If f(x) be a polynomial of degree n such that $1 + f(x) = \frac{f(x-1)+f(x+1)}{2}$ $\forall x \in R$, then the possible value of n is

Let $f: R - \{2\} \to R$ be a function satisfying the following functional equation,
$2f(x) + 3f\left(\frac{2x+29}{x-2}\right) = 100x + 80$ $\forall x \in R - \{2\}$. Determine f(x)

A function $f: R \to R$ satisfies
$\sin 2x \sin 2y \{f(3x+3y) + f(3x-3y)\} = \cos 2x \cos 2y \{f(3x-3y) - f(3x+3y)\}$ if $f(0) = 1$ then

(A) $4f(x) + 9f''(x) = 0$
(B) $f(x) + f''(x) = 0$
(C) $4\{f(x)\}^2 + 9\{f'(x)\}^2 = 4$
(D) $9\{f(x)\}^2 + 4\{f'(x)\}^2 = 4$

	Column – I		Column – II
(A)	$g: R \to Q$ (Rational number); $f: R \to Q$ (Rational number); f and g are continuous function such that $\sqrt{3}f(x)+g(x)=3$ then $(1-f(x))^3+(g(x)-3)^3$ is	(p)	1
(B)	If f(x), g(x) and h(x) are continuous and positive function such that $f(x)+g(x)+h(x)=\sqrt{f(x)g(x)}+\sqrt{g(x)h(x)}+\sqrt{h(x)f(x)}$ then $f(x) + g(x) - 2h(x)$ is	(q)	0
(C)	y = f(x) satisfy the equation $y^3-2y^2(x+1)+4xy+(x^2-1)(y-2)=0$, than $y'(1) + y(1)$ would be equal to	(r)	2
(D)	If y = f(x) satisfy $(xf(x))^{99}+(xf(x))^{98}....(xf(x))+1=0$ then $(1 + f(1))$ is	(s)	3
		(t)	–1

24. Let $f: \mathbb{R} \to [0, \infty)$ be a continuous function such that

$$f(x+y) = f(x)f(y),$$

for all $x, y \in \mathbb{R}$. Suppose that f is differentiable at $x = 1$ and

$$\left.\frac{df(x)}{dx}\right|_{x=1} = 2.$$

Then, the value of $f(1)\log_e f(1)$ is

(A) e.
(B) 2.
(C) $\log_e 2$.
(D) 1.

2. Let $f : \mathbb{Z} \to \mathbb{Z}$ be a function satisfying $f(0) \neq 0 = f(1)$. Assume also that f satisfies equations **(A)** and **(B)** below.

$$f(xy) = f(x) + f(y) - f(x)f(y) \qquad \text{(A)}$$

$$f(x-y)f(x)f(y) = f(0)f(x)f(y) \qquad \text{(B)}$$

for all integers x, y.

(i) Determine explicitly the set $\{f(a) : a \in \mathbb{Z}\}$.

(ii) Assuming that there is a non-zero integer a such that $f(a) \neq 0$, prove that the set $\{b : f(b) \neq 0\}$ is infinite.

Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that $f(x+y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$, and $g : \mathbb{R} \to (0, \infty)$ be a function such that $g(x+y) = g(x)g(y)$ for all $x, y \in \mathbb{R}$. If $f\left(\frac{-3}{5}\right) = 12$ and $g\left(\frac{-1}{3}\right) = 2$, then the value of $\left(f\left(\frac{1}{4}\right) + g(-2) - 8\right)g(0)$ is ________.

Q.12 Let $\mathbb{R}$ denote the set of all real numbers. Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that $f(x) > 0$ for all $x \in \mathbb{R}$, and $f(x+y) = f(x)f(y)$ for all $x, y \in \mathbb{R}$.

Let the real numbers $a_1, a_2, \ldots, a_{50}$ be in an arithmetic progression. If $f(a_{31}) = 64 f(a_{25})$, and

$$\sum_{i=1}^{50} f(a_i) = 3(2^{25} + 1),$$

then the value of

$$\sum_{i=6}^{30} f(a_i)$$

is ____________________.

Answer: 96

Question 3

A real-valued function f satisfies the relation

$f(x)f(y) = f(2xy + 3) + 3f(x+y) - 3f(y) + 6y$, for all real numbers x and

Then the value of $f(8)$ is____

If f(1) = 1 and f(n + 1) = 2f(n) + 1 if n ≥ 1, then f(n) is equal to
(A) $2^n + 1$ (B) 2^n (C*) $2^n - 1$ (D) $2^{n-1} - 1$

A function f : R → R satisfies the condition, $x^2 f(x) + f(1 - x) = 2x - x^4$. Then f(x) is:
(A) $-x^2 - 1$ (B*) $-x^2 + 1$ (C) $x^2 - 1$ (D) $-x^4 + 1$

Let f(x) be a polynomial function satisfying the relation $f(x).\ f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)\ \forall\ x \in R - \{0\}$ and f(3) = –26. Determine f′(1).

Ans. – 3

Determine a function f satisfying the functional relation $f(x) + f\left(\frac{1}{1-x}\right) = \frac{2(1-2x)}{x(1-x)}$.

Ans. $\frac{x+1}{x-1}$

Let a function $f: R \to R$ such that $f(1) = 2$ and $f(x+y) = 2^x f(y) + 4^y f(x)\ \forall\ x, y \in R$. If $f'(2) = k \ln 2$, then find the value of k.

(a) Find a function f, other than a constant function, such that $|f(y) - f(x)| \le |y - x|$.
(b) Suppose that $f(y) - f(x) \le (y - x)^2$ for all x and y. (Why does this imply that $|f(y) - f(x)| \le (y - x)^2$?) Prove that f is a constant function. Hint: Divide the interval from x to y into n equal pieces.

Prove that there do *not* exist functions f and g with either of the following properties:

(i) $f(x) + g(y) = xy$ for all x and y.
(ii) $f(x) \cdot g(y) = x + y$ for all x and y.

Determine all functions f : R → R such that f(x) f(y) – f(xy) = x + y for all x, y ∈ R.

Let f, g : R → R be functions satisfying, for all real numbers x and y, the equality f(x + g(y)) = 2x + y + 5. Find an expression for g(x + f(y)).

Find all the functions g : R → R satisfying $g(x + y) + g(x - y) = 2x^2 + 2y^2$ for all x and y.

Let a function f satisfy f(x + 1) = f(x) + x ∀ x ∈ N, where f(1) = 0. Find f(3) and a formula for f(x).

Let a polynomial function $f(x^2) = x^3 f(x) + x^3 - 1\ \forall\ x \in R$, where f(2) = 7. Find f(x).

Let a polynomial function f satisfy f(x) + f(1/x) = f(x) · f(1/x) ∀ x ∈ R – {0}. Prove that $f(x) = \pm x^n + 1$, where n ∈ W.

Let f satisfy $f(n + 1) = (-1)^{n+1} n - 2f(n)$, n ≥ 1. If f(1) = f(1001), find f(1) + f(2) + f(3) + ... + f(1000).

Find all polynomials P(x) such that xP(x – 1) = (x – 15) P(x).

Find all polynomials $P(x)$ such that $P(x)\,P(x+1) = P(x^2)$.

Find all functions $f : N \to N$ such that

(a) $f(2) = 2$;

(b) $f(mn) = f(m)\,f(n)$ for all m, n in N;

(c) $f(m) < f(n)$ whenever $m < n$.

Find all functions $f : R \to R$ such that

(a) $f(-x) = -f(x)$ for all real x;

(b) $f(x + 1) = f(x) + 1$, for all real x;

(c) $f(1/x) = f(x)/x^2$ for all $x \neq 0$.

Find all functions $f : Q \to Q$ such that $f(x + y) + f(x - y) = 2f(x) + 2f(y)$, for all rational x, y.

Let $f : [1, \infty) \to [1, \infty)$ satisfy

(a) $f(x) \leq 2(1 + x)$ for all $x \in [1, \infty)$;

(b) $x\,f(x + 1) = f(x)^2 - 1$ for all $x \in [1, \infty)$.

Prove that $f(x) \leq x + 1$.

Let f : I → I satisfy the equations

(1) $f(x^2) = [f(x)]^2$,

(2) $f(x + 1) = f(x) + 1$.

Prove that f(x) = x for all integral x.

If a function f satisfies $f(x + y) \le f(x) + f(y)$ for all real x and y, and $f(x) \le x$ for all real x, find f.

Prove that f(n) = 1 – n is the only integer valued function f defined on integers such that

(i) $f(f(n)) = n$ for all $n \in I$, and

(ii) $f(f(n + 2) + 2) = n$ for all $n \in I$, and

(iii) $f(0) = 1$.

How many polynomials p(x) of degree at least one with integer coefficients satisfy $16\, p(x^2) = (p(2x))^2$, for all real numbers x?

Suppose that f(x) is a function satisfying $|f(m + n) - f(m)| \le n/m$ for all positive rational numbers m and n. Prove that, for all natural numbers k,

$$\Sigma_{i=1}^{k} |f(2^k) - f(2^i)| \le k(k - 1)/2.$$

In each of the following cases, find all polynomials with real coefficients which satisfy the equation.

(a) $f(x^2 + x) = f(x)\, f(x + 1)$.

(b) $f(g(x)) = f(x)\, g(x)$.

Let $f\left(x + \frac{1}{y}\right) + f\left(x - \frac{1}{y}\right) = 2f(x) \cdot f\left(\frac{1}{y}\right)$ $\forall\, x, y \in R$ and $y \neq 0$. If $f(0) = 0$, then show that $f(1) = f(2) = 0$.

Then find the value of

In the equation below f is a function from R to R. Find f:

(i) $f(x + y) - 2f(x - y) + f(x) - 2f(y) = y - 2$

(ii) $f(x + y) + 2f(x - y) + f(x) + 2f(y) = 4x + y$

(iii) $f(x)\, f(x + y) = f(y)^2 f(x - y)^2 e^{y + 4}$

(iv) $f(x + y) + f(x - y) - (y + 2)\, f(x) + y(x^2 - 2y) = 0$

Suppose $f : R \to R$ is such that $f(xy) = xf(x) + yf(y)$, for all $x, y \in R$. Prove that $f(x) = 0$ for all $x \in R$.

Suppose $f : R \to R$ is such that $f(xf(z) + f(y)) = zf(x) + y$, for all real numbers x, y, z. Prove that $f(x) = x$ for all real x.

Find all pairs of functions f, g : R → R which satisfy

(a) g is an one-one function;

(b) $f(g(x) + y) = g(x + f(y))$, for all x, y ∈ R.

Let $f : R \to R$ satisfy $f(3) = 1$, $\forall x \in R$ $f(x + 3) \geq f(x) + 3$, $f(x + 1) \leq f(x) + 1$. Put $g(x) = f(x) - x + 1$. Determine g(2008)

Prove that $f(n) = 1 - n$ is the only integer valued function defined on the integers that satisfies the following conditions.

(a) $f(f(n)) = n$, for all integers n;

(b) $f(f(n + 2) + 2) = n$, for all integers n;

(c) $f(0) = 1$.

Problem

Let $f : \mathbb{R} \to \mathbb{R}$ be continuous and satisfy

$$f(3x - 2) = 3f(x) - 2$$

and

$$f(x + y - 1) = f(x) + f(y) - 1.$$

Find all f.

2. This problem asks the reader to fill in the details of the proof of Theorem 2.4, above. Let $f : \mathbf{R} \to \mathbf{R}$ be a continuous function satisfying Cauchy's equation. Suppose in addition that there exists some interval $[c, d]$ of real numbers, where $c < d$, such that f is bounded below on $[c, d]$.
   a) Show that $f(n\,x) = n\,f(x)$ for all real x.
   b) Define $p = d - c$. Show that f is bounded below on the interval $[0, p]$. (However, it need not be bounded below by the same constant as on the interval $[c, d]$.)
   c) Define the function
   $$g(x) = f(x) - \frac{f(p)}{p}\,x\,.$$
   Prove that g is also bounded below on the interval $[0, p]$ and satisfies Cauchy's equation.
   d) Show that g is periodic with period p in the sense that $g(x+p) = g(x)$ for all real x. Conclude from this, and the fact that g is bounded below on the interval $[0, p]$ that g is bounded below on the entire real line $(-\infty, +\infty)$.
   e) Suppose that there exists some x_0 for which $g(x_0) \neq 0$. Prove a contradiction, by showing that the sequence of values $g(n\,x_0)$, $n = \pm1, \pm2, \pm3, \ldots$ is not bounded below.
   f) Conclude that $g(x) = 0$ for all real x, and therefore that $f(x) = a\,x$ for all real x, where $a = f(p)/p$.

P2. State the domain, codomain, and any regularity hypothesis in each of the following problems, without solving them.

(a) Find all $f : \mathbb{N} \to \mathbb{N}$ such that $f(2) = 2$,

$$f(mn) = f(m)f(n) \quad \text{whenever } \gcd(m,n) = 1,$$

and $f(m) < f(n)$ whenever $m < n$.

(b) Find all continuous $f : \mathbb{R} \to \mathbb{R}$ such that

$$f(x+y) = f(x)f(y) \qquad \forall x, y \in \mathbb{R},$$

with f differentiable at $x = 1$ and $f'(1) = 2$.

(c) Find all $f : \mathbb{R} \setminus \{0,1\} \to \mathbb{R}$ such that

$$f(x) + f\left(\frac{1}{1-x}\right) = \frac{2(1-2x)}{x(1-x)}.$$

(d) Find all $f : \mathbb{Z} \to \mathbb{Z}$ with $f(0) = f(1) = 0$ satisfying

$$f(x+y) + f(x-y) = f(x)f(y)$$

and

$$f(x-y)f(x)f(y) = f(0)f(x)f(y)$$

for all integers x, y.

P3. Show that there is no function $f : \mathbb{R}_+ \to \mathbb{R}_+$ such that

$$(x+y)f(f(x)y) = x^2 f(f(x) + f(y))$$

for all $x, y > 0$.

P4. A student claims that

$$f(x+1) = f(x) + 1 \qquad \forall x \in \mathbb{R}$$

together with $f(1) = 1$ forces $f(x) = x$. Find an explicit counterexample and identify the missing hypothesis that would make the claim correct.

P5. Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that for all $x, y \in \mathbb{R}$,

$$|f(x+y) - f(x-y) - y| \le y^2.$$

Determine the exact form of f, identifying any constant.

P6. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function satisfying

$$f(x + f(y)) = f(x) + y \qquad \forall x, y \in \mathbb{R}.$$

Show that f is a bijection and find every such function.

P7. Let $f : \mathbb{R} \to \mathbb{R}$ be a function satisfying

$$f(x) = f(1 - x)$$

and

$$f(x + 1) = f(x) + 1$$

for all $x \in \mathbb{R}$. Show that there is no such function.

P8. Suppose $f : \mathbb{N}_0 \to \mathbb{N}_0$ satisfies

$$f(0) = 0, \qquad f(2n) = f(n), \qquad f(2n + 1) = f(n) + 1.$$

Compute $f(13)$, writing $13 = 1101_2$.

P9. Find all $f : \mathbb{N} \to \mathbb{N}$ such that $f(2) = 2$,

$$f(mn) = f(m)f(n)$$

for all coprime $m, n \in \mathbb{N}$, and

$$f(m) < f(n) \qquad \text{whenever } m < n.$$

P10. Let $f : \mathbb{N}_0 \to \mathbb{N}_0$ satisfy

$$f(2n) = f(f(n)), \qquad f(2n + 1) = f(2n) + 1.$$

(a) Assuming $f(0) = 0$, determine $f(n)$ for every n.

(b) Show that $f(0)$ cannot equal 1.

(c) For which nonnegative integers k, if any, can $f(0) = 2^k$?

P11. Let $\mathbb{N}_0 = \{0, 1, 2, \ldots\}$ and let $f : \mathbb{N}_0 \to \mathbb{N}_0$ be a function such that

$$f(f(f(n))) < f(n + 1) \qquad \forall n \in \mathbb{N}_0.$$

Prove that $f(n) = n$ for all $n \in \mathbb{N}_0$.

P12. Let $f : \mathbb{R} \to \mathbb{R}$ be a twice differentiable function such that

$$\frac{1}{2y} \int_{x-y}^{x+y} f(t)\, dt = f(x)$$

for all $x \in \mathbb{R}$ and all $y > 0$. Show that there exist $a, b \in \mathbb{R}$ such that

$$f(x) = ax + b \qquad \forall x \in \mathbb{R}.$$

P13. Let $f, g : \mathbb{R} \to \mathbb{R}$ be continuous functions satisfying

$$f(x+y) = f(x)f(y) - g(x)g(y)$$

and

$$g(x+y) = g(x)f(y) + f(x)g(y)$$

for all $x, y \in \mathbb{R}$, with $f(0) \neq 0$. Show that $f(0) = 1$, $g(0) = 0$, and that

$$h(x) = f(x)^2 + g(x)^2$$

satisfies

$$h(x+y) = h(x)h(y) \qquad \forall x, y \in \mathbb{R}.$$

P14. Find every continuous $f : \mathbb{R} \to \mathbb{R}$ satisfying

$$f(x+y) = f(x) + f(y) + 2xy \qquad \forall x, y \in \mathbb{R}.$$

P15. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function satisfying

$$f(x)f(y) = f(x+y) + f(x-y) \qquad \forall x, y \in \mathbb{R}.$$

Show that either $f \equiv 0$, or there exist constants $\alpha, \beta \in \mathbb{R}$ such that

$$f(x) = 2\cosh(\alpha x)$$

or

$$f(x) = 2\cos(\beta x).$$

P16. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function satisfying

$$f(x) = f(e^t x) \qquad \forall x \in \mathbb{R},\ \forall t \geq 0.$$

Show that f is constant.

P17. Let $f : (0, \infty) \to \mathbb{R}$ be a continuous function satisfying

$$f(2x) = f(x) \qquad \forall x > 0.$$

Define

$$g(x) = \int_x^{2x} \frac{f(t)}{t}\, dt \qquad (x > 0).$$

Show that g is constant.

P18. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function such that

$$f(x+1) = f(x) \qquad \forall x \in \mathbb{R},$$

and let

$$g(t) = \int_0^t f(x)\,dx \qquad (t \in \mathbb{R}).$$

Define

$$h(t) = \lim_{n\to\infty} \frac{g(t+n)}{n}$$

whenever the limit exists. Show that h is defined for every $t \in \mathbb{R}$ and that h is constant in t.

P19. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function such that

$$f(x+1) = f(x) \qquad \forall x \in \mathbb{R},$$

and let

$$a_n = \int_0^n f(t)\,dt \qquad (n \geq 1).$$

Determine

$$\lim_{n\to\infty} \frac{a_n}{n}.$$

Find all continuous functions $f : (-1,\infty) \to (-1,\infty)$ satisfying

$$f(x+y+xy) = f(x) + f(y) + f(x)f(y) \qquad \forall x, y > -1.$$

Prove that there exists a unique function $f : (0,\infty) \to (0,\infty)$ such that

$$f(f(x)) = 6x - f(x) \qquad \forall x > 0.$$

Find all continuous functions $f : \mathbb{R} \to \mathbb{R}$ such that

$$f(2x+3) = 2f(x) + 3$$

and

$$f(x+y+3) = f(x) + f(y) + 3$$

for all real x, y.

8. Consider the real-valued function $h : \{0, 1, 2, \ldots, 100\} \to \mathbb{R}$ such that $h(0) = 5$, $h(100) = 20$ and satisfying $h(i) = \frac{1}{2}(h(i+1) + h(i-1))$, for every $i = 1, 2, \ldots, 99$. Then, the value of $h(1)$ is:

(A) 5.15 (B) 5.5 (C) 6 (D) 6.15.

If f(x + y) = f(x) + f(y) – xy – 1 for all x, y, and f(1) = 1 then the number of solutions of f(n) = n, $n \in N$, is

(a) one (b) two (c) four (d) none of these

Q.2 Solve the following problems from (a) to (e) on functional equation.

(a) The function $f(x)$ defined on the real numbers has the property that $f(f(x))\cdot(1+f(x)) = -f(x)$ for all x in the domain of f. If the number 3 is in the domain and range of f, compute the value of $f(3)$.

(b) Suppose f is a real function satisfying $f(x + f(x)) = 4f(x)$ and $f(1) = 4$. Find the value of $f(21)$.

(c) Let 'f be a function defined from $R^+ \to R^+$. If $[f(xy)]^2 = x(f(y))^2$ for all positive numbers x and y and $f(2) = 6$, find the value of $f(50)$.

(d) Let $f(x)$ be a function with two properties

(i) for any two real number x and y, $f(x + y) = x + f(y)$ and

(ii) $f(0) = 2$.

Find the value of $f(100)$.

(e) Let f be a function such that $f(3) = 1$ and $f(3x) = x + f(3x - 3)$ for all x. Then find the value of $f(300)$.

In a function $2f(x) + xf\left(\frac{1}{x}\right) - 2f\left(\left|\sqrt{2}\sin\left(\pi\left(x+\frac{1}{4}\right)\right)\right|\right) = 4\cos^2\frac{\pi x}{2} + x\cos\frac{\pi}{x}$

Prove that (i) $f(2) + f(1/2) = 1$ and (ii) $f(2) + f(1) = 0$

A function f, defined for all $x, y \in R$ is such that $f(1) = 2$; $f(2) = 8$ & $f(x+y) - kxy = f(x) + 2y^2$, where k is some constant. Find f(x) & show that :

$$f(x+y)\,f\left(\frac{1}{x+y}\right) = k \text{ for } x + y \neq 0.$$

Let 'f' be a real valued function defined for all real numbers x such that for some positive constant 'a' the equation $f(x+a) = \frac{1}{2} + \sqrt{f(x) - (f(x))^2}$ holds for all x. Prove that the function f is periodic.

If for all real values of u & v, $2f(u)\cos v = f(u + v) + f(u - v)$, prove that, for all real values of x

(i) $f(x) + f(-x) = 2a\cos x$ (ii) $f(\pi - x) + f(-x) = 0$

(iii) $f(\pi - x) + f(x) = -2b\sin x$. Deduce that $f(x) = a\cos x - b\sin x$, a, b are arbitrary constants.

If f(x + ay, x – ay) = axy, then f (x,y) equals-

(A) $\frac{x^2 + y^2}{4}$ (B) $\frac{x^2 - y^2}{4}$

(C) x^2 (D) y^2

Let f be a real valued function such that

$$f(x) + 2f\left(\frac{2002}{x}\right) = 3x$$

for all $x > 0$. Find $f(2)$.

(A) 1000 (B) 2000 (C) 3000 (D) 4000

Let f be a function satisfying $f(xy) = \frac{f(x)}{y}$ for all positive real numbers x and y. If $f(30) = 20$, then the value of $f(40)$ is

(A) 15 (B) 20 (C) 40 (D) 60

Let $f(x)$ and $g(x)$ be functions which take integers as arguments. Let $f(x+y) = f(x) + g(y) + 8$ for all integer x and y. Let $f(x) = x$ for all negative integers x, and let $g(8) = 17$. The value of $f(0)$ is

(A) 17 (B) 9 (C) 25 (D) – 17

Consider a real-valued function $f(x)$ satisfying $2f(xy) = (f(x))^y + (f(y))^x\ \forall\ x, y \in R$ and $f(1) = a$ where $a \neq 1$. Prove that $(a-1)\sum_{i=1}^{n} f(i) = a^{n+1} - a$.

If $f(x+y+1) = (\sqrt{f(x)} + \sqrt{f(y)})^2$ and $f(0) = 1$, $\forall\ x, y \in R$. Determine $f(n), n \in N$.

A real-valued function $f(x)$ satisfies the functional equation $f(x-y) = f(x)f(y) - f(a-x)f(a+y)$, where a is a given constant and $f(0) = 1$. $f(2a-x)$ is equal to

a. $f(x)$ b. $-f(x)$

c. $f(-x)$ d. $f(a)+f(a-x)$

None of these

If the function f satisfies the relation $f(x+y) + f(x-y) = 2f(x)\,f(y)\ \forall\ x, y \in R$ and $f(0) \neq 0$, then

a. $f(x)$ is an even function

b. $f(x)$ is an odd function

c. If $f(2) = a$ then $f(-2) = a$

d. If $f(4) = b$ then $f(-4) = -b$

Let $f(x) + f(y) = f(x\sqrt{1-y^2} + y\sqrt{1-x^2})$ ($f(x)$ is not identically zero). Then

a. $f(4x^3-3x)+3f(x)=0$

b. $f(4x^3-3x)=3f(x)$

c. $f(2x\sqrt{1-x^2})+2f(x)=0$

d. $f(2x\sqrt{1-x^2})=2f(x)$

If $f: R^+ \to R^+$ is a polynomial function satisfying the functional equation $f(f(x)) = 6x - f(x)$, then $f(17)$ is equal to

a. 17

b. -51

c. 34

d. -34

c. 34

d. -34

Let $f: R \to R$ be a function defined by $f(x+1) = \dfrac{f(x)-5}{f(x)-3}$ $\forall\, x \in R$. Then which of the following statement(s) is/are true

a. $f(2008) = f(2004)$

b. $f(2006) = f(2010)$

c. $f(2006) = f(2002)$

d. $f(2006) = f(2018)$

Consider the function $f(x)$ satisfying the identity $f(x)+f\left(\frac{x-1}{x}\right)$ $=1+x, \forall\ x \in R-\{0, 1\}$ and $g(x)=2f(x)-x+1$.

4. The domain of $y=\sqrt{g(x)}$ is

a. $\left(-\infty, \frac{1-\sqrt{5}}{2}\right] \cup \left[1, \frac{1+\sqrt{5}}{2}\right]$

b. $\left(-\infty, \frac{1-\sqrt{5}}{2}\right] \cup (0, 1) \cup \left[\frac{1+\sqrt{5}}{2}, \infty\right)$

c. $\left[\frac{-1-\sqrt{5}}{2}, 0\right] \cup \left[\frac{-1+\sqrt{5}}{2}, 1\right]$

d. None of these

5. The range of $y=g(x)$ is

a. $(-\infty, 5]$ b. $[1, \infty)$

c. $(-\infty, 1] \cup [5, \infty)$ d. None of these

6. The number of roots of the equation $g(x)=1$ is

a. 2 b. 1 c. 3 d. 0

Let $f: N \to R$ be a function satisfying the following conditions, $f(1)=1/2$ and $f(1)+2 f(2)+3 f(3)+\cdots+n f(n)=n(n+1)$, $f(n)$ for $n \geq 2$.

7. The value of $f(1003)=\frac{1}{K}$, where K equals

a. 1003 b. 2003 c. 2005 d. 2006

8. The value of $f(999)$ is $\frac{1}{K}$, where K equals

a. 999 b. 1000 c. 1998 d. 2000

9. $f(1), f(2), f(3), f(4), \ldots$ represents a series of

a. an A.P b. a G.P.

c. a H.P. d. An arithmetico-geometric

If $(f(x))^2 \times f\left(\frac{1-x}{1+x}\right) = 64x, \ \forall x \in Df$, then

10. $f(x)$ is equal to

a. $4x^{2/3}\left(\frac{1+x}{1-x}\right)^{1/3}$
b. $x^{1/3}\left(\frac{1-x}{1+x}\right)^{1/3}$
c. $x^{2/3}\left(\frac{1-x}{1+x}\right)^{1/3}$
d. $x\left(\frac{1+x}{1-x}\right)^{1/3}$

11. The domain of $f(x)$ is

a. $[0, \infty)$
b. $R - \{1\}$
c. $(-\infty, \infty)$
d. None of these

12. The value of $f(9/7)$ is

a. $8(7/9)^{2/3}$
b. $4(9/7)^{1/3}$
c. $-8(9/7)^{2/3}$
d. None of these

Let $f: R \to R$ is a function satisfying $f(2-x) = f(2+x)$ and $f(20-x) = f(x), \forall x \in R$. For this function f, answer the following.

25. If $f(0) = 5$, then the minimum possible number of values of x satisfying $f(x) = 5$, for $x \in [0, 170]$, is

a. 21
b. 12
c. 11
d. 22

26. The graph of $y = f(x)$ is not symmetrial about

a. symmetrical about $x = 2$
b. symmetrical about $x = 10$
c. symmetrical about $x = 8$
d. None of these

27. If $f(2) \neq f(6)$, then the

a. fundamental period of $f(x)$ is 1
b. fundamental period of $f(x)$ may be 1
c. period of $f(x)$ cannot be 1
d. fundamental period of $f(x)$ is 8

$f: R \to R\, f(x^2+x+3)+2f(x^2-3x+5)=6x^2-10x+17\ \forall$
$x \in R$, then the value of $f(5)$ is

If $f(x)$ is an odd function and $f(1)=3$, and $f(x+2)=f(x)+f(2)$, then the value of $f(3)$ is

Let $f: R \to R$ be a continuous onto function satisfying $f(x)+f(-x)=0,\ \forall x \in R$.
If $f(-3)=2$ and $f(5)=4$ in $[-5, 5]$, then the minimum number of roots of the equation $f(x)=0$ is

The function f is continuous and has the property $f(f(x))=1-x$, then the value of $f\left(\frac{1}{4}\right)+f\left(\frac{3}{4}\right)$ is

For the function $f(x)$ satisfying $2.f(\sin x)+f(\cos x)=x, \forall x \in R$

a) Domain is $[0,1]$

b) range is $\left[\frac{-2\pi}{3}, \frac{\pi}{3}\right]$

c) Domain is $[-1,1]$

d) range is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

Let p(x) be a quadratic polynomial with real coefficient such that for all real 'x' the relation $2(1+p(x))=p(x-1)+p(x+1)$ holds of p(0) = 8 and p(2) = 32 then

Sum of all the coefficients of $p(x)$ is

a) 15 b) 17 c) 19 d) 21

If the range of $p(x)$ in $[m,\infty)$ then the value of 'm' is

a) -5 b) -12 c) -15 d) -17

. f is a function defined as $\sum_{k=1}^{n} f(a+k)=16(2^n-1)$ and $f(x+y)=f(x).f(y)$ and $f(1)=2$ then integral value of a

a) 3 **b) 0** **c) 2** **d) 1**

If 'p' and 'q' are +ve integers, f is a function defined for +ve numbers and attains only +ve valu such that $f(x, f(y))=x^p y^q$ then $p^2=$

a) $2q$ b) q c) $3q$ d) $4q$

Let $f(x)$ be a polynomial with positive degree satisfying the relation $f(x)f(y)=f(x)+f(y)+f(xy)-2$ and $f(4)=65$, then

a) $f(2)=9$

b) $f(3\sqrt{3})=27$

c) $f(5)=126$

d) The roots of the equation $f(x)=2x^2$ are all real

If $f(x)$ is an even funciton and $g(x)$ is an odd funciotn and satisfies the relation $x^2 f(x) - 2f\left(\frac{1}{x}\right) = g(x)$ then

a) $f(x) = 0, \forall x \in R$ b) $f(2009) = 0$

c) f is constant function d) $g(x) = 0, \forall x \in R$

ssage - II :

Let $f : R \to R$ be a continuous function such that $f(x) - 2f\left(\frac{x}{2}\right) + f\left(\frac{x}{4}\right) = x^2$.

Now answer the following

31. $f(3)$=

a) $f(0)$ b) $4 + f(0)$ c) $9 + f(0)$ d) $16 + f(0)$

32. The equation $f(x) - x - f(0) = 0$ have exactly

a) no solution b) one solution c) two solution d) infinite solutions

If the periodic function $f(x)$ satisfies the equation $f(x+1) + f(x-1) = \sqrt{3} f(x)$ $\forall x \in R$ and the perio of $f(x)$ is 4λ then $\lambda =$

If 'f' is a polynomial such that $f\left(\frac{1-x}{1+x}\right) f\left(\frac{1+x}{1-x}\right) = f\left(\frac{1-x}{1+x}\right) + f\left(\frac{1+x}{1-x}\right)$ $(where\ x \neq 0, \pm 1)$ and $f(3)=28$ then the value of $\frac{1}{605}\left(\sum_{n=1}^{10}(f(n)-1)\right)$ is

2. Let $f : \mathbb{R} \to \mathbb{R}$ be a function, where $\mathbb{R}$ is the set of real numbers. For each statement below, write whether it is TRUE or FALSE.

a) If $|f(x) - f(y)| \leq 39|x - y|$ for all x, y then f must be continuous everywhere.

Answer: ________

b) If $|f(x) - f(y)| \leq 39|x - y|$ for all x, y then f must be differentiable everywhere.

Answer: ________

c) If $|f(x) - f(y)| \leq 39|x - y|^2$ for all x, y then f must be differentiable everywhere.

Answer: ________

d) If $|f(x) - f(y)| \leq 39|x - y|^2$ for all x, y then f must be constant.

Answer: ________

4. Suppose $f(x)$ is a function from $\mathbb{R}$ to $\mathbb{R}$ such that $f(f(x)) = f(x)^{2013}$. Show that there are infinitely many such functions, of which exactly four are polynomials. (Here $\mathbb{R}$ = the set of real numbers.)

3. Let f be a function on nonnegative integers defined as follows

$$f(2n) = f(f(n)) \quad \text{and} \quad f(2n+1) = f(2n) + 1.$$

(a) If $f(0) = 0$, find $f(n)$ for every n. [2 marks]
(b) Show that $f(0)$ cannot equal 1. [4 marks]
(c) For what nonnegative integers k (if any) can $f(0)$ equal 2^k? [9 marks]

B2. [12 points] Let f be a function from natural numbers to natural numbers that satisfies

$$f(n) = n - 2 \quad \text{for } n > 3000;$$
$$f(n) = f(f(n+5)) \quad \text{for } n \leq 3000.$$

Show that $f(2022)$ is uniquely decided and find its value.

B4. **[12 points]** The domain of f is the set of *positive integers* and $f(xy) = f(x) + f(y)$ for all x, y. Answer the *independent* questions below. (Data from (a) are not valid for the rest.)

(a) Suppose $f(2025) = 0$, $f(20) = 10$ and $f(25) = 20$. What is the smallest n for which $f(n)$ is not uniquely determined? Write values of $f(x)$ for each positive integer $x < n$.

(b) Is there such a function f for which $f(x) = 0$ for all positive integers $x < 2025^{2025}$ but f is not identically 0? Show how to define such f or show that it is not possible.

Q.3(a) A function f is defined for all positive integers and satisfies $f(1) = 2005$ and $f(1)+f(2)+ \ldots +f(n) = n^2 f(n)$ for all $n > 1$. Find the value of $f(2004)$.