JEE Advanced 2024 Mock Test 4

Section 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question have **FOUR** options. **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If ONLY the correct option is chosen.
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : –1 In all other cases.

1. 6 white and 6 black balls of the same size are distributed among 10 different urns. Balls are alike except for the colour and each urn can hold any number of balls. The number of different distribution of the balls so that there is at least one ball in each urn is

(a) 14700 (b) 17850 (c) 26250 (d) 29400

2. Let $A(z_1)$, $B(z_2)$, $C(z_3)$ and $D(z_4)$ be the vertices of a trapezium in an argand plane. Let $|z_1 - z_2| = 4$, $|z_3 - z_4| = 10$ and the diagonals AC and BD intersect at P. It is given that $\arg\left(\frac{z_4 - z_2}{z_3 - z_1}\right) = \frac{\pi}{2}$ and $\arg\left(\frac{z_3 - z_2}{z_4 - z_1}\right) = \frac{\pi}{4}$. The area of the trapezium ABCD is equal to

(a) $\frac{130}{3}$ (b) $\frac{140}{3}$ (c) $\frac{160}{3}$ (d) $\frac{190}{3}$

3. If $\alpha, \beta, \gamma,$ and δ be the eccentric angles of the four points of the intersection of the ellipse and any circle then for an integer n, the value of $\alpha + \beta + \gamma + \delta$ must be

(a) $n\pi$ (b) $(-1)^n n\pi$ (c) $(2n-1)\pi$ (d) $2n\pi$

4. A slip of paper is given to a person "A" who marks it with either a (+)ve or a (-)ve sign, the probability of his writing a (+)ve sign being $\frac{1}{3}$. "A" passes the slip to "B" who may leave it alone or change the sign before passing it to "C". Similarly, "C" passes on the slip to "D" and "D" passes on the slip to the Referee, who finds a (+)ve sign on the slip. If it is known that B, C and D each change the sign with a probability $\frac{2}{3}$, then the probability that "A" originally wrote a (+)ve sign is

(a) $\frac{13}{28}$ (b) $\frac{28}{81}$ (c) $\frac{41}{81}$ (d) $\frac{13}{41}$

Section 2 (maximum marks: 32)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme.
  Full Marks : +4 If only (all) the correct option(s) is (are) chosen;
  Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;
  Partial marks : +2 if three or more options are correct but ONLY two options are chosen and both of which are correct;
  Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : –1 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get –1 mark.

5. Let $f:[0,8]\to\mathbb{R}$ be a differentiable function such that $f(0)=0, f(4)=1, f(8)=1$ then which of the following hold(s) good.

(a) There exists some $c_1\in(0,8)$ where $f'(c_1)=\frac{1}{4}$

(b) There exists some $c\in(0,8)$ where $f'(c)=\frac{1}{12}$

(c) There exists some $c_1,c_2\in[0,8]$ where $8f'(c_1)f(c_2)=1$

(d) There exists some $\alpha,\beta\in(0,2)$ such that $\int_0^8 f(t)dt=3(\alpha^2 f(\alpha^3)+\beta^2 f(\beta^3))$

6. If [.] represents greatest integer function, |.| represents modulus function, then which of the following option(s) is/are correct.

(a) The number of solution of the equation $|\ln|x||=|\sin\pi x|$ is 10.

(b) The number of solution of the equation $\sin x=[x]$ is 3.

(c) The number of solution of the equation $x^2-3x+[x]=0$ is 2.

(d) The number of solution of the equation $[\tan^{-1}x]+[\cot^{-1}x]=2$ is 1.

7. If $P_1(x)=(x-2)^2$; $P_2(x)=((x-2)^2-2)^2$; $P_3(x)=(((x-2)^2-2)^2-2)^2$ i.e. in general $P_k(x)=(\mathrm{P}_{k-1}(x)-2)^2$ then

(a) In $P_k(x)$ the constant term is 4.

(b) In $P_k(x)$ the coefficient of x is 4^k .

(c) In $P_k(x)$ the coefficient of x is -4^k .

(d) In $P_k(x)$ the coefficient of x^2 is $\dfrac{4^{2k-1}-4^{k-1}}{3}$.

8. For the vectors $\vec{x}\,and\,\vec{y}$, given that $\vec{x}+\vec{c}\times\vec{y}=\vec{a}$ and $\vec{y}+\vec{c}\times\vec{x}=\vec{b}$ where $\vec{c}$ is a non-zero vector, then

(a) $\vec{x}=\dfrac{\vec{a}+\vec{b}\times\vec{c}+(\vec{c}.\vec{a})\vec{c}}{1+|\vec{c}|^2}$

(b) $\vec{y}=\dfrac{\vec{b}+\vec{c}\times\vec{a}+(\vec{c}.\vec{b})\vec{c}}{1+|\vec{c}|^2}$

(c) $\vec{x}=\dfrac{\vec{a}-\vec{b}\times\vec{c}+(\vec{c}.\vec{a})\vec{c}}{1+|\vec{c}|^2}$

(d) $\vec{y}=\dfrac{\vec{b}-\vec{c}\times\vec{a}+(\vec{c}.\vec{b})\vec{c}}{1+|\vec{c}|^2}$

9. The equation(s) of the plane(s) through the origin which is/are parallel to the line $\dfrac{x-1}{2}=\dfrac{y+3}{-1}=\dfrac{z+1}{-2}$ and distance $\dfrac{5}{3}$ from it is/are

(a)2x+2y+z=0 (b) x+2y+2z=0 (c) 2x-2y+z=0 (d) x-2y+2z=0

10. Let a, b and c be three distinct real numbers and $f(x)$ be a quadratic polynomial satisfying the equation $\begin{bmatrix}4a^2 & 4a & 1\\ 4b^2 & 4b & 1\\ 4c^2 & 4c & 1\end{bmatrix}\begin{bmatrix}f(-1)\\ f(1)\\ f(2)\end{bmatrix}=\begin{bmatrix}3a^2+3a\\ 3b^2+3b\\ 3c^2+3c\end{bmatrix}$. Let V be the point of local maxima of $y=f(x)$ and A be the point where $y=f(x)$ meets the x-axis and B be a point on $y=f(x)$ such that AB subtends a right angle at V, then which of the following is/are correct.

(a) The coordinate of V is (0,1).

(b) The coordinate of A is (1,0).

(c) The coordinate of B is (-8, -15).

(d) The area of region lying between the curve and the chord AB is $\frac{125}{3}$ square units.

11. Let $S_1 = x^2 - 10x + y^2 + 16 = 0$ and S_2 be the image of S_1 w.r.t. the line $x - y = 0$ and S_3 be the image of S_2 w.r.t. $y = 0$. Let S′ be the circle which cut all three circles orthogonally and S″ be the circle of minimum radius which contains all three circles then

(a) The equation of S′ is $x^2 + y^2 = 9$

(b) The equation of S″ is $x^2 + y^2 = 64$

(c) The equation of S′ is $x^2 + y^2 = 16$

(d) The equation of S″ is $x^2 + y^2 = 81$

12. A ΔABC is inscribed in the parabola $y^2 = 4x$ with A as its vertex and it's orthocentre at the focus, then

(a) distance of BC from A is 5.

(b) distance of circumcentre from A is 4.

(c) distance of from centroid A is 3.

(d) the inradius of ΔABC is 2.

SECTION 3 (Maximum Marks: 18)

- This section contains SIX (06) questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If ONLY the correct numerical value is entered;
  Zero Marks : 0 In all other cases.

13. Let $f(x)$ be a differentiable function in $[-1, \infty)$ and $f(0) = 1$ such that $\lim_{t \to x+1} \frac{t^2 f(x+1) - (x+1)^2 f(t)}{f(t) - f(x+1)} = 1$. The value of $\lim_{x \to 1} \frac{\ln(f(x)) - \ln 2}{x - 1}$ is ______.

14. The integral value of k for which the system of equations:

$$\begin{cases} \cos^{-1}x + (\sin^{-1}y)^2 = \dfrac{k\pi^2}{4} \\ (\cos^{-1}x).(\sin^{-1}y)^2 = \dfrac{\pi^4}{16} \end{cases}$$

possesses solutions is ______.

15. The set of natural numbers is divided into array of rows and columns in the form of matrices as $A_1 = [1], A_2 = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}, A_3 = \begin{bmatrix} 6 & 7 & 8 \\ 9 & 10 & 11 \\ 12 & 13 & 14 \end{bmatrix}$ and so on. Then the trace of A_{10} is ______.

(Note: trace is sum of diagonal elements)

16. If $I_n = \int_0^\infty e^{-x}(\sin x)^n \, dx$ for $n > 1$, then the value of $\dfrac{101 I_{10}}{10 I_8}$ is equal to ______.

17. Consider a set of $3n$ numbers having variance 4. In this set, the mean of first $2n$ numbers is 6 and the mean of the remaining n numbers is 3. A new set is constructed by adding 1 into each of first $2n$ numbers, and subtracting 1 from each of the remaining n numbers. If the variance of the new set is k, then $9k$ is equal to ______.

18. Let $a_1, a_2, ..., a_n$ be sequence of real numbers, with $a_{n+1} = a_n + \sqrt{1 + a_n^{\ 2}}$ and $a_0 = 0$ if $\lim_{n \to \infty} \dfrac{a_n}{2^{n-1}} = \dfrac{k}{\pi}$, then k is equal to ______.