MATHEMATICS JEE ADVANCED 2024 MOCK 3

Section 1

Four questions | +3/-1 Marks. ONLY ONE of the four options is correct in each question.

1. For the differential equation

$$y\cos\left(\frac{y}{x}\right)(xdy-ydx)+x\sin\left(\frac{y}{x}\right)(xdy+ydx)=0$$

when $y(1)=\frac{\pi}{2}$, the solution is

(a) $xy\sin\left(\frac{y}{x}\right)=\frac{\pi}{2}$

(b) $xy=\frac{\pi}{2}\sin\left(\frac{y}{x}\right)$

(c) $xy\sin xy=\frac{\pi}{2}$

(d) $\frac{y}{x}\sin\frac{y}{x}=\frac{\pi}{2}$

2. The function $f:\mathrm{R}\to\mathrm{R}$ is defined by

$$f(x)=\begin{cases}e^{-\frac{1}{x}}, & x>0\\ 0, & x\le 0.\end{cases}$$

then at x=0 which of the following is correct

(a) f is not continuous

(b) f is continuous, but not differentiable

(c) f is differentiable, but f' is not continuous

(d) f is differentiable, and f' is continuous

3. If $p\vec{x}+(\vec{x}\times\vec{a})=\vec{b};(p\neq 0)$, then which of the following is/are true

(a) $\vec{x}=\frac{p^2\vec{a}+(\vec{a}.\vec{b})\vec{b}-p(\vec{a}\times\vec{b})}{p(p^2+|\vec{a}|^2)}$

(b) $\vec{x}=\frac{p^2\vec{b}+(\vec{a}.\vec{b})\vec{a}-p(\vec{b}\times\vec{a})}{p(p^2+|\vec{a}|^2)}$

(c) $\vec{x}=\frac{p^2\vec{a}+(\vec{a}.\vec{b})\vec{b}+p(\vec{a}\times\vec{b})}{p(p^2+|\vec{a}|^2)}$

(d) $\vec{x}=\frac{p^2\vec{b}+(\vec{a}.\vec{b})\vec{a}+p(\vec{b}\times\vec{a})}{p(p^2+|\vec{a}|^2)}$

4. Find a sum of the series:

$$S = \frac{\cos 2x}{1.3} + \frac{\cos 4x}{3.5} + \frac{\cos 6x}{5.7} + \ldots = \sum_{n=1}^{\infty} \frac{\cos(2nx)}{(2n-1)(2n+1)}$$

(a) $-\frac{\pi}{2}\sin x + \frac{1}{3}$ (b) $-\frac{\pi}{4}\sin x + \frac{1}{2}$ (c) $\frac{\pi}{2}\sin x + \frac{1}{3}$ (d) $\frac{\pi}{4}\sin x + \frac{1}{2}$

Section 2

Four questions | +4/0 Marks. Each question has Four options. One or more than one of these four options is/are correct answer(s).

5. The value of $\sum\sum\sum\sum\sum_{1\le i\le j<k\le w\le n} 1$ is

(a) ${}^{n+2}C_4$ (b) ${}^{n+1}C_4 + 2{}^{n}C_2$ (c) ${}^{n}C_4 + {}^{n}C_3 + 2{}^{n}C_2$ (d) ${}^{n}C_4 + 2{}^{n}C_3 + {}^{n}C_2$

6. Let $\vec{v_0} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ be a fixed vector and for $n \ge 0$ a sequence is defined as $\vec{v}_{n+1} = \vec{v}_n + \left(\frac{1}{2}\right)^{n+1} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}^{n+1} \vec{v_0}$. If $\lim_{n\to\infty} \vec{v}_n = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$, then which of the following options is/are correct?

(a) $\alpha + \beta = 2$ (b) $\alpha - \beta = 2$ (c) $\alpha\beta = 2$ (d) $\frac{\alpha}{\beta} = 2$

7. Suppose A, B and C are events and the probability of each of these two events is strictly between 0 and 1. Which of the following statement(s) is/are true

(a) A and B are mutually exclusive if and only if not A and not B are exhaustive

(b) If A, B and C are pairwise independent and A is independent of $B \cup C$, then A, B and C are mutually independent.

(c) A and B cannot be simultaneously independent and exhaustive.

(d) An event A is known to be independent of the events B, $B \cup C$ and $B \cap C$, it is also independent of C.

8. If $t_1 = (\sin^{-1} x)^{\sin^{-1} x}$, $t_2 = (\sin^{-1} x)^{\cos^{-1} x}$, $t_3 = (\cos^{-1} x)^{\sin^{-1} x}$ and $t_4 = (\cos^{-1} x)^{\cos^{-1} x}$

Then which of the following option(s) is/are correct

(a) If $x \in (0, \cos 1)$ then $t_4 > t_3 > t_1 > t_2$ (b) If $x \in \left(\cos 1, \frac{1}{\sqrt{2}}\right)$ then $t_3 > t_4 > t_2 > t_1$

(c) If $x \in \left(\frac{1}{\sqrt{2}}, \sin 1\right)$ then $t_2 > t_1 > t_4 > t_3$ (d) If $x \in (\sin 1, 1)$ then $t_1 > t_2 > t_4 > t_3$

Section 3

Six questions. +4/0 Marks. The answer to each question is a NON-NEGATIVE INTEGER.

9. Let $a_1 < a_2 < a_3 < a_4$ be positive integers such that $\sum_{i=1}^{4} \frac{1}{a_i} = \frac{11}{6}$. Then $a_4 - a_2$ equals _____?

10. If $f(x) = \int_0^2 |x - t|\, dt$ then number of solutions of the equation $x \sin x + \cos x = f(x)$ is _____?

11. Let $X_1, X_2, ..., X_{18}$ be eighteen observations such that $\sum_{i=1}^{18} (X_i - \alpha) = 36$ and $\sum_{i=1}^{18} (X_i - \beta)^2 = 90$, where α and β are distinct real numbers. If the standard deviation of these observations is 1, then the value of $|\alpha - \beta|$ is _____?

12. If the tangent at the point $\left(\frac{a}{\sqrt{2}}, \frac{b}{\sqrt{2}}\right)$ on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets the auxiliary circle in two points and the chord joining them subtends a right angle at the centre. It e is eccentricity of the ellipse, then the value of $3e^2$ is _____?

13. Let $S=\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}+...+\sqrt{1+\frac{1}{1999^2}+\frac{1}{2000^2}}$, then the value of $|2000(S-2000)|$ is _____?

14. Four quadrants are drawn inside a square with each side (r =3 units) as radius (*fig* 1).

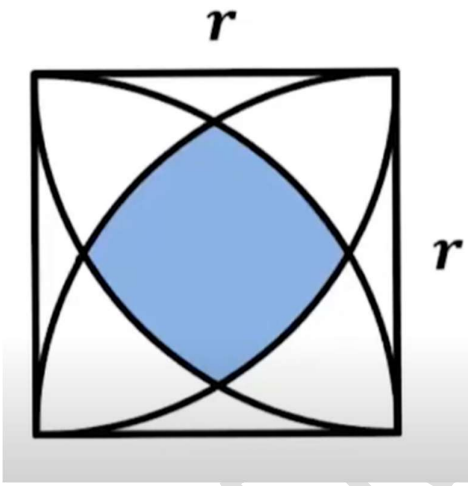

fig 1

If the area common in all the four quadrants (shaded region) is A, then the integral part of A is _____?

Section 4

This section contains TWO paragraphs. Based on each paragraph, there are TWO questions. ONLY ONE of these four options is correct in each question. +3/-1 marks.

Passage I

If l and m are variable real numbers such that $5l^2+6m^2-4lm+3l=0$, then variable line $lx+my=1$ always touches a fixed parabola, whose axis is parallel to x-axis.

14. Focus of the parabola is

(a) $\left(\frac{1}{6},-\frac{7}{6}\right)$ (b) $\left(\frac{1}{3},\frac{4}{3}\right)$ (c) $\left(\frac{3}{2},-\frac{3}{2}\right)$ (d) $\left(\frac{3}{4},-\frac{3}{4}\right)$

15. Directrix of the parabola is

(a) $6x+7=0$ (b) $4x+11=0$ (c) $3x+11=0$ (d) $x+1=0$

Passage II

If n and k are integers with $1 \le k \le n$, let $f(n,k)$ be the number of ways of partitioning a set of n elements into k subsets, each subset with at least two and at most n elements. For example, $f(5,2) = 10$ because the allowable partitions of $\{1,2,3,4,5\}$ are

$\{1,2\}\{3,4,5\}; \{1,3\}\{2,4,5\}; \{1,4\}\{2,3,5\}; \{1,5\}\{2,3,4\};$
$\{2,3\}\{1,4,5\}; \{2,4\}\{1,3,5\}; \{2,5\}\{1,3,4\};$
$\{3,4\}\{1,2,5\}; \{3,5\}\{1,2,4\};$
$\{4,5\}\{1,2,3\}$

16. For $2 \le k < n$, $f(n+1,k)$ can be expressed in terms of $f(n-1,k-1)$ and $f(n,k)$ as

(a) $f(n+1,k) = n\big(f(n-1,k-1) + f(n,k)\big)$ (b) $f(n+1,k) = k\big(f(n-1,k-1) + f(n,k)\big)$

(c) $f(n+1,k) = kf(n-1,k-1) + nf(n,k)$ (d) $f(n+1,k) = \mathrm{n}\, f(n-1,k-1) + \mathrm{k}\, f(n,k)$

17. The value of $f(7,3)$ is

(a) 25 (b) 35 (c) 70 (d) 105