) Centre of S is $(-8, 1)$
6. The common tangents to the circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ touch the circle at the points P, Q and the parabola at the points R, S . Then the area of the quadrilateral $PQRS$ is:
- (A) 3 (B) 6 (C) 9 (D) 15
7. If the normals of the parabola $y^2 = 4x$ drawn at the end points of its latus rectum are tangents to the circle

$(x - 3)^2 + (y + 2)^2 = r^2$, then the value of r^2 is:

§ Page 14

1. The circle $C_1 : x^2 + y^2 = 3$, with centre at O , intersects the parabola $x^2 = 2y$ at the point P in the first quadrant. Let the tangent to the circle C_1 at P touch other two circles C_2 and C_3 at R_2 and R_3 , respectively. Suppose C_2 and C_3 have equal radii $2\sqrt{3}$ and centres Q_2 and Q_3 , respectively. If Q_2 and Q_3 lie on the y -axis, then:

- (A) $Q_2Q_3 = 12$ (B) $R_2R_3 = 4\sqrt{6}$
 (C) Area of $\Delta OR_2R_3 = 6\sqrt{2}$ (D) Area of $\Delta PQ_2Q_3 = 4\sqrt{2}$

2. Let RS be the diameter of the circle $x^2 + y^2 = 1$ where S is the point $(1, 0)$. Let P be a variable point (other than R and S) on the circle and tangents to the circle at S and P meet at the point Q . The normal to the circle at P intersects a line drawn through Q parallel to RS at point E . Then the locus of E passes through the point(s):

- (A) $\left(\frac{1}{3}, \frac{1}{\sqrt{3}}\right)$ (B) $\left(\frac{1}{4}, \frac{1}{2}\right)$
 (C) $\left(\frac{1}{3}, -\frac{1}{\sqrt{3}}\right)$ (D) $\left(\frac{1}{4}, -\frac{1}{2}\right)$

3. Let P be the point on the parabola $y^2 = 4x$ which is at the shortest distance from the center S of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$. Let Q be the point on the circle dividing the line segment SP internally. Then:

- (A) $SP = 2\sqrt{5}$ (B) $SQ : QP = (\sqrt{5} + 1) : 2$
 (C) The x -intercept of normal at P is 6 (D) Slope of tangent to circle at Q is $\frac{1}{2}$

4. For how many values of p , the circle $x^2 + y^2 + 2x + 4y - p = 0$ and the coordinate axes have exactly three common points?

5. Columns 1, 2, and 3 contain conics, equations of tangents to the conics, and points of contact, respectively:

Column 1 (Conic)	Column 2 (Tangent)	Column 3 (Point of Contact)
(I) $x^2 + y^2 = a^2$	(i) $my = m^2x + a$	(P) $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$
(II) $x^2 + a^2y^2 = a^2$	(ii) $y = mx + a\sqrt{m^2 + 1}$	(Q) $\left(\frac{-ma}{\sqrt{m^2 + 1}}, \frac{a}{\sqrt{m^2 + 1}}\right)$
(III) $y^2 = 4ax$	(iii) $y = mx + \sqrt{a^2m^2 - 1}$	(R) $\left(\frac{-a^2m}{\sqrt{a^2m^2 + 1}}, \frac{1}{\sqrt{a^2m^2 + 1}}\right)$
(IV) $x^2 - a^2y^2 = a^2$	(iv) $y = mx + \sqrt{a^2m^2 + 1}$	(S) $\left(\frac{-a^2m}{\sqrt{a^2m^2 - 1}}, \frac{-1}{\sqrt{a^2m^2 - 1}}\right)$

§ Page 15

1. The tangent to a suitable conic (Column 1) at $\left(\sqrt{3}, \frac{1}{2}\right)$ is found to be $\sqrt{3}x + 2y = 4$, then which of the following options is the only CORRECT combination?

- [A] (II) (iii) (R) [B] (IV) (iv) (S) [C] (IV) (iii) (S) [D] (II) (iv) (R)

2. If a tangent to a suitable conic (Column 1) is found to be $y = x + 8$ and its point of contact is $(8, 16)$, then which of the following options is the only CORRECT combination?

- [A] (III) (i) (P) [B] (III) (ii) (Q) [C] (II) (iv) (R) [D] (I) (ii) (Q)

3. For $a = \sqrt{2}$, if a tangent is drawn to a suitable conic (Column 1) at the point of contact $(-1, 1)$, then which of the following options is the only CORRECT combination for obtaining its equation?

- [A] (II) (ii) (Q) [B] (III) (i) (P) [C] (I) (i) (P) [D] (I) (ii) (Q)

4. Let E_1E_2 and F_1F_2 be the chords of S passing through the point $P_0(1, 1)$ and parallel to the x -axis and the y -axis, respectively. Let G_1G_2 be the chord of S passing through P_0 and having slope -1 . Let the tangents to S at E_1 and E_2 meet at E_3 , the tangents to S at F_1 and F_2 meet at F_3 , and the tangents to S at G_1 and G_2 meet at G_3 . Then, the points E_3, F_3 , and G_3 lie on the curve:

- (a) $x + y = 4$ (b) $(x - 4)^2 + (y - 4)^2 = 16$
 (c) $(x - 4)(y - 4) = 4$ (d) $xy = 4$

5. Let P be a point on the circle S with both coordinates being positive. Let the tangent to S at P intersect the coordinate axes at the points M and N . Then, the midpoint of the line segment MN must lie on the curve:

- (a) $(x + y)^2 = 3xy$ (b) $x^{2/3} + y^{2/3} = 2^{4/3}$
 (c) $x^2 + y^2 = 2xy$ (d) $x^2 + y^2 = x^2y^2$

6. Let T be the line passing through the points $P(-2, 7)$ and $Q(2, -5)$. Let $\mathcal{F}_1$ be the set of all pairs of circles (S_1, S_2) such that T is tangent to S_1 at P and tangent to S_2 at Q , and also such that S_1 and S_2 touch each other at a point, say, M . Let E_1 be the set representing the locus of M as the pair (S_1, S_2) varies in $\mathcal{F}_1$. Let the set of all straight line segments joining a pair of distinct points of E_1 and passing through the point $R(1, 1)$ be $\mathcal{F}_2$. Let E_2 be the set of the midpoints of the line segments in the set $\mathcal{F}_2$. Then, which of the following statement(s) is (are) TRUE?

- (A) The point $(-2, 7)$ lies in E_1 (B) The point $(\frac{4}{5}, \frac{7}{5})$ does NOT lie in E_2
 (C) The point $(\frac{1}{2}, 1)$ lies in E_2 (D) The point $(0, \frac{3}{2})$ does NOT lie in E_1

§ Page 16

1. Consider two straight lines, each of which is tangent to both the circle $x^2 + y^2 = \frac{1}{2}$ and the parabola $y^2 = 4x$. Let these lines intersect at the point Q . Consider the ellipse whose center is at the origin $O(0, 0)$ and whose semi-major axis is OQ . If the length of the minor axis of this ellipse is $\sqrt{2}$, then which of the following statement(s) is (are) TRUE?

- (A) For the ellipse, the eccentricity is $\frac{1}{\sqrt{2}}$ and the length of the latus rectum is 1.
 (B) For the ellipse, the eccentricity is $\frac{1}{2}$ and the length of the latus rectum is $\frac{1}{2}$.
 (C) The area of the region bounded by the ellipse between the lines $x = \frac{1}{\sqrt{2}}$ and $x = 1$ is $\frac{1}{4\sqrt{2}}(\pi - 2)$.
 (D) The area of the region bounded by the ellipse between the lines $x = \frac{1}{\sqrt{2}}$ and $x = 1$ is $\frac{1}{16}(\pi - 2)$.

2. In a non-right-angled triangle ΔPQR , let p, q, r denote the lengths of the sides opposite to the angles at P, Q, R respectively. The median from R meets the side PQ at S , the perpendicular from P meets the side QR at E , and RS and PE intersect at O . If $p = \sqrt{3}, q = 1$, and the radius of the circumcircle of the ΔPQR equals 1, then which of the following options is/are correct?

3. Let the point B be the reflection of the point $A(2, 3)$ with respect to line $8x - 6y - 23 = 0$. Let Γ_A and Γ_B be circles of radii 2 and 1 with centres A and B respectively. Let T be a common tangent to the circles Γ_A and Γ_B such that both the circles are on the same side of T . If C is the point of intersection of T and the line passing through A and B , then the length of the line segment AC is:

§ Page 17

Comprehension Passage / Instructions:

Let the circles $C_1 : x^2 + y^2 = 9$ and $C_2 : (x - 3)^2 + (y - 4)^2 = 16$ intersect at the points X and Y . Suppose that another circle $C_3 : (x - h)^2 + (y - k)^2 = r^2$ satisfies the following conditions:

- (i) Centre of C_3 is collinear with the centres of C_1 and C_2
 (ii) C_1 and C_2 both lie inside C_3
 (iii) C_3 touches C_1 at M and C_2 at N

Let the line through X and Y intersect C_3 at Z and W , and let a common tangent of C_1 and C_3 be a tangent to the parabola $x^2 = 8\alpha y$.

List-I	List-II
(I) $2h + k$	(P) 6
(II) Length of ZW	(Q) $\sqrt{6}$
(III) $\frac{\text{Area of triangle } MZN}{\text{Area of triangle } ZMW}$	(R) $\frac{5}{4}$
(IV) α	(S) $\frac{21}{5}$
	(T) $2\sqrt{6}$
	(U) $\frac{10}{3}$

1. Which of the following is the only CORRECT combination?

- (A) (II), (T) (B) (II), (Q) (C) (I), (U) (D) (I), (S)

2. Which of the following is the only INCORRECT combination?

- (A) (IV), (S) (B) (III), (R) (C) (IV), (U) (D) (I), (P)

3. Let O be the centre of the circle $x^2 + y^2 = r^2$, where $r > \frac{\sqrt{5}}{2}$. Suppose PQ is a chord of this circle and the equation of the line passing through P and Q is $2x + 4y = 5$. If the centre of the circumcircle of the triangle OPQ lies on the line $x + 2y = 4$, then the value of r is:

4. Consider a triangle Δ whose two sides lie on the x -axis and the line $x + y + 1 = 0$. If the orthocentre of Δ is $(1, 1)$, then the equation of the circle passing through the vertices of the triangle Δ is:

(a) $x^2 + y^2 - 3x + y = 0$

(b) $x^2 + y^2 + x + 3y = 0$

(c) $x^2 + y^2 + 2y - 1 = 0$

(d) $x^2 + y^2 + x + y = 0$

§ Page 18

1. Consider the region $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x \geq 0 \text{ and } y^2 \leq 4 - x\}$. Let $\mathcal{F}$ be the family of all circles that are contained in R and have centers on the x -axis. Let C be the circle that has largest radius among the circles in $\mathcal{F}$. Let (α, β) be a point where the circle C meets the curve $y^2 = 4 - x$.

(i) The radius of the circle C is:

(ii) The value of α is:

2. Let $M = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x^2 + y^2 \leq r^2\}$, where $r > 0$. Consider the geometric progression $a_n = \frac{1}{2^{n-1}}$, $n = 1, 2, 3, \dots$. Let $S_0 = 0$ and for $n \geq 1$, let S_n denote the sum of the first n terms of this progression. For $n \geq 1$, let C_n denote the circle with center $(S_{n-1}, 0)$ and radius a_n , and D_n denote the circle with center (S_{n-1}, S_{n-1}) and radius a_n .

(i) Consider M with $r = \frac{1025}{513}$. Let k be the number of all those circles C_n that are inside M . Let l be the maximum possible number of circles among these k circles such that no two circles intersect. Then:

(A) $k + 2l = 22$

(B) $2k + l = 26$

(C) $2k + 3l = 34$

(D) $3k + 2l = 40$

(ii) Consider M with $r = \frac{(2^{199} - 1)\sqrt{2}}{2^{198}}$. The number of all those circles D_n that are inside M is:

(A) 198

(B) 199

(C) 200

(D) 201

3. Let ABC be the triangle with $AB = 1$, $AC = 3$, and $\angle BAC = \frac{\pi}{2}$. If a circle of radius $r > 0$ touches the sides AB , AC , and also touches internally the circumcircle of the triangle ABC , then the value of r is:

4. Let G be a circle of radius $R > 0$. Let $G_1, G_2, \dots, G_n$ be n circles of equal radius $r > 0$. Suppose each of the n circles $G_1, G_2, \dots, G_n$ touches the circle G externally. Also, for $i = 1, 2, \dots, n - 1$, the circle G_i touches G_{i+1} externally, and G_n touches G_1 externally. Then, which of the following statements is/are TRUE?

(A) If $n = 4$, then $(\sqrt{2} - 1)r < R$

(B) If $n = 5$, then $r < R$

(C) If $n = 8$, then $(\sqrt{2} - 1)r < R$

(D) If $n = 12$, then $\sqrt{2}(\sqrt{3} + 1)r > R$

§ Page 19

1. A circle of radius r passes through the origin O and cuts the axes at A and B . If the locus of the foot of the perpendicular from O to AB is $(x^2 + y^2)^2 \left(\frac{1}{x^2} + \frac{1}{y^2} \right) = \lambda r^2$, the value of λ is:

(A) 1

(B) 2

(C) 4

(D) 8

2. It is given that two circles, both of which pass through the point $(0, a)$ and $(0, -a)$ and touch the line $y = mx + c$, will intersect orthogonally if $c^2 = a^2(\lambda + m^2)$. The value of λ is:

(A) 1

(B) 2

(C) 3

(D) 4

(E) None of these

3. Prove that the locus of the centres of the circles cutting two given circles orthogonally is their radical axis.

4. Let S be the sum of this (decreasing) infinite sequence of shaded areas between an angle of 30° and successive tangent circles with radii $r_1 = 1, r_2, r_3, \dots$. The value of $[S]$ (the greatest integer part of S) is:

(A) 0

(B) 1

(C) 2

(D) 3

5. Three circles with radii r_1, r_2, r_3 (where $r_1 < r_2 < r_3$) touch each other externally. If they have a common tangent, the value of $\sqrt{\frac{r_1}{r_2}} + \sqrt{\frac{r_1}{r_3}}$ is:

(A) 1

(B) $\sqrt{2}$

(C) 2

(D) $2\sqrt{2}$

6. Let $L_1 : 2x + 3y + p - 3 = 0$ and $L_2 : 2x + 3y + p + 3 = 0$ be two lines and $p \in \mathbb{Z}$. Let $C : x^2 + y^2 + 6x + 10y + 30 = 0$. If it is given that at least one of the lines L_1, L_2 is a chord of C , the probability that both are chords of C is:
- (A) $\frac{2}{7}$ (B) $\frac{3}{7}$ (C) $\frac{4}{11}$ (D) $\frac{5}{11}$ (E) None of these
7. Consider a family of circles passing through the intersection point of the lines $\sqrt{3}(y - 1) = x - 1$ and $y - 1 = \sqrt{3}(x - 1)$ and having its centre on the acute angle bisector of the given lines.
- (a) Show that the common chords of each member of the family and the circle $x^2 + y^2 + 4x - 6y + 5 = 0$ are concurrent.
- (b) If the point of concurrency is (a, b) , the value of $a + b$ is:
- (A) 0 (B) 1 (C) 2 (D) 3

§ Page 20

1. Consider two fixed circles $x^2 + y^2 + 4|x| + 3 = 0$. A triangle ABC is initially located so that its vertices have positions: $A(0, 2), B(2, 2\sqrt{3} + 2), C(-2, 2\sqrt{3} + 2)$. It starts translating downwards perpendicular to the x -axis, and stops when its edges hit the circles (AB at P_1 and AC at P_2). The ratio in which P_1 divides AB is:
- (A) $\frac{3-\sqrt{3}}{\sqrt{3}}$ (B) $\frac{4-\sqrt{3}}{\sqrt{3}}$ (C) $\frac{1+\sqrt{3}}{\sqrt{3}}$ (D) $\frac{2+\sqrt{3}}{\sqrt{3}}$ (E) None of these
2. Consider the straight line $x + y = 0$ and the circle $x^2 + y^2 - 2x + \frac{1}{2} = 0$. At $t = 0$, the line starts rotating anti-clockwise about the origin with angular speed ω . The circle too starts moving (anti-clockwise) such that the locus of its center is a circle of radius 1 centered at the origin. The angular speed of the circle's center is 2ω . Find the equation of the line and the circle at the instant when they first meet after $t = 0$.
3. Tangents are drawn from the point (h, k) to the circle $x^2 + y^2 = a^2$; prove that the area of the triangle formed by them and the straight line joining their points of contact is:

$$\frac{a(h^2 + k^2 - a^2)^{3/2}}{h^2 + k^2}$$

4. Find the locus of the middle points of chords of the circle $x^2 + y^2 = a^2$ which pass through the fixed point (h, k) .
5. Find the locus of the middle points of chords of the circle $x^2 + y^2 = a^2$ which subtend a right angle at the point $(c, 0)$.
6. Whatever be the value of α , prove that the locus of the intersection of the straight lines $x \cos \alpha + y \sin \alpha = a$ and $x \sin \alpha - y \cos \alpha = b$ is a circle.
7. From a point P on a circle perpendiculars PM and PN are drawn to two radii of the circle which are not at right angles; find the locus of the middle point of MN .
8. Show that the locus of a point, which is such that the tangents from it to two given concentric circles are inversely as the radii, is a concentric circle, the square of whose radius is equal to the sum of the squares of the radii of the given circles.
9. Prove that a common tangent to two circles of a coaxial system subtends a right angle at either limiting point of the system.

§ Page 21

1. Prove that the circle of similitude of the two circles $x^2 + y^2 - 2kx + \delta = 0$ and $x^2 + y^2 - 2k'x + \delta = 0$ (i.e., the locus of the points at which the two circles subtend the same angle) is the coaxial circle:

$$x^2 + y^2 - 2 \frac{kk' + \delta}{k + k'} x + \delta = 0$$

2. Find the equation to the circle cutting orthogonally the three circles $x^2 + y^2 = a^2, (x - c)^2 + y^2 = a^2$, and $x^2 + (y - b)^2 = a^2$.
3. Three concentric circles of which the biggest is $x^2 + y^2 = 1$ have their radii in A.P. If the line $y = x + 1$ cuts all the circles in real and distinct points, the interval in which the common difference of the A.P. will lie is:
- (a) $(0, \frac{1}{4})$ (b) $(0, \frac{1}{2\sqrt{2}})$ (c) $(0, \frac{2-\sqrt{2}}{4})$ (d) None

4. A circle of radius unity is centred at origin. Two particles start moving at the same time from the point $(1, 0)$ and move around the circle in opposite directions. One of the particles moves counter clockwise with constant speed v and the other moves clockwise with constant speed $3v$. After leaving $(1, 0)$ the two particles meet first at a point P , and continue until they meet next at point Q . The coordinates of the point Q are:

- (a) $(1, 0)$ (b) $(0, 1)$ (c) $(0, -1)$ (d) $(-1, 0)$

5. If $a = \max\{(x+2)^2 + (y-3)^2\}$ and $b = \min\{(x+2)^2 + (y-3)^2\}$ where x, y satisfy $x^2 + y^2 + 8x - 10y - 40 = 0$, then:

- (a) $a + b = 18$ (b) $a + b = 178$
(c) $a - b = 4\sqrt{2}$ (d) $a - b = 72\sqrt{2}$

6. The locus of points of intersection of the tangents to $x^2 + y^2 = a^2$ at the extremities of a chord of circle $x^2 + y^2 = a^2$ which touches the circle $x^2 + y^2 - 2ax = 0$ is/are:

- (a) $y^2 = a(a - 2x)$ (b) $x^2 = a(a - 2y)$
(c) $x^2 + y^2 = (x - a)^2$ (d) $x^2 + y^2 = (y - a)^2$

§ Page 22

Comprehension Passage / Instructions:

Paragraph for Questions 52 to 54:

Let each of the circles:

$$S_1 \equiv x^2 + y^2 + 4y - 1 = 0, \quad S_2 \equiv x^2 + y^2 + 6x + y + 8 = 0, \quad S_3 \equiv x^2 + y^2 - 4x - 4y - 37 = 0$$

touch the other two. Let P_1, P_2, P_3 be the points of contact of S_1 and S_2 , S_2 and S_3 , S_3 and S_1 respectively, and C_1, C_2, C_3 be the centres of S_1, S_2, S_3 respectively.

1. The coordinates of P_1 are:

- (a) $(2, -1)$ (b) $(2, 1)$ (c) $(-2, 1)$ (d) $(-2, -1)$

2. The ratio $\frac{\text{area}(\Delta P_1 P_2 P_3)}{\text{area}(\Delta C_1 C_2 C_3)}$ is equal to:

- (a) $3 : 2$ (b) $2 : 5$ (c) $5 : 3$ (d) $2 : 3$

3. P_2 and P_3 are images of each other with respect to the line:

- (a) $y = x + 1$ (b) $y = -x$ (c) $y = x$ (d) $y = -x + 2$

Comprehension Passage / Instructions:

Paragraph for Questions 55 to 57:

Let $A(3, 7)$ and $B(6, 5)$ be two points. $C : x^2 + y^2 - 4x - 6y - 3 = 0$ is a circle.

4. The chords in which the circle C cuts the members of the family S of circles passing through A and B are concurrent at:

- (a) $(2, 3)$ (b) $(2, \frac{23}{3})$ (c) $(3, \frac{23}{2})$ (d) $(3, 2)$

5. Equation of the member of the family of circles S that bisects the circumference of C is:

- (a) $x^2 + y^2 - 5x - 1 = 0$ (b) $x^2 + y^2 - 5x + 6y - 1 = 0$
(c) $x^2 + y^2 - 5x - 6y - 1 = 0$ (d) $x^2 + y^2 + 5x - 6y - 1 = 0$

6. If O is the origin and P is the center of C , then the absolute value of the difference of the squares of the lengths of the tangents from A and B to the circle C is equal to:

- (a) $(AB)^2$ (b) $(OP)^2$
(c) $|(AP)^2 - (BP)^2|$ (d) $(AP)^2 + (BP)^2$

§ Page 23

1. Let L_1, L_2 , and L_3 be the lengths of tangents drawn from a point P to the circles $x^2 + y^2 = 4$, $x^2 + y^2 - 4x = 0$, and $x^2 + y^2 - 4y = 0$ respectively. If $L_1^4 = L_2^2 L_3^2 + 16$, then the loci of P are the curves C_1 (a straight line) and C_2 (a circle).

- (i) Circumcentre of the triangle formed by C_1 and two other lines which are at angle of 45° with C_1 and tangent to C_2 is:
 (a) $(1, 1)$ (b) $(0, 0)$ (c) $(-1, -1)$ (d) $(2, 2)$
- (ii) If S_1, S_2 and S_3 are three circles congruent to C_2 and touch both C_1 and C_2 , then the area of triangle formed by joining centres of the circles S_1, S_2 and S_3 is (in square units):
 (a) 2 (b) 4 (c) 8 (d) 16

2. Match the following Column-I with Column-II:

Column-I	Column-II
(A) A ray of light coming from the point $(1, 2)$ is reflected at a point A on the x -axis then passes through the point $(5, 3)$. The coordinates of the point A are:	(P) $\left(\frac{13}{5}, 0\right)$
(B) The equation of three sides of triangle ABC are $x + y = 3, x - y = 5$ and $3x + y = 4$. Considering the sides as diameter, three circles S_1, S_2, S_3 are drawn whose radical centre is at:	(Q) $(4, -1)$
(C) If the straight line $x - 2y + 1 = 0$ intersects the circle $x^2 + y^2 = 25$ at the points P and Q , then the coordinate of the point of intersection of tangents drawn at P and Q to the circle is:	(R) $(-25, 50)$
(D) The equation of three sides of a triangle are $4x + 3y + 9 = 0, 2x + 3 = 0$ and $3y - 4 = 0$. The circumcentre of the triangle is:	(S) $(-1, 2)$

§ Page 24

1. Tangents are drawn to circle $x^2 + y^2 = 1$ at its intersection points (distinct) with the circle $x^2 + y^2 + (2\lambda - 3)x + (2\lambda + 2)y + 2 = 0$. The locus of intersection of tangents is a straight line, then the slope of that straight line is:
2. Let $S = \{(x, y) \mid x, y \in \mathbb{R}, x^2 + y^2 - 10x + 16 = 0\}$. The largest value of $\frac{y}{x}$ can be put in the form $\frac{m}{n}$ where m, n are relatively prime natural numbers, then $m^2 + n^2 =$
3. Three circles with radii 3 cm, 4 cm and 5 cm touch each other externally. If A is the point of intersection of tangents to these circles at their points of contact, then the distance of A from the points of contact is:
 (a) $\sqrt{3}$ (b) 2 (c) $\sqrt{5}$ (d) $\sqrt{6}$
4. A line meets the coordinate axes in A and B . A circle is circumscribed about the triangle OAB . If m and n are the distances of the tangent to the circle at the origin from the points A and B respectively, the diameter of the circle is:
 (a) $m(m + n)$ (b) $m + n$ (c) $n(m + n)$ (d) $\frac{1}{2}(m + n)$
5. If O is the origin and OP, OQ are distinct tangents to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, the circumcentre of the triangle OPQ is:
 (a) $(-g, -f)$ (b) (g, f) (c) $(-f, -g)$ (d) None of these

PART 3: ADVANCED PROBLEM SET

Pages 25 to 36 • Tangents, Common Chords, Orthogonality & Loci

§ Page 25

1. Tangents drawn from the point $P(1, 8)$ to the circle $x^2 + y^2 - 6x - 4y - 11 = 0$ touch the circle at the points A and B . The equation of the circumcircle of the triangle PAB is:
 (a) $x^2 + y^2 + 4x - 6y + 19 = 0$ (b) $x^2 + y^2 - 4x - 10y + 19 = 0$
 (c) $x^2 + y^2 - 2x + 6y - 29 = 0$ (d) $x^2 + y^2 - 6x - 4y + 19 = 0$

2. C_1 and C_2 are circles of unit radius with centres at $(0,0)$ and $(1,0)$ respectively. C_3 is a circle of unit radius, passes through the centres of the circles C_1 and C_2 , and has its centre above the x -axis. Equation of the common tangent to C_1 and C_3 which does not pass through C_2 is:

- (a) $x - \sqrt{3}y + 2 = 0$ (b) $\sqrt{3}x - y + 2 = 0$
 (c) $\sqrt{3}x - y - 2 = 0$ (d) $x + \sqrt{3}y + 2 = 0$

§ Page 26

1. A circle C_1 of radius b touches the circle $x^2 + y^2 = a^2$ externally and has its centre on the positive x -axis; another circle C_2 of radius c touches the circle C_1 externally and has its centre on the positive x -axis. Given $a < b < c$, then the three circles have a common tangent if a, b, c are in:

- (a) A.P. (b) G.P. (c) H.P. (d) None of these

2. A and B are two points on the x -axis and y -axis respectively. Two circles are drawn passing through the origin and having centres at A and B .

- (a) Equation of the common chord is $ax - by = 0$ (b) Midpoint of common chord is $\left(\frac{ab^2}{a^2+b^2}, \frac{a^2b}{a^2+b^2}\right)$
 (c) AB bisects the common chord (d) AB is perpendicular to the common chord

3. Equation of the straight line which meets the circle $x^2 + y^2 = a^2$ at points which are at a distance d from a point $A(\alpha, \beta)$ on the circle is:

- (a) $2\alpha x + 2\beta y = 2a^2 - d^2$ (b) $2\alpha x - 2\beta y = 2a^2 + d^2$
 (c) $2\alpha x + 2\beta y = 2a^2 + d^2$ (d) $2\alpha x + 2\beta y + 2a^2 = d^2$

§ Page 27

Comprehension Passage / Instructions:

$P(a, 5a)$ and $Q(4a, a)$ are two points. Two circles are drawn through these points touching the axis of y .

1. The centres of these circles are at:

- (a) (a, a) (b) $\left(\frac{205a}{18}, \frac{29a}{3}\right)$
 (c) $\left(\frac{5a}{2}, 3a\right)$ (d) $\left(3a, \frac{29a}{3}\right)$

2. Angle of intersection of these circles is:

- (a) $\tan^{-1}(4/3)$ (b) $\tan^{-1}(40/9)$ (c) $\tan^{-1}(84/187)$ (d) None of these

3. If C_1, C_2 are the centres of these circles, then the area of $\triangle OC_1C_2$, where O is the origin, is:

- (a) a^2 (b) $5a^2$ (c) $10a^2$ (d) $20a^2$

§ Page 28

Comprehension Passage / Instructions:

A circle C of radius 1 is inscribed in an equilateral triangle PQR . The points of contact of C with the sides PQ, QR, RP are D, E, F respectively. The line PQ is given by the equation $\sqrt{3}x + y - 6 = 0$ and the point D is $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$. Further, it is given that the origin and the centre of C are on the same side of PQ .

1. The equation of circle C is:

- (a) $(x - 2\sqrt{3})^2 + (y - 1)^2 = 1$ (b) $(x - 2\sqrt{3})^2 + \left(y + \frac{1}{2}\right)^2 = 1$
 (c) $(x - \sqrt{3})^2 + (y + 1)^2 = 1$ (d) $(x - \sqrt{3})^2 + (y - 1)^2 = 1$

2. Points E and F are given by:

- (a) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), (\sqrt{3}, 0)$ (b) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), (\sqrt{3}, 0)$
 (c) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ (d) $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

3. Equations of the sides QR, RP are:

- (a) $y = \frac{2}{\sqrt{3}}x + 1, y = -\frac{2}{\sqrt{3}}x - 1$ (b) $y = \frac{1}{\sqrt{3}}x, y = 0$
 (c) $y = \frac{\sqrt{3}}{2}x + 1, y = -\frac{\sqrt{3}}{2}x - 1$ (d) $y = \sqrt{3}x, y = 0$

4. A triangle has two of its sides along the axes, its third side touches the circle $x^2 + y^2 - 2ax - 2ay + a^2 = 0$. If the locus of the circumcentre of the triangle passes through the point $(38, -37)$, then $a^2 - 2a$ is equal to:

§ Page 29

1. The centres of two circles C_1 and C_2 , each of unit radius, are at a distance of 6 units from each other. Let P be the midpoint of the line segment joining the centres of C_1 and C_2 , and C be a circle touching C_1 and C_2 externally. If a common tangent to C_1 and C passing through P is also a common tangent to C_2 and C_1 , then the radius of the circle C is:

2. Match the following geometric properties of circles:

(a) The radical axis of two circles	(q) is perpendicular to the line joining the centres
(b) The common tangent to two intersecting circles of equal radii	(r) is parallel to the line joining the centres
(c) The common chord of two intersecting circles	(s) is bisected by the line joining the centres
(d) The line joining the centres of two circles intersecting orthogonally	(p) subtends a right angle at a point of intersection

3. **Statement-1:** From a point P on the circle with centre O , the chord $PA = 8$ cm is drawn. The radius of the circle is 24 cm. Let PB be drawn parallel to OA . Suppose BO extended meets PA extended at M . The length of MA is 9 cm.

Statement-2: OA is a radius of a circle with centre at O . R' is a point on OA through which a chord CD perpendicular to OA is drawn. Let a chord through A meet the chord CD at M and the circle at B . Also OS is perpendicular from O on chord AB . The radius of the circle is 18 cm. R is the midpoint of AO and $AM/MB = 1/2$. The length of OS is 9 cm.

§ Page 30

1. Lines $5x + 12y - 10 = 0$ and $5x - 12y - 40 = 0$ touch a circle C_1 of diameter 6. If the centre of C_1 lies in the first quadrant, find the equation of the circle C_2 which is concentric with C_1 and cuts intercepts of length 8 on these lines.

2. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = k^2$ orthogonally, then the equation of the locus of its centre is:

- | | |
|---|---|
| (a) $2ax + 2by - (a^2 + b^2 + k^2) = 0$ | (b) $2ax + 2by - (a^2 - b^2 + k^2) = 0$ |
| (c) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - k^2) = 0$ | (d) $x^2 + y^2 - 2ax - 3by + (a^2 - b^2 + k^2) = 0$ |

3. If two circles $(x - 1)^2 + (y - 3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then:

- | | | | |
|-----------------|-------------|-------------|-------------|
| (a) $2 < r < 8$ | (b) $r < 2$ | (c) $r = 2$ | (d) $r > 2$ |
|-----------------|-------------|-------------|-------------|

4. Find the intervals of values of a for which the line $y + x = 0$ bisects two chords drawn from a point $\left(\frac{1+\sqrt{2}a}{2}, \frac{1-\sqrt{2}a}{2}\right)$ to the circle $2x^2 + 2y^2 - (1 + \sqrt{2}a)x - (1 - \sqrt{2}a)y = 0$.

§ Page 31

1. Let $ABCD$ be a quadrilateral with area K , with side AB parallel to side CD and $AB = 2CD$. Let AD be perpendicular to AB and CD . If a circle is drawn inside the quadrilateral $ABCD$ touching all the sides, then its radius is:

- | | | | |
|-------|-------|-----------|-------|
| (a) 3 | (b) 2 | (c) $3/2$ | (d) 1 |
|-------|-------|-----------|-------|

2. Tangents are drawn from the point $(17, 7)$ to the circle $x^2 + y^2 = 169$.

- **Statement-1:** The tangents are mutually perpendicular.
- **Statement-2:** The locus of the point from which mutually perpendicular tangents can be drawn to the given circle is $x^2 + y^2 = 338$.

§ Page 32

1. The number of lattice points that are interior to the circle $x^2 + y^2 = 25$ is:

- | | | | |
|--------|--------|--------|--------|
| (a) 81 | (b) 69 | (c) 12 | (d) 70 |
|--------|--------|--------|--------|

2. The set of values of c so that $y = |x| + c$ and $x^2 + y^2 - 8|x| - 9 = 0$ have no solution is:

- (a) $(-\infty, -3) \cup (3, \infty)$ (b) $(-3, 3)$
 (c) $(-\infty, -5\sqrt{2}) \cup (5\sqrt{2}, \infty)$ (d) $(5\sqrt{2} - 4, \infty)$

§ Page 33

1. A circle with centre at the origin and radius equal to a meets the axis of X at A and B . $P(\alpha)$ and $Q(\beta)$ are two points on this circle so that $\alpha - \beta = 2\gamma$ where γ is a constant. The locus of the point of intersection of AP and BQ is:

- (a) $x^2 - y^2 - 2ay \tan \gamma = a^2$ (b) $x^2 + y^2 - 2ay \tan \gamma = a^2$
 (c) $x^2 + y^2 + 2ay \tan \gamma = a^2$ (d) $x^2 - y^2 + 2ay \tan \gamma = a^2$

2. A variable straight line through $A(-1, 1)$ is drawn to cut the circle $x^2 + y^2 = 1$ at the points B and C . A point P is chosen on the line ABC satisfying the conditions given in Column-I. Let d be the minimum distance of the origin from the locus of P given in Column-II:

Column-I	Column-II
(A) AB, AP, AC are in A.P.	(p) 0
(B) AB, AP, AC are in G.P.	(q) $1/\sqrt{2}$
(C) AB, AP, AC are in H.P.	(r) $\sqrt{2}$
(D) $AB, \frac{AP}{2}, AC$ are in A.P.	(s) $\sqrt{2} - 1$

3. The base of a triangle $AB = 6$, the third vertex C moves such that $\frac{\sin A}{\sin B} = 2$. If the locus of C is a circle, then its radius is:

4. A ray of light incident at the point $(-2, -1)$ gets reflected from the tangent at $(0, -1)$ to the circle $x^2 + y^2 = 1$. The reflected ray touches the circle; the equation of the line along which the incident ray moved is:

- (a) $4x - 3y + 11 = 0$ (b) $4x + 3y + 11 = 0$
 (c) $3x + 4y + 11 = 0$ (d) None of these

5. If C_1 and C_2 are two concentric circles with radii r_1 and r_2 ($r_1 < r_2$). If the tangents drawn from any point of C_2 to C_1 meet again C_2 at the ends of its diameter, then:

- (a) $r_2 = 2r_1$ (b) $r_2 = \sqrt{2}r_1$ (c) $r_2^2 < 2r_1^2$ (d) $r_2 = 3r_1$

§ Page 34

1. Two circles of radii r_1 and r_2 ($r_1 > r_2$) touch each other externally, then the radius of the circle which touches both of them externally and also their direct common tangent is:

- (a) $\frac{r_1 r_2}{(\sqrt{r_1} + \sqrt{r_2})^2}$ (b) $\sqrt{r_1 r_2}$
 (c) $\frac{r_1 r_2}{2}$ (d) $r_1 - r_2$

2. If C, C_1, C_2 be circles of radii 5, 3, 2 respectively. If C_1 and C_2 touch externally and they touch internally with C . The radius of circle C_3 which touches externally with C_1 and C_2 and internally with C is:

- (a) $\frac{30}{19}$ (b) 1 (c) 3 (d) Cannot be determined

3. A square $OABC$ is formed by line pairs $xy = 0$ and $xy + 1 = x + y$ where O is the origin. A circle with centre C_1 inside the square is drawn to touch the line pair $xy = 0$, and another circle with centre C_2 and radius twice that of C_1 is drawn to touch circle C_1 and the other line pair. The radius of the circle with centre C_1 is:

- (a) $\frac{\sqrt{2}}{\sqrt{3}(\sqrt{2}+1)}$ (b) $\frac{2\sqrt{2}}{3(\sqrt{2}+1)}$
 (c) $\frac{\sqrt{2}}{3(\sqrt{2}+1)}$ (d) $\frac{\sqrt{2}+1}{3\sqrt{2}}$

§ Page 35

1. If PQR is the triangle formed by the common tangents to the circles $x^2 + y^2 + 6x = 0$ and $x^2 + y^2 - 2x = 0$, then:

- (a) Centroid of ΔPQR is $(1, 0)$ (b) Incentre of ΔPQR is $(1, 0)$
 (c) Circumradius of ΔPQR is 2 units (d) Inradius of ΔPQR is 1 unit

2. Two circles of radii a and b touching each other externally are inscribed in the area bounded by $y = \sqrt{1 - x^2}$ and the x -axis. If $b = 1/2$, then $4a$ is equal to:
3. From a point P outside a circle with centre at O , tangent segments PA and PB are drawn. If $\frac{1}{(AB)^2} + \frac{1}{(PA)^2} = \frac{1}{16}$, then the length of the chord AB is:
4. Inside a semicircle of radius 1 unit, two circles of radii r_1 and r_2 are drawn, each touching the circumference and the diameter of the semicircle, and also touching each other externally. Then find $[\max(r_1 + r_2)]$ where $[\cdot]$ denotes the nearest integer.
5. If the line $x + y = n$, $n \in \mathbb{N}$ is a chord of the circle $x^2 + y^2 = 4$, then:
- (a) Sum of squares of chord lengths is 22
(b) Number of such chords is 4
(c) Number of such chords is 2
(d) None of these
6. Let $f(x, y) = 0$ be the equation of a circle, such that $f(0, y) = 0$ has equal roots and $f(x, 0) = 0$ has two distinct real roots. The angle between the tangents from P to $f(x, y) = 0$ is $\pi/3$. The locus of P is the circle $g(x, y) = x^2 + y^2 - 8x - 4y + c = 0$. Let Q be a point from which the tangents to $g(x, y) = 0$ are perpendicular, and AB is the chord of contact of Q w.r.t. $g(x, y) = 0$.
- (i) Equation of $f(x, y) = 0$ is:
(a) $x^2 + y^2 - 5x - 4y + 5 = 0$
(b) $x^2 + y^2 - 5x - 4y - 4 = 0$
(c) $x^2 + y^2 - 5x - 4y - 5 = 0$
(d) $x^2 + y^2 - 5x - 4y + 4 = 0$
- (ii) Area of ΔQAB is:
(A) $25/2$
(B) $25/4$
(C) $25/8$
(D) $25/12$
- (iii) The length of x -intercept made by $g(x, y) = 0$ is:
(a) $2\sqrt{80}$
(b) $\sqrt{84}$
(c) $\sqrt{74}$
(d) $\sqrt{78}$

§ Page 36

1. Consider the relation $4l^2 - 5m^2 + 6l + 1 = 0$ where $l, m \in \mathbb{R}$. Then the line $lx + my + 1 = 0$ touches a fixed circle.
- (i) Centre and radius of the circle are:
(a) $(2, 0), 3$
(b) $(-3, 0), \sqrt{3}$
(c) $(3, 0), \sqrt{5}$
(d) $(3, 0), \sqrt{3}$
- (ii) Number of tangents which can be drawn from the point $(2, -3)$ are:
(a) 0
(b) 1
(c) 2
(d) 1 or 2
- (iii) From a point $P(2, -3)$, two tangents are drawn to the circle, and A, B are points of contact, then $PA \cdot PB$ is:
(a) 26
(b) 5
(c) 15
(d) 25
2. Five circles C_1, C_2, C_3, C_4, C_5 with radii $r_1 < r_2 < r_3 < r_4 < r_5$ be such that C_i and C_{i+1} touch each other externally for all $i = 1, 2, 3, 4$. If all five circles touch each of two straight lines L_1 and L_2 , with $r_1 = 2$ and $r_5 = 32$, then r_3 is equal to:
(a) 8
(b) 17
(c) $64/17$
(d) Depends upon r_2 and r_4
3. Two circles with centres at A and B touch at T . BD is tangent at D , and TC is a common tangent. AT has length 3 and BT has length 2. The length of CD is:
(a) 4
(b) $3/2$
(c) $5/3$
(d) $7/4$
4. C_1 and C_2 are two circles of radii a, b ($a < b$) respectively, touching both coordinate axes, with centres in the first quadrant. The true statements are:
(a) If C_1, C_2 touch each other, $\frac{b}{a} = 3 + 2\sqrt{2}$
(b) If C_1, C_2 are orthogonal, $\frac{b}{a} = 2 + \sqrt{3}$
(c) If chord has max length, $\frac{b}{a} = 2$
(d) If C_2 passes through centre of C_1 , $\frac{b}{a} = 2 + \sqrt{2}$
5. Let x, y be real variables satisfying $x^2 + y^2 + 8x - 10y - 40 = 0$. Let $a = \max\left(\sqrt{(x+2)^2 + (y-3)^2}\right)$ and $b = \min\left(\sqrt{(x+2)^2 + (y-3)^2}\right)$, then:
(a) $a + b = 18$
(b) $a + b = 4\sqrt{2}$
(c) $a - b = 4\sqrt{2}$
(d) $ab = 73$

PART 4: ADVANCED PROBLEM SET

Pages 37 to 46 • Conjugate Points, Radical Pencils, Inscribed Circles & Extremal Loci

§ Page 37

1. In parallelogram $ABCD$ with $\angle A = 60^\circ$, the bisector of $\angle B$ cuts CD at E . A circle S_1 of radius r is inscribed in $\triangle ECB$. Another circle S_2 is inscribed in trapezoid $ABED$.

Column-I	Column-II
(A) The radius of S_2 is	(p) $2\sqrt{3}r$
(B) Distance between the centres of S_1 and S_2 is	(q) $\frac{\sqrt{3}}{2}r$
(C) Length of common tangent of S_1 and S_2 is	(r) $\sqrt{7}r$
(D) Length of CE is	(s) $\frac{3}{2}r$

2. $A(-2, 0)$ and $B(2, 0)$ are two fixed points and P satisfies $PA - PB = 2$. Let S be $x^2 + y^2 = r^2$:

Column-I	Column-II
(a) If $r = 2$, number of points P on $x^2 + y^2 = r^2$ is	(p) 2
(b) If $r = 1$, number of points P on $x^2 + y^2 = r^2$ is	(q) 4
(c) For $r = 2$, number of common tangents is	(r) 0
(d) For $r^2 = 1/2$, number of common tangents is	(s) 1

3. Circle C_1 touches the x -axis and $y = x \tan \theta$, $\theta \in (0, \pi/2)$ in the first quadrant, and circle C_2 touches $y = x \tan \theta$, the y -axis, and C_1 such that the ratio of radii of C_1 to C_2 is $2 : 1$. If $\tan \frac{\theta}{2} = \frac{\sqrt{a-b}}{c}$ where a, b, c are coprime natural numbers, then $\frac{a+b+c}{11}$ is:

4. If the circles $x^2 + y^2 + 2ax + cy + a = 0$ and $x^2 + y^2 - 3ax + dy - 1 = 0$ intersect at distinct points P and Q , then the line $5x + by - a = 0$ passes through P and Q for:

- (1) Exactly one value of a (2) No value of a
 (3) Infinitely many values of a (4) Exactly two values of a

5. If r and r' are radii of $S = 0$ and $S' = 0$ respectively, the circles $\frac{S}{r} \pm \frac{S'}{r'} = 0$ intersect at an angle of:

- (1) $\pi/3$ (2) $\pi/4$ (3) $\pi/2$ (4) $\pi/6$

§ Page 38

1. Points $A(2, 3)$ and $B(-7, -12)$ are conjugate points w.r.t. $x^2 + y^2 - 6x - 8y - 1 = 0$. The centre of the circle through A and B orthogonal to the given circle is:

- (1) $(-5, -9)$ (2) $(-9, -15)$ (3) $(-5/2, -9/2)$ (4) $(1/2, 3/2)$

2. If $x^2 + y^2 + 2gx + 2fy + c = 0$ bisects the circumference of $x^2 + y^2 + 2g'x + 2f'y + c' = 0$, the length of their common chord is:

- (1) $2\sqrt{g^2 + f^2 - c}$ (2) $2\sqrt{g'^2 + f'^2 - c'}$
 (3) $3\sqrt{g^2 + f^2 + c}$ (4) 0

3. $x^2 + y^2 = a^2$ and $(x - c)^2 + y^2 = b^2$ are intersecting circles. If a, b, c are sides BC, CA, AB of $\triangle ABC$, and p_1, p_2, p_3 are altitudes from A, B, C respectively, then the length of their common chord is:

- (1) $2p_1$ (2) $2p_2$ (3) $2p_3$ (4) p_1

4. Circles of radii 1, 2, 3 touch each other externally. The radius of the circle cutting all three orthogonally is:

- (1) 1 (2) $3/2$ (3) 2 (4) 3

5. Circle C_1 of radius 2 rolls externally around $C : x^2 + y^2 + 4x = 0$. The line joining the centres makes an angle of 60° with the x -axis:

- (i) Locus of the centre of C_1 is:
 (a) $x^2 + y^2 + 4x - 12 = 0$
 (b) $x^2 + y^2 - 4\sqrt{3}y + 8 = 0$
 (c) $x^2 + y^2 + 4x + 12 = 0$
 (d) $x^2 + y^2 - 4\sqrt{3}y - 12 = 0$
- (ii) Circle with the line of centres as diameter is:
 (a) $x^2 + y^2 - 2x + 2\sqrt{3}y = 0$
 (b) $x^2 + y^2 - 2x - 2\sqrt{3}y = 0$
 (c) $x^2 + y^2 + 2x - 2\sqrt{3}y = 0$
 (d) None of these
- (iii) The equation of the smallest circle containing both circles is:
 (a) $x^2 + y^2 - 4x + 12 = 0$
 (b) $x^2 + y^2 - 4x - 12 = 0$
 (c) $x^2 + y^2 - 2x + 2\sqrt{3}y + 12 = 0$
 (d) $x^2 + y^2 + 2x - 2\sqrt{3}y - 12 = 0$

§ Page 39

1. Tangents PA and PB are drawn to $(x - 4)^2 + (y - 5)^2 = 4$ from P on $y = \sin x$. Let $y = f(x)$ be the locus of the circumcentre of $\triangle PAB$:
 (i) Range of $y = f(x)$ is:
 (a) $[-2, 1]$
 (b) $[-1, 4]$
 (c) $[0, 2]$
 (d) $[2, 3]$
 (ii) Period of $y = f(x)$ is:
 (a) 2π
 (b) 3π
 (c) π
 (d) None
 (iii) Which of the following is true?
 (a) $f(x) = 4$ has real roots
 (b) $f(x) = 1$ has real roots
 (c) Range of $y = f^{-1}(x)$ is $[-\pi/4 + 2, \pi/4 + 2]$
 (d) None
2. Orthocentre and centroid of a triangle are $A(-3, 5)$ and $B(3, 3)$. If C is the circumcentre, the radius of the circle with diameter AC is:
 (a) $\sqrt{10}$
 (b) $2\sqrt{10}$
 (c) $3\sqrt{5/2}$
 (d) $\frac{3\sqrt{5}}{2}$
3. Centre of the circle through $(0, 1)$ touching $y = x^2$ at $(2, 4)$ is:
 (a) $(-16/5, 27/10)$
 (b) $(-16/7, 53/10)$
 (c) $(-16/5, 53/10)$
 (d) None of the above
4. The abscissae of A, B are roots of $x^2 + 2ax - b^2 = 0$ and ordinates are roots of $y^2 + 2py - q^2 = 0$. Find the equation and radius of the circle with diameter AB .
5. The line $2x - 3y = 1$ divides $x^2 + y^2 \leq 6$ into two parts. How many points of the set S lie inside the smaller part, where $S = \{(2, 3/4), (5/2, 3/4), (1/4, -1/4), (1/8, 1/4)\}$?

§ Page 40

1. $S : x^2 + y^2 = 4$. E_1E_2 and F_1F_2 are chords through $P_0(1, 1)$ parallel to the axes. G_1G_2 has slope -1 through P_0 . The tangents at the endpoints meet at E_3, F_3, G_3 . These points lie on:
 (a) $x + y = 4$
 (b) $(x - 4)^2 + (y - 4)^2 = 16$
 (c) $(x - 5)(y - 4) = 4$
 (d) $xy = 4$
2. The tangent at P on S (in the first quadrant) meets the coordinate axes at M and N . The midpoint of MN lies on:
 (a) $(x + y)^2 = 3xy$
 (b) $x^{2/3} + y^{2/3} = 2^{4/3}$
 (c) $x^2 + y^2 = 2xy$
 (d) $x^2 + y^2 = x^2y^2$
3. If $3x + 4y - \lambda = 0$ separates $x^2 + y^2 - 2x - 2y + 1 = 0$ and $x^2 + y^2 - 18x - 2y + 78 = 0$ to opposite sides, then λ lies in:
 (a) $[13, 23]$
 (b) $(2, 17)$
 (c) $[12, 21]$
 (d) $(23, 31)$

§ Page 41

1. T passes through $P(-2, 7)$ and $Q(2, -5)$. $\mathcal{F}_1$ is the family of pairs (S_1, S_2) tangent to T at P, Q and touching each other at M . E_1 is the locus of M . Chords of E_1 through $R(1, 1)$ have midpoints in E_2 . Which statements are true?
 (a) $(-2, 7) \in E_1$
 (b) $(4/5, 7/5) \notin E_2$
 (c) $(1/2, 1) \in E_2$
 (d) $(0, 3/2) \notin E_1$

2. $ABCD$ is a square of side 2. C_2 circumscribes $ABCD$ and C_1 is inscribed in it. $P \in C_1, Q \in C_2$.

(i) Value of ratio of distance sums:

$$\frac{PA^2 + PB^2 + PC^2 + PD^2}{QA^2 + QB^2 + QC^2 + QD^2} =$$

- (a) 0.75 (b) 1.25 (c) 1 (d) 0.5

(ii) Locus of centre of circle touching C_1 externally and line L through A on the same side is:

- (a) Ellipse (b) Hyperbola (c) Parabola (d) Straight line parts

(iii) Line M through A is parallel to BD . Distance of S from BD equals distance from A . If locus of S cuts M at T_2, T_3 and AC at T_1 , area of $\Delta T_1 T_2 T_3$ is:

- (a) $1/2$ sq unit (b) $2/3$ sq unit (c) 1 sq unit (d) 2 sq units

§ Page 42

1. Match the geometric configurations:

Column-I	Column-II
A. Two intersecting circles	p. Have a common tangent
B. Two mutually external circles	q. Have a common normal
C. Two circles, one strictly inside the other	r. Do not have a common tangent
D. Two branches of a hyperbola	s. Do not have a common normal

2. Tangent and normal to $x^2 + y^2 = 4$ at $(\sqrt{3}, 1)$ and the x -axis form a triangle. Its area is:

- (a) $1/3$ (b) $4/\sqrt{3}$ (c) $2/\sqrt{3}$ (d) $1/\sqrt{3}$

3. RS is a diameter of $x^2 + y^2 = 1$ with $S(1, 0)$. Tangents at S and P meet at Q . Normal at P intersects the line through Q parallel to RS at E . Locus of E passes through:

- (a) $(1/3, 1/\sqrt{3})$ (b) $(1/4, 1/2)$
 (c) $(1/3, -1/\sqrt{3})$ (d) $(1/4, -1/2)$

§ Page 43

1. Circle $C_1 : x^2 + y^2 = 3$ intersects $x^2 = 2y$ at P in the first quadrant. Tangent at P touches circles C_2, C_3 of radius $2\sqrt{3}$ with centres on the y -axis. Then:

- (a) $Q_2 Q_3 = 12$ (b) $R_2 R_3 = 4\sqrt{6}$
 (c) Area of $\Delta O R_2 R_3 = 6\sqrt{2}$ (d) Area of $\Delta P Q_2 Q_3 = 4\sqrt{2}$

2. Tangent PT to $x^2 + y^2 = 4$ at $(\sqrt{3}, 1)$. Line $L \perp PT$ touches $(x - 3)^2 + y^2 = 1$.

(i) A possible equation of L is:

- (a) $x - \sqrt{3}y = 1$ (b) $x + \sqrt{3}y = 1$
 (c) $x - \sqrt{3}y = -1$ (d) $x + \sqrt{3}y = 5$

(ii) A common tangent of the two circles is:

- (a) $x = 4$ (b) $y = 2$ (c) $x + \sqrt{3}y = 4$ (d) $x + 2\sqrt{2}y = 6$

§ Page 44

1. Two circles of equal radii intersect at $(0, 1)$ and $(0, -1)$. The tangent at $(0, 1)$ to one circle passes through the centre of the other. Distance between their centres is:

- (a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) 1 (d) 2

2. Three circles of radii $a < b < c$ touch each other externally and have the x -axis as a common tangent. Then:

- (a) a, b, c are in A.P. (b) $\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}$
 (c) $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in A.P. (d) $\frac{1}{\sqrt{b}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{c}}$

§ Page 45

1. Find the equation of the circle passing through $(2, 0)$ whose centre is $\lim_{c \rightarrow 1}$ of the intersection of $3x + 5y = 1$ and $(2 + c)x + 5c^2y = 1$.
2. Locus of the midpoint of the chord of contact from points on $4x - 5y = 20$ to $x^2 + y^2 = 9$ is:

(a) $20(x^2 + y^2) - 36x + 45y = 0$ (c) $36(x^2 + y^2) - 20x + 45y = 0$	(b) $20(x^2 + y^2) + 36x - 45y = 0$ (d) $36(x^2 + y^2) + 20x - 45y = 0$
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3. Equations of tangents from the origin to $x^2 + y^2 + 2rx + 2hy + h^2 = 0$ are:

(a) $x = 0$ (c) $(h^2 - r^2)x - 2rhy = 0$	(b) $y = 0$ (d) $(h^2 - r^2)x + 2rhy = 0$
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4. Two parallel chords of a circle of radius 2 are at distance $\sqrt{3} + 1$ apart. If they subtend π/k and $2\pi/k$ at the centre ($k > 0$), find $[k]$.
5. In $\triangle ABC$, $A(1, 10)$, circumcentre is $(-1/3, 2/3)$, and orthocentre is $(11/3, 4/3)$. The midpoint of the side opposite to A is:

(1) $(1, -11/3)$	(2) $(1, 5)$	(3) $(1, -3)$	(4) $(1, 6)$
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§ Page 46

1. In $\triangle ABC$, side BC is $x - y = 0$. Circumcentre is $(2, 3)$ and orthocentre is $(5, 8)$. Equation of the circumcircle is:

(A) $x^2 + y^2 - 4x + 6y - 27 = 0$ (C) $x^2 + y^2 + 4x + 6y - 27 = 0$	(B) $x^2 + y^2 - 4x - 6y - 27 = 0$ (D) $x^2 + y^2 + 4x - 6y - 27 = 0$
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End of Complete Problem Bank (Pages 1 to 46)

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