① Solve $\dfrac{2x^2+3y^2-7}{y}dx = \dfrac{3x^2+2y^2-8}{x}dy.$

$x^2+y^2-3 = k(x^2-y^2-1)^5$

② **Let $u(x)$ and $v(x)$ satisfy the differential equations $\dfrac{du}{dx}+p(x)u = f(x)$ and $\dfrac{dv}{dx}+p(x)v = g(x)$ respectively where $p(x), f(x)$ and $g(x)$ are continuous functions. If $u(x_1) > v(x_1)$ for some x_1 and $f(x) > g(x)$ for all $x > x_1$, prove that any point (x, y), where $x > x_1$, does not satisfy the equations $y = u(x)$ and $y = v(x)$.**

③ $$y = 2x\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2.$$

$4y(y+x^2)^2 = 4x(xy+c)(y+x^2)+(xy+c)^2$

④ **A curve passing through (1, 2) has its slope at any point (x, y) equal to $\dfrac{2}{y-2}$. Find the area of the region bounded by the curve and the line $2x-y-4=0$.** ⑨

~~⑨~~ $$\frac{x\,dx - y\,dy}{x\,dy - y\,dx} = \sqrt{\frac{1+x^2-y^2}{x^2-y^2}}$$

$\sqrt{x^2-y^2}+\sqrt{1+x^2-y^2} = \dfrac{c(x+y)}{\sqrt{x^2+y^2}}$

⑯ A normal is drawn at a point $P(x, y)$ of a curve. It meets the x-axis at Q. If PQ is of constant length k, then show that the differential equation describing such curves is, $y\dfrac{dy}{dx} = \pm\sqrt{k^2-y^2}$. Find the equation of such a curve passing through $(0, k)$. $x^2+y^2=k^2$

⑲ A curve is such that the length of the polar radius of any point on the curve is equal to the length of the tangent drawn at this point. Form the differential equation and solve it to find the equation of the curve.

$y = kx$ or $xy = c$

$\frac{y^2 \pm y\sqrt{y^2-x^2}}{x^2} = \ln\left|(y \pm \sqrt{y^2-x^2})\frac{c}{x^3}\right|$ take same sign

(20) Find the curve $y = f(x)$ where $f(x) \geq 0$, $f(0) = 0$, bounding a curvilinear trapezoid with the base $[0, x]$ whose area is proportional to $(n+1)^{th}$ power of $f(x)$. It is known that $f(1) = 1$. $y = x^{1/n}$

II (2) Find the equation of a curve such that the projection of its ordinate upon the normal is equal to its abscissa.

(3) The light rays emanating from a point source situated at origin when reflected from the mirror of a search light are reflected as beam parallel to the x – axis. Show that the surface is parabolic, by first forming the differential equation and then solving it.

(7) Find the isogonal trajectories for the family of rectangular hyperbolas $x^2 - y^2 = a^2$ which makes with it an angle of 45°. $x^2-y^2+2xy=c$; $x^2-y^2-2xy=c$

(8) $$(x^3 - 3xy^2)\,dx = (y^3 - 3x^2y)\,dy$$

$y^2-x^2 = c(y^2+x^2)^2$

(9) Show that every homogeneous differential equation of the form $\frac{dy}{dx} = \frac{f(x, y)}{g(x, y)}$ where f and g are homogeneous function of the same degree can be converted into variable separable by the substitution $x = r\cos\theta$ and $y = r\sin\theta$.

(10) $$\left[x\cos\frac{y}{x} + y\sin\frac{y}{x}\right]y - \left[y\sin\frac{y}{x} - x\cos\frac{y}{x}\right]x\frac{dy}{dx} = 0$$ $xy\cos\frac{y}{x} = c$

(11) Find the curve for which any tangent intersects the y–axis at the point equidistant from the point of tangency and the origin. $x^2+y^2 = cx$

(17) $$\frac{dy}{dx} + \frac{\cos x\,(3\cos y - 7\sin x - 3)}{\sin y\,(3\sin x - 7\cos y + 7)} = 0$$

$(\cos y - \sin x - 1)^2(\cos y + \sin x - 1)^5 = c$

(19) If the normal drawn to a curve at any point P intersects the x-axis at G and the perpendicular from P on the x-axis meets at N, such that the sum of the lengths of PG and NG is proportional to the abscissa of the point P, the constant of proportionality being k. Form the differential equation and solve it to show that the equation of the curve is, $y^2 = cx^{\frac{1}{k}} - \frac{k^2x^2}{2k-1}$ or $y^2 = \frac{k^2x^2}{2k+1} - cx^{-\frac{1}{k}}$, where c is any arbitrary constant.

(20) Show that the curve such that the distance between the origin and the tangent at an arbitrary point is equal to the distance between the origin and the normal at the same point,

$\sqrt{x^2+y^2} = ce^{\pm\tan^{-1}\frac{y}{x}}$

III (5) Find the curve such that the area of the trapezium formed by the co–ordinate axes, ordinate of an arbitrary point & the tangent at this point equals half the square of its abscissa. $y = cx^2 \pm x$

(6) $$x(x-1)\frac{dy}{dx} - (x-2)y = x^3(2x-1)$$

$y(x-1) = x^2(x^2-x+c)$

(7) $$(1 + y + x^2y)\,dx + (x + x^3)\,dy = 0$$

$xy = c - \tan^{-1}x$

(8) Find the curve possessing the property that the intercept, the tangent at any point of a curve cuts off on the y–axis is equal to the square of the abscissa of the point of tangency. $y = cx - x^2$

(10) $$x(x^2 + 1)\,\frac{dy}{dx} = y\,(1 - x^2) + x^3 . \ln x$$

$4(x^2+1)y + x^3(1-2\ln x) = cx$

(12) $$(1 + y^2)\,dx = (\tan^{-1}y - x)dy$$

$x = c\,e^{-\tan^{-1}y} + \tan^{-1}y - 1$

(13) Find the curve such that the area of the rectangle constructed on the abscissa of any point and the initial ordinate of the tangent at this point is equal to a^2. (Initial ordinate means y intercept of the tangent). $y = cx \pm \frac{a^2}{2x}$

(14) Let the function $\ln f(x)$ is defined where $f(x)$ exists for $x \geq 2$ & k is fixed positive real number, prove that if $\frac{d}{dx}(x . f(x)) \leq -k f(x)$ then $f(x) \leq A x^{-1-k}$ where A is independent of x.

(15) Find the differentiable function which satisfies the equation $f(x) = -\int_0^x f(t)\tan t\,dt + \int_0^x \tan(t - x)\,dt$ where $x \in (-\pi/2, \pi/2)$

$(\cos x - 1$

(18) If y_1 & y_2 be solutions of the differential equation $\frac{dy}{dx} + Py = Q$, where P & Q are functions of x alone, and $y_2 = y_1 z$, then prove that

$$z = 1 + a\,e^{-\int \frac{Q}{y_1} dx}, \text{ 'a' being an arbitrary constant.}$$

(19) $$\frac{dy}{dx} + \frac{y}{x}\,\ln y = \frac{y}{x^2}(\ln y)^2$$

$x = \ln y\,(cx^2 + \frac{1}{2})$

(20) $$\frac{dy}{dx} + xy = y^2 e^{x^2/2} . \sin x$$

$e^{-x^2/2} = y(c + \cos x)$

(22) $$x^2 y - x^3\,\frac{dy}{dx} = y^4 \cos x$$

$x^3 y^{-3} = 3\sin x + c$

(23) $y\,(2\,xy + e^{x})\,dx - e^{x}\,dy = 0$ $\frac{1}{y}e^{x} = c - x^2$

(24) Find the curve for which the area of the triangle formed by the x–axis, the tangent line and radius vector of the point of tangency is equal to a^2.

$x = cy \pm a^2/y$

(25) A tank contains 100 litres of fresh water. A solution containing 1 gm/litre of soluble lawn fertilizer runs into the tank at the rate of 1 lit/min, and the mixture is pumped out of the tank at the rate of 3 litres/min. Find the time when the amount of fertilizer in the tank is maximum.

$27\,7/9$ min

IV (2) $(x^3 + y^2 + 2)\,dx + 2y\,dy = 0$

$y^2 = 3x^2 - 6x - x^3 + ce^{-x} + 4$

(4) $$\frac{dy}{dx} - \frac{\tan y}{1+x} = (1 + x)\,e^{x} \sec y$$

$\sin y = (e^{x} + c)(1+x)$

(7) $$\frac{dy}{dx} = \frac{y^2 - x}{2y\,(x+1)}$$

$y = -1 + (x+1)\ln\frac{c}{x+1}$

or $x + (x+1)\ln\frac{c}{x+1}$

(8) $(1 - xy + x^2y^2)\,dx = x^2\,dy$ $y = \frac{1}{x}\tan(\ln|cx|)$

(9) $$\frac{dy}{dx} = e^{x-y}(e^{x} - e^{y})$$

$e^{y} = c\,e^{(-e^{x})} + e^{x} - 1$

(10) $y\,y'\sin x = \cos x\,(\sin x - y^2)$ $y^2 = \frac{2}{3}\sin x + \frac{c}{\sin^2 x}$

I (1) $\frac{dy}{dx} - y\ln 2 = 2^{\sin x}.\,(\cos x - 1)\ln 2$, y being bounded when $x \to +\infty$.

$y = 2^{\sin x}$

$y = \frac{1}{3-e}(2e^x - e + 1)$

(2) $$\frac{dy}{dx} = y + \int_0^1 y\,dx \quad \text{given } y = 1\text{, where } x = 0$$

(3) Given two curves $y = f(x)$ passing through the points $(0, 1)$ & $y = \int_{-\infty}^{x} f(t)\,dt$ passing through the points $(0, 1/2)$. The tangents drawn to both curves at the points with equal abscissas intersect on the x – axis. Find the curve $f(x)$. $f(x) = e^{2x}$

(4) Consider the differential equation

$$\frac{dy}{dx} + P(x)y = Q(x)$$

$y = v(x) + K(u(x) - v(x))$; K is const.

(1) If two particular solutions of given equation $u(x)$ and $v(x)$ are known, find the general solution of the same equation in terms of $u(x)$ and $v(x)$.

(2) If α and β are constants such that the linear combinations $\alpha \cdot u(x) + \beta \cdot v(x)$ is a solution of the given equation, find the relation between α and β. $\alpha + \beta = 1$.

(3) If $w(x)$ is the third particular solution different from $u(x)$ and $v(x)$ then find the ratio $\frac{v(x) - u(x)}{w(x) - u(x)}$. constant

(5) $$x^3 \frac{dy}{dx} = y^3 + y^2\sqrt{y^2 - x^2} \qquad xy = c(y + \sqrt{y^2 - x^2})$$

(6) Find the curve which passes through the point $(2, 0)$ such that the segment of the tangent between the point of tangency & the y – axis has a constant length equal to 2.

$y = \pm\left(\sqrt{4-x^2} + 2\ln\frac{2-\sqrt{4-x^2}}{x}\right)$

(7) $$x\,dy + y\,dx + \frac{x\,dy - y\,dx}{x^2 + y^2} = 0 \qquad xy + \tan^{-1}\frac{y}{x} = c$$

(8) $$\frac{y\,dx - x\,dy}{(x - y)^2} = \frac{dx}{2\sqrt{1 - x^2}}, \text{ given that } y = 2 \text{ when } x = 1$$

$\frac{\sin^{-1}x}{2} + \frac{y}{x-y} = \frac{\pi}{4} - 2$

(9) Find the equation of the curve passing through the orgin if the middle point of the segment of its normal from any point of the curve to the x-axis lies on the parabola $2y^2 = x$. $y^2 = 2x + 1 - e^{2x}$

(10) Find the continuous function which satisfies the relation, $\int_0^x t\,f(x - t)\,dt = \int_0^x f(t)\,dt + \sin x + \cos x - x - 1$, for all real number x.

$f(x) = e^x - \cos x$

(1) $$(x^2 + y^2 + a^2)\, y \frac{dy}{dx} + x\,(x^2 + y^2 - a^2) = 0$$

$(x^2+y^2)^2 + 2a^2(y^2-x^2) = c$

(12) $$(1 - x^2)^2\, dy + \left(y\sqrt{1 - x^2} - x - \sqrt{1 - x^2}\right) dx = 0.$$

$y = \frac{x}{\sqrt{1-x^2}} + c\, e^{-\frac{x}{\sqrt{1-x^2}}}$

(13) $$3x^2y^2 + \cos(xy) - xy\sin(xy) + \frac{dy}{dx}\{2x^3y - x^2\sin(xy)\} = 0.$$

$x(x^2y^2 + \cos xy) = c$

(14) Find the integral curve of the differential equation, $x(1 - x \ln y)\cdot\frac{dy}{dx} + y = 0$ which passes through $\left(1, \frac{1}{e}\right)$.

$x(ey + \ln y + 1) = 1$

(15) Find all the curves possessing the following property; the segment of the tangent between the point of tangency & the x-axis is bisected at the point of intersection with the y-axis. $y^2 = cx$

(16) $$y^2(y\,dx + 2x\,dy) - x^2(2y\,dx + x\,dy) = 0$$

$x^2y^2(y^2 - x^2) = c$

(17) A perpendicular drawn from any point P of the curve on the x–axis meets the x–axis at A. Length of the perpendicular from A on the tangent line at P is equal to 'a'. If this curve cuts the y-axis orthogonally, find the equation to all possible curves, expressing the answer explicitly. $y = \pm a \cdot \frac{e^{x/a} + e^{-x/a}}{2}$; $y = \pm a$

(18) Find the orthogonal trajectories for the given family of curves when 'a' is the parameter.
(i) $y = ax^2$ $\quad$ (ii) $\cos y = a\,e^{-x}$ $\quad$ (iii) $x^k + y^k = a^k$

$x^2 + 2y^2 = c$ $\quad$ $\sin y = c e^{-x}$ $\quad$ $y = cx$ if $k = 2$; $\frac{1}{x^{k-2}} - \frac{1}{y^{k-2}} = \frac{1}{c}$ if $k \neq 2$

(19) A curve passing through (1, 0) such that the ratio of the square of the intercept cut by any tangent off the y-axis to the subnormal is equal to the ratio of the product of the co-ordinates of the point of tangency to the product of square of the slope of the tangent and the subtangent at the same point. Determine all such possible curves.

$x = e^{2\sqrt{y/x}}$; $x = e^{-2\sqrt{y/x}}$

(20) A & B are two separate reservoirs of water. Capacity of reservoir A is double the capacity of reservoir B. Both the reservoirs are filled completely with water, their inlets are closed and then the water is released simultaneously from both the reservoirs. The rate of flow of water out of each reservoir at any instant of time is proportional to the quantity of water in the reservoir at that time. One hour after the water is released, the quantity of water in reservoir A is 1.5 times the quantity of water in reservoir B. After how many hours do both the reservoirs have the same quantity of water? $T = \log_{4/3}(2)$ hr

VI (2) Solve the differential equation ; $\cos^2 x \frac{dy}{dx} - (\tan 2x)\, y = \cos^4 x$, $|x| < \frac{\pi}{4}$, when $y(\pi/6) = 3\sqrt{3}/8$.

$y = \frac{1}{2}\tan 2x \cos^2 x$

[REE '96, 6]

(3) Solve the diff. equation ; $y\cos\frac{y}{x}(x\,dy - y\,dx) + x\sin\frac{y}{x}(x\,dy + y\,dx) = 0$, when $y(1) = \frac{\pi}{2}$.

$xy\sin\frac{y}{x} = \frac{\pi}{2}$

[REE '97, 6]

(6) Solve the differential equation $(1 + \tan y)(dx - dy) + 2x\,dy = 0$

$x e^y(\cos y + \sin y) = e^y \sin y + c$

(8) Solve the differential equation, $(x^2 + 4y^2 + 4xy)\,dy = (2x + 4y + 1)\,dx$.

$y = \ln\left((x+2y)^2 + 4(x+2y) + 2\right) - \frac{3}{2\sqrt{2}}\ln\left(\frac{x+2y+2-\sqrt{2}}{x+2y+2+\sqrt{2}}\right) + c$

9. A country has a food deficit of 10 %. Its population grows continuously at a rate of 3 %. Its annual food production every year is 4 % more than that of the last year. Assuming that the average food requirement per person remains constant, prove that the country will become self-sufficient in food after 'n' years, where 'n' is the smallest integer bigger than or equal to,

$$\frac{\ln 10 - \ln 9}{\ln(1.04) - 0.03}.$$ [JEE '2000 (Mains) 10]

10. A hemispherical tank of radius 2 metres is initially full of water and has an outlet of 12 cm^2 cross sectional area at the bottom. The outlet is opened at some instant. The flow through the outlet is according to the law $V(t) = 0.6\sqrt{2gh(t)}$, where V(t) and h(t) are respectively the velocity of the flow through the outlet and the height of water level above the outlet at time t, and g is the acceleration due to gravity. Find the time it takes to empty the tank. [JEE '2001 (Mains) 10]

$\frac{7\pi \times 10^5}{135\sqrt{g}}$ sec

11. Find the equation of the curve which passes through the origin and the tangent to which at every point (x, y) has slope equal to $\frac{x^4 + 2xy - 1}{1+x^2}$. [REE '2001 (Mains) 3]

$y = (x - 2\tan^{-1}x)(1+x^2)$

12. Let f(x), $x \geq 0$, be a nonnegative continuous function, and let $F(x) = \int_0^x f(t)dt$, $x \geq 0$. If for some $c > 0$, $f(x) \leq cF(x)$ for all $x \geq 0$, then show that $f(x) = 0$ for all $x \geq 0$. [JEE 2001 (Mains) 5 out of 100]

1 12 **General solution of differential equation of $f(x)\frac{dy}{dx} = f^2(x) + f(x)y + f'(x)y$ is :**

(c being arbitary constant.)

(a) $y = f(x) + ce^x$ **(b)** $y = -f(x) + ce^x$

(c) $y = -f(x) + ce^x f(x)$ **(d)** $y = cf(x) + e^x$

The ord... (c) has an asymptote parallel to x-axis

17. The solution of the differential equation $(x^2+1)\frac{d^2y}{dx^2} = 2x\left(\frac{dy}{dx}\right)$ under the conditions $y(0) = 1$ and $y'(0) = 3$, is :

(a) $y = x^2 + 3x + 1$ (b) $y = x^3 + 3x + 1$

(c) $y = x^4 + 3x + 1$ (d) $y = 3\tan^{-1}x + x^2 + 1$

... (where c is an arbitrary constant.)

21. **Let $f(x)$ be differentiable function on the interval $(0, \infty)$ such that $f(1) = 1$ and $\lim_{t \to x}\left(\frac{t^3 f(x) - x^3 f(t)}{t^2 - x^2}\right) = \frac{1}{2}$ $\forall x > 0$, then $f(x)$ is :**

(a) $\frac{1}{4x} + \frac{3x^2}{4}$ **(b)** $\frac{3}{4x} + \frac{x^3}{4}$ **(c)** $\frac{1}{4x} + \frac{3x^3}{4}$ **(d)** $\frac{1}{4x^3} + \frac{3x}{4}$

The population $p(t)$ at time 't' of a certain mouse species satisfies ... (d) 27

27. **Let $y = f(x)$ and $\frac{x}{y}\frac{dy}{dx} = \frac{3x^2 - y}{2y - x^2}$; $f(1) = 1$ then the possible value of $\frac{1}{3}f(3)$ equals :**

(a) 9 **(b) 4** **(c) 3** **(d) 2**

Suppose f and g are differentiable functions such that $xg(f(x))f'(g(x))g'(x) = f(g(x))$ $g'(f(x))f'(x)\ \forall x \in R$ and f is positive, g is positive $\forall x \in R$. Also $\int_0^x f(g(t))\,dt = \frac{1}{2}(1-e^{-2x})$ $\forall x \in R$, $g(f(0)) = 1$ and $h(x) = \dfrac{g(f(x))}{f(g(x))}\ \forall x \in R$.

3 . The graph of $y = h(x)$ is symmetric with respect to line :
(a) $x = -1$ (b) $x = 0$ (c) $x = 1$ (d) $x = 2$

4 . The value of $f(g(0)) + g(f(0))$ is equal to :
(a) 1 (b) 2 (c) 3 (d) 4

5 . The largest possible value of $h(x)\ \forall x \in R$ is :
(a) 1 (b) $e^{1/3}$ (c) e (d) e^2

Given a function 'g' which has a derivative $g'(x)$ for every real x and which satisfy $g'(0) = 2$ and $g(x+y) = e^y g(x) + e^x g(y)$ for all x and y.

6 The function $g(x)$ is :
(a) $x(2 + xe^x)$ (b) $x(e^x + 1)$ (c) $2xe^x$ (d) $x + \ln(x+1)$

7 The range of function $g(x)$ is :
(a) R (b) $\left[-\frac{2}{e}, \infty\right)$ (c) $\left[-\frac{1}{e}, \infty\right)$ (d) $[0, \infty)$

4. Let $f(x)$ be a differentiable function in $[-1, \infty)$ and $f(0) = 1$ such that $\lim\limits_{t \to x+1} \dfrac{t^2 f(x+1) - (x+1)^2 f(t)}{f(t) - f(x+1)} = 1$. Find the value of $\lim\limits_{x \to 1} \dfrac{\ln(f(x)) - \ln 2}{x - 1}$. 1

The general solution of the differential equation, $y' + y\phi'(x) - \phi(x) . \phi'(x) = 0$ where $\phi(x)$ is a known function is :
(A*) $y = ce^{-\phi(x)} + \phi(x) - 1$ (B) $y = ce^{+\phi(x)} + \phi(x) - 1$
(C) $y = ce^{-\phi(x)} - \phi(x) + 1$ (D) $y = ce^{-\phi(x)} + \phi(x) + 1$

Water is drained from a vertical cylindrical tank by opening a valve at the base of the tank. It is known that the rate at which the water level drops is proportional to the square root of water depth y, where the constant of proportionality $k > 0$ depends on the acceleration due to gravity and the geometry of the hole. If t is measured in minutes and $k = \frac{1}{15}$ then the time to drain the tank if the water is 4 meter deep to start with is
(A) 30 min (B) 45 min (C*) 60 min (D) 80 min

The differential equation of all parabolas each of which has a latus rectum '4a' & whose axes are parallel to x-axis is :
(A) of order 1 & degree 2 (B) of order 2 & degree 3
(C*) of order 2 and degree 1 (D) of order 2 and degree 2

Number of straight lines which satisfy the differential equation $\frac{dy}{dx} + x\left(\frac{dy}{dx}\right)^2 - y = 0$ is:

(A) 1 (B*) 2 (C) 3 (D) 4

The solution of the differential equation, $x^2 \frac{dy}{dx} . \cos\frac{1}{x} - y \sin\frac{1}{x} = -1$, where $y \to -1$ as $x \to \infty$ is

(A*) $y = \sin\frac{1}{x} - \cos\frac{1}{x}$

(B) $y = \frac{x+1}{x \sin\frac{1}{x}}$

(C) $y = \cos\frac{1}{x} + \sin\frac{1}{x}$

(D) $y = \frac{x+1}{x \cos\frac{1}{x}}$

The real value of m for which the substitution, $y = u^m$ will transform the differential equation, $2x^4y\frac{dy}{dx} + y^4 = 4x^6$ into a homogeneous equation is :

(A) $m = 0$ (B) $m = 1$ (C*) $m = 3/2$ (D) no value of m

The solution of the differential equation, $2x^2y\frac{dy}{dx} = \tan(x^2y^2) - 2xy^2$ given $y(1) = \sqrt{\frac{\pi}{2}}$ is

(A*) $\sin x^2y^2 = e^{x-1}$ (B) $\sin(x^2y^2) = x$ (C) $\cos x^2y^2 + x = 0$ (D) $\sin(x^2y^2) = e.e^x$

The differential equation whose general solution is given by,
$y = (c_1 \cos(x + c_2)) - (c_3 e^{(-x+c_4)}) + (c_5 \sin x)$, where c_1, c_2, c_3, c_4, c_5 are arbitrary constants, is

(A) $\frac{d^4y}{dx^4} - \frac{d^2y}{dx^2} + y = 0$

(B*) $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$

(C) $\frac{d^5y}{dx^5} + y = 0$

(D) $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0$

If $\int_a^x t\, y(t)dt = x^2 + y(x)$ then y as a function of x is

(A*) $y = 2 - (2 + a^2)e^{\frac{x^2-a^2}{2}}$

(B) $y = 1 - (2 + a^2)e^{\frac{x^2-a^2}{2}}$

(C) $y = 2 - (1 + a^2)e^{\frac{x^2-a^2}{2}}$

(D) none

The substitution $y = z^\alpha$ transforms the differential equation $(x^2y^2 - 1)dy + 2xy^3dx = 0$ into a homogeneous differential equation for

(A*) $\alpha = -1$ (B) 0 (C) $\alpha = 1$ (D) no value of α.

A curve passing through (2, 3) and satisfying the differential equation $\int_0^x t\, y(t)dt = x^2y(x), (x > 0)$ is

(A) $x^2 + y^2 = 13$ (B) $y^2 = \frac{9}{2}x$ (C) $\frac{x^2}{8} + \frac{y^2}{18} = 1$ (D*) $xy = 6$

Solution of the differential equation

$$\left(e^{x^2}+e^{y^2}\right)y\frac{dy}{dx}+e^{x^2}(xy^2-x)=0,\ \text{is}$$

(A*) $e^{x^2}(y^2-1)+e^{y^2}=C$

(B) $e^{y^2}(x^2-1)+e^{x^2}=C$

(C) $e^{y^2}(y^2-1)+e^{x^2}=C$

(D) $e^{x^2}(y-1)+e^{y^2}=C$

A function $y=f(x)$ satisfying the differential equation $\frac{dy}{dx}\cdot\sin x - y\cos x+\frac{\sin^2 x}{x^2}=0$ is such that, $y\to 0$ as $x\to\infty$ then the statement which is correct is

(A*) $\lim_{x\to 0} f(x)=1$

(B*) $\int_0^{\pi/2} f(x)\,dx$ is less than $\frac{\pi}{2}$

(C*) $\int_0^{\pi/2} f(x)\,dx$ is greater than unity

(D) f(x) is an odd function

L215 If $y=\frac{x}{\log|cx|}$ where c is an arbitrary constant is the general solution of the differential equation $\frac{dy}{dx}=\left(\frac{y}{x}\right)+\phi\left(\frac{y}{x}\right)$ then the function $\phi\left(\frac{x}{y}\right)$ is ....

a) $\frac{x^2}{y^2}$

b) $-\frac{x^2}{y^2}$

c) $\frac{y^2}{x^2}$

d) $-\frac{y^2}{x^2}$

20. The degree of the differential equation satisfying $\sqrt{1+x^2}+\sqrt{1+y^2}=A(x\sqrt{1+y^2}+y\sqrt{1+x^2})$ 1

If the DE of all straight lines which

E.S.I L2A

29. If $x^m + y^m = cx^n$ is the solution of $(x^3-2y^3)dx +$ [redacted] dy [redacted]0, th[redacted]

a) $m-n=1$

b) m [redacted] $=5$

c) [redacted] n [redacted]

d) $2m=3n$

P1 A right cicular cone with radius R and height H contains a liquid which evaporates at a rate proportional to its surface area in contact with air (proportionality constant = k > 0). Suppose that r(t) is the radius of liquid cone at time t.

31 The time after which the cone is empty is

a) $H/2k$

b) H/k

c) $H/3k$

d) $2H/k$

The radius of water cone at $t=1$ is

a) $R[1-k/H]$

b) $R[1-H/k]$

c) $R[1+H/k]$

d) $R[1+k/H]$

32 The value of $\sum_{i=1}^{10} r(i)$ is equal to

a) $10R\left[2-\frac{k}{H}\right]$

b) $5R\left[2+\frac{11k}{H}\right]$

c) $5R\left[2-\frac{11k}{H}\right]$

d) $4R\left[2-\frac{11k}{H}\right]$

33.

31. If a curve satisfying $xy_1-4y-x^2\sqrt{y}=0$ passes through $\left(1,(\log 4)^2\right)$ then the value of $\frac{y(2)}{(\log 32)^2}=$ 4

dy

34 Let $f:[1,\infty)\to[2,\infty)$ be a differential function such that $f(1)=2$. If $6\int_1^x f(t)\,dt = 3x f(x) - x^3$ for all $x>1$, then the value of $f(2)$ is 6

11 The solution of $(1+y^2)+(x-e^{Tan^{-1}y})\frac{dy}{dx}=0$ is $2x=e^{f(y)}+kf(y)$ then $f(0)=$ 0

13 A curve $y=f(x)$ passes through (2, 0) and the slope at (x, y) as $\frac{(x+1)^2+(y-3)}{x+1}$. then $f(3)=$ 3

8 The function $f(\theta)=\frac{d}{d\theta}\int_0^\theta \frac{dx}{1-\cos\theta\cos x}$ satisfies the differential equation.

a) $\frac{df}{d\theta}+2f(\theta)\cot\theta=0$ b) $\frac{df}{d\theta}-2f(\theta)\cot\theta=0$ c) $\frac{df}{d\theta}+2f(\theta)=0$ d) $\frac{df}{d\theta}-2f(\theta)=0$

32 Let $f:[1,\infty)\to[2,\infty)$ be a differentiable function such that f(1)=2. If $6\int_1^x f(t)dt=3xf(x)-x^3$ for all $x\geq 1$, then the value of $f(2)$ is —— 6

8. 24 A wet porous substance loses its moisture in the open air at a rate proportional to the moisture content. if a sheet, made of the substance, hung in the wind loses half its moisture content during the first hour, then find the time in which it will lose 99% moisture, weather conditions remaining the same $\frac{2\ln 10\ hr}{\ln 2}$

48 A function f(x) defined on $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ with real values and has a primitive F(x) such that $f(x)+\cos x\cdot F(x)=\frac{\sin 2x}{(1+\sin x)^2}$, then f(x) is $f(x)+\cos x.F(x)=\frac{\sin 2x}{(1+\sin x)^2}$

(A) $\frac{2\cos x}{(1+\sin x)^2}+ce^{-\sin x}\cos x$ (B) $\frac{2\cos x}{(1-\sin x)^2}+ce^{-\sin x}\cos x$

(C) $\frac{-2\cos x}{(1+\sin x)^2}-ce^{-\sin x}\cos x$ (D) $\frac{-2\cos x}{(1-\sin x)^2}-ce^{\sin x}\cos x$

$xdy-y\log_e y\,dx+2yx^3\sin x\,dx+yx^4\cos x\,dx=0$

51 If $x\,dy-y\log_e y\,dx+2yx^3\sin x\,dx+y\,x^4\cos x\,dx=0$, then solution is

(A) $y=e^{x(c-x^2\sin x)}$ (B) $y=e^{x^2(c+x\sin x)}$

(C) $y=e^{x(c+x^2\sin x)}$ (D) $y=e^{x^2(c-x\sin x)}$

52 If differential equation of family of curves $y=(\sin^{-1}x)^2+A\cos^{-1}x+B$ (where A and B are arbitrary constants) is $(p-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=q$ then $(p+q)$ is equal to

(A) 1 (B) 2
(C) 3 (D) 4

53 The possible solution of the differential equation $y(y^2-2x^2)dx+x(2y^2-x^2)dy=0$ is

(A) $x^2y^2(x^2+y^2)=c$ (B) $x^2(y^2-x^2)=c$
(C) $x^2y^2(y^2-x^2)=c$ (D) none of these

$y=(\sin^{-1}x)^2+A\cos^{-1}x+B.$

$xdy-y\ln y\,dx+2yx^3\sin x\,dx+yx^4\cos x\,dx=0$

$y(y^2-2x^2)dx+x(2y^2-x^2)dy=0.$

(54) $(2xy^4e^y + 2xy^3 + y)dx + (x^2y^4e^y - x^2y^2 - 3x)dy = 0$

(21b) $(x^7y^2 + 3y)dx + (3x^8y - x)dy = 0$

(54) The solution of differential equatin $(2xy^4e^y + 2xy^3 + y)dx + (x^2y^4e^y - x^2y^2 - 3x)dy = 0$ is

(A) $x^2e^y + \frac{x^2}{y} + \frac{x}{y^3} = c$
(B) $x^2e^y - \frac{x^2}{y} + \frac{x}{y^3} = c$
(C) $x^2e^y + \frac{x^2}{y} - \frac{x}{y^3} = c$
(D) $x^2e^y - \frac{x^2}{y} - \frac{x}{y^3} = c$

(201) Solution of differential equation $\left(\frac{1}{y}\sin\frac{x}{y} - \frac{y}{x^2}\cos\frac{y}{x} + 1\right)dx + \left(\frac{1}{x}\cos\frac{y}{x} - \frac{x}{y^2}\sin\frac{x}{y} + \frac{1}{y^2}\right)dy = 0$ is

(A) $-\frac{1}{y} + \cos\frac{x}{y} = \sin\frac{y}{x} - x + c$
(B) $\frac{1}{y} + \cos\frac{x}{y} = \sin\frac{y}{x} + x + c$
(C) $-\frac{1}{y} + \cos\frac{x}{y} = -\sin\frac{y}{x} + x + c$
(D) none of these

(21b) The solution of the differential equation $(x^7y^2 + 3y)dx + (3x^8y - x)dy = 0$, is

(A) $cy^6 = x(2x^7y - 1)$
(B) $cx^6 = y(2x^7y - 1)$
(C) $cy^2 = x(2x^6y - 1)$
(D) $cx^6 = y^2(2x^7y - 1)$

(249) If the curve whose differential equation is $\left(\frac{dy}{dx}\right)^2 + (2y\cot x)\frac{dy}{dx} - y^2 = 0$, passes through $\left(\frac{\pi}{2}, 1\right)$ Then the equation of the curve is

(A) $\frac{1}{2}\operatorname{cosec}^2\frac{x}{2}$
(B) $\frac{1}{2}\sec^2\frac{x}{2}$
(C) $\frac{1}{2}\cos^2\frac{x}{2}$
(D) $\frac{1}{2}\sin^2\frac{x}{2}$

(378) If solution of equation $\frac{dy}{dx} = y + \int_0^2 y\,dx$, is $y(x)$. Then $[|y(2)|]$ is equal to ____ (where [.] greatest integer function) (given $y(0) = 1$) (4)

* Solution of the differential equation $\left(x\frac{dy}{dx} + y\right) = e^{xy - \ln x^2}\left(x\frac{dy}{dx} - y\right)$

(c)

(A) $\frac{y}{x} - e^{-xy} = c$
(B) $\frac{x}{y} + e^{-xy} = c$
(C) $\frac{y}{x} + e^{-xy} = c$
(D) $-\frac{x}{y} + e^{-xy} = c$

Let $f : \left[\frac{1}{2}, 1\right] \to R$ (the set of all real numbers) be a positive, non-constant and differentiable function such that $f'(x) < 2f(x)$ and $f\left(\frac{1}{2}\right) = 1$. Then the value of $\int_{1/2}^{1} f(x)dx$ lies in the interval

(A) $(2e - 1, 2e)$
(B) $(e - 1, 2e - 1)$
(C) $\left(\frac{e-1}{2}, e-1\right)$
(D) $\left(0, \frac{e-1}{2}\right)$

(D)

A curve passes through the point $\left(1, \frac{\pi}{6}\right)$. Let the slope of the curve at each point (x, y) be $\frac{y}{x} + \sec\left(\frac{y}{x}\right)$, x > 0. Then the equation of the curve is

(A) $\sin\left(\frac{y}{x}\right) = \log x + \frac{1}{2}$ (B) $\operatorname{cosec}\left(\frac{y}{x}\right) = \log x + 2$

(C) $\sec\left(\frac{2y}{x}\right) = \log x + 2$ (D) $\cos\left(\frac{2y}{x}\right) = \log x + \frac{1}{2}$

(A)

The value of $\int_0^1 4x^3 \left\{\frac{d^2}{dx^2}\left(1-x^2\right)^5\right\} dx$ is ______ 2

47 The function $y = f(x)$ is the solution of the differential equation $\frac{dy}{dx} + \frac{xy}{x^2-1} = \frac{x^4+2x}{\sqrt{1-x^2}}$ in (−1, 1) satisfying $f(0) = 0$. Then $\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x)dx$ is

(A) $\frac{\pi}{3} - \frac{\sqrt{3}}{2}$ (B) $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$

(C) $\frac{\pi}{6} - \frac{\sqrt{3}}{4}$ (D) $\frac{\pi}{6} - \frac{\sqrt{3}}{2}$

55 Let $y(x)$ be a solution of the differential equation $(1 + e^x)y' + ye^x = 1$. If $y(0) = 2$, then which of the following statements is (are) true ?

(A) $y(-4) = 0$ (B) $y(-2) = 0$

(C) $y(x)$ has a critical point in the interval (−1, 0) (D) $y(x)$ has no critical point in the interval (−1, 0)

56 Consider the family of all circles whose centers lie on the straight line y = x. If this family of circles is represented by the differential equation $Py'' + Qy' + 1 = 0$, where P, Q are functions of x, y and y' (here $y' = \frac{dy}{dx}$, $y'' = \frac{d^2y}{dx^2}$), then which of the following statements is (are) true ?

(A) $P = y + x$ (B) $P = y - x$

(C) $P + Q = 1 - x + y + y' + (y')^2$ (D) $P - Q = x + y - y' - (y')^2$

53 Consider the hyperbola $H : x^2 - y^2 = 1$ and a circle S with center $N(x_2, 0)$. Suppose that H and S touch each other at a point $P(x_1, y_1)$ with $x_1 > 1$ and $y_1 > 0$. The common tangent to H and S at P intersects the x-axis at point M. If (l, m) is the centroid of the triangle ΔPMN, then the correct expression(s) is(are)

(A) $\frac{dl}{dx_1} = 1 - \frac{1}{3x_1^2}$ for $x_1 > 1$ (B) $\frac{dm}{dx_1} = \frac{x_1}{3\left(\sqrt{x_1^2 - 1}\right)}$ for $x_1 > 1$

(C) $\frac{dl}{dx_1} = 1 + \frac{1}{3x_1^2}$ for $x_1 > 1$ (D) $\frac{dm}{dy_1} = \frac{1}{3}$ for $y_1 > 0$

PARAGRAPH 1

Let $F : \mathbb{R} \to \mathbb{R}$ be a thrice differentiable function. Suppose that F(1) = 0, F(3) = –4 and $F'(x) < 0$ for all $x \in (1/2, 3)$. Let $f(x) = xF(x)$ for all $x \in \mathbb{R}$.

. The correct statement(s) is(are)

(A) $f'(1) < 0$ (B) $f(2) < 0$

(C) $f'(x) \neq 0$ for any $x \in (1, 3)$ (D) $f'(x) = 0$ for some $x \in (1, 3)$

. If $\int_1^3 x^2 F'(x)dx = -12$ and $\int_1^3 x^3 F''(x)dx = 40$, then the correct expression(s) is(are)

(A) $9f'(3) + f'(1) - 32 = 0$. (B) $\int_1^3 f(x)dx = 12$

(C) $9f'(3) - f'(1) + 32 = 0$ (D) $\int_1^3 f(x)dx = -12$

Let $f:(0, \infty) \to \mathbb{R}$ be a differentiable function such that $f'(x) = 2 - \frac{f(x)}{x}$ for all $x \in (0, \infty)$ and $f(1) \neq 1$. Then

(A) $\lim_{x \to 0+} f'\left(\frac{1}{x}\right) = 1$ (B) $\lim_{x \to 0+} xf\left(\frac{1}{x}\right) = 2$

(C) $\lim_{x \to 0+} x^2 f'(x) = 0$ (D) $|f(x)| \leq 2$ for all $x \in (0, 2)$

(A)

A solution curve of the differential equation $\left(x^2 + xy + 4x + 2y + 4\right)\frac{dy}{dx} - y^2 = 0,\ x > 0$, passes through the point (1, 3). Then the solution curve

(A) intersects $y = x + 2$ exactly at one point
(B) intersects $y = x + 2$ exactly at two points
(C) intersects $y = (x + 2)^2$
(D) does NOT intersect $y = (x + 3)^2$

If $y = y(x)$ satisfies the differential equation

$$8\sqrt{x}\left(\sqrt{9+\sqrt{x}}\right)dy = \left(\sqrt{4+\sqrt{9+\sqrt{x}}}\right)^{-1} dx, \qquad x > 0$$

and $y(0) = \sqrt{7}$, then $y(256) =$

[A] 3 [B] 9

[C] 16 [D] 80

If $f : R \to R$ is a differentiable function such that $f'(x) > 2f(x)$ for all $x \in R$, and $f(0) = 1$, then

[A] $f(x)$ is increasing in $(0, \infty)$ [B] $f(x)$ is decreasing in $(0, \infty)$

[C] $f(x) > e^{2x}$ in $(0, \infty)$ [D] $f'(x) < e^{2x}$ in $(0, \infty)$

A, C

Let $f : R \to R$ and $g : R \to R$ be two non-constant differentiable functions. If

$$f'(x) = (e^{(f(x)-g(x))})\, g'(x) \text{ for all } x \in R,$$

and $f(1) = g(2) = 1$, then which of the following statement(s) is (are) TRUE ?

(A) $f(2) < 1 - \log_e 2$

(B) $f(2) > 1 - \log_e 2$

(C) $g(1) > 1 - \log_e 2$

(D) $g(1) < 1 - \log_e 2$

Let $f : [0, \infty) \to R$ be a continuous function such that

$$f(x) = 1 - 2x + \int_0^x e^{x-t} f(t)\,dt$$

for all $x \in [0, \infty)$. Then, which of the following statement(s) is (are) TRUE ?

(A) The curve $y = f(x)$ passes through the point $(1, 2)$

(B) The curve $y = f(x)$ passes through the point $(2, -1)$

(C) The area of the region $\{(x, y) \in [0, 1] \times R : f(x) \le y \le \sqrt{1-x^2}\}$ is $\frac{\pi - 2}{4}$

(D) The area of the region $\{(x, y) \in [0, 1] \times R : f(x) \le y \le \sqrt{1-x^2}\}$ is $\frac{\pi - 1}{4}$

Let $f : (0, \pi) \to R$ be a twice differentiable function such that

$$\lim_{t \to x} \frac{f(x)\sin t - f(t)\sin x}{t - x} = \sin^2 x \quad \text{for all } x \in (0, \pi)$$

If $f\left(\frac{\pi}{6}\right) = -\frac{\pi}{12}$, then which of the following statement(s) is (are) TRUE ?

(A) $f\left(\frac{\pi}{4}\right) = \frac{\pi}{4\sqrt{2}}$

(B) $f(x) < \frac{x^4}{6} - x^2$ for all $x \in (0, \pi)$

(C) There exists $\alpha \in (0, \pi)$ such that $f'(\alpha) = 0$

(D) $f''\left(\frac{\pi}{2}\right) + f\left(\frac{\pi}{2}\right) = 0$

B, C, D

Let $f : R \to R$ be a differentiable function with $f(0) = 0$. If $y = f(x)$ satisfies the differential equation

$$\frac{dy}{dx} = (2 + 5y)(5y - 2),$$

then the value of $\lim_{x \to -\infty} f(x)$ is ______ 0.4

Let $f : R \to R$ be a differentiable function with $f(0) = 1$ and satisfying the equation

$$f(x + y) = f(x)f'(y) + f'(x)f(y) \text{ for all } x, y \in R.$$

Then, the value of $\log_e(f(4))$ is ______ 2

2

Let Γ denote a curve $y = y(x)$ which is in the first quadrant and let the point $(1, 0)$ lie on it. Let the tangent to Γ at a point P intersect the y–axis at Y_P. If PY_P has length 1 for each point P on Γ, then which of the following option is/are correct?

A. $y = \log_e\left(\frac{1+\sqrt{1-x^2}}{x}\right) - \sqrt{1-x^2}$

B. $xy' + \sqrt{1-x^2} = 0$

C. $y = -\log_e\left(\frac{1+\sqrt{1-x^2}}{x}\right) + \sqrt{1-x^2}$

D. $xy' - \sqrt{1-x^2} = 0$

A, B

Let b be a nonzero real number. Suppose f L $\mathbb{R} \to \mathbb{R}$ is a differentiable function such that $f(0) = 1$. If the derivative f' of f satisfies the equation

$$f'(x) = \frac{f(x)}{b^2 + x^2}$$

for all $x \in \mathbb{R}$, then which of the following statements is/are TRUE?

(A) If $b > 0$, then f is an increasing function
(B) If $b < 0$, then f is a decreasing function
(C) $f(x) f(-x) = 1$ for all $x \in \mathbb{R}$
(D) $f(x) - f(-x) = 0$ for all $x \in \mathbb{R}$

A, C

Let $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$ be functions satisfying

$$f(x + y) = f(x) + f(y) + f(x)f(y) \text{ and } f(x) = xg(x)$$

for all $x, y \in \mathbb{R}$. If $\lim_{x\to 0} g(x) = 1$, then which of the following statements is/are TRUE?

(A) f is differentiable at every $x \in \mathbb{R}$
(B) If $g(0) = 1$, then g is differentiable at every $x \in \mathbb{R}$
(C) The derivative $f'(1)$ is equal to 1
(D) The derivative $f'(0)$ is equal to 1

A, B, D

For any real numbers α and β, let $y_{\alpha,\beta}(x)$, $x \in R$, be the solution of the differential equation

$$\frac{dy}{dx} + \alpha y = xe^{\beta x}, \quad y(1) = 1.$$

Let $S = \{y_{\alpha,\beta}(x) : \alpha, \beta \in R\}$. Then which of the following functions belong(s) the set S?

(A) $f(x) = \frac{x^2}{2}e^{-x} + \left(e - \frac{1}{2}\right)e^{-x}$

(B) $f(x) = -\frac{x^2}{2}e^{-x} + \left(e + \frac{1}{2}\right)e^{-x}$

(C) $f(x) = \frac{e^x}{2}\left(x - \frac{1}{2}\right) + \left(e - \frac{e^2}{4}\right)e^{-x}$

(D) $f(x) = \frac{e^x}{2}\left(\frac{1}{2} - x\right) + \left(e + \frac{e^2}{4}\right)e^{-x}$

If y(x) is the solution of the differential equation $xdy - (y^2 - 4y)dx = 0$ for $x > 0$, $y(1) = 2$, and the slope of the curve $y = y(x)$ is never zero, then the value of $10y(\sqrt{2})$ is 8.

02

For x ∈ R, let the function y(x) be the solution of the differential equation

$$\frac{dy}{dx}+12y=\cos\left(\frac{\pi}{12}x\right),\ y(0)=0.$$

Then, which of the following statements is/are TRUE?

(A) y(x) is an increasing function

(B) y(x) is a decreasing function

(C) There exists a real number β such that the line y = β intersects the curve y = y(x) at infinitely many points

(D) y(x) is a periodic function

Let $f:[1,\infty)\to\mathbb{R}$ be a differentiable function such that $f(1)=\frac{1}{3}$ and

$3\int_1^x f(t)dt = x\,f(x)-\frac{x^3}{3},\ x\in[1,\infty)$. Let e denote the base of the natural logarithm. Then the value of $f(e)$ is

(A) $\frac{e^2+4}{3}$ (B) $\frac{\log_e 4+e}{3}$ (C) $\frac{4e^2}{3}$ (D) $\frac{e^2-4}{3}$

For $x\in\mathbb{R}$, let $y(x)$ be a solution of the differential equation

$$(x^2-5)\frac{dy}{dx}-2xy=-2x(x^2-5)^2 \text{ such that } y(2)=7.$$

Then the maximum value of the function $y(x)$ is 16

Show that (4x + 3y + 1) dx + (3x + 2y + 1) dy = 0 represents a hyperbola having the lines x + y = 0 and 2x + y + 1 = 0 as asymptotes

Solve : $y = mx + m - m^3$ where, $m=\frac{dy}{dx}$

$27y^2=4(x+1)^3$

~~(c) $y=x+1$ (d) $x+y+1=0$~~

. Let g be a differentiable function satisfying $\int_0^x (x-t+1)g(t)\,dt = x^4+x^2$ for all $x\geq 0$. The value of $\int_0^1 \frac{12}{g'(x)+g(x)+10}dx$ is equal to :

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

. If y = $\frac{x}{\ldots}$ (where ...

21. Let I be the purchase value of an equipment and $V(t)$ be the value after it has been used for t years. The value $V(t)$ depreciates at a rate given by differential equation $\frac{dV(t)}{dt} = -k\,(T-t)$, where $k > 0$ is a constant and T is the total life in years of the equipment. Then the scrap value $V(T)$ of the equipment is :

(a) $I - \frac{kT^2}{2}$ (b) $I - \frac{k(T-t)^2}{2}$

(c) e^{-kT} (d) $T^2 - \frac{I}{k}$

22. Let $y'(x) + \frac{g'(x)}{g(x)}\, y(x) = \frac{g'(x)}{1+g^2(x)}$ where $f'(x)$ denotes $\frac{df(x)}{dx}$ and $g(x)$ is a given non-constant differentiable function an R. If $g(1) = y(1) = 1$ and $g(e) = \sqrt{(2e-1)}$ then $y(e)$ equals :

(a) $\frac{3}{2g(e)}$ (b) $\frac{1}{2g(e)}$ (c) $\frac{2}{3g(e)}$ (d) $\frac{1}{3g(e)}$

24. A continuous function $f : R \to R$ satisfy the differential equation $f(x) = (1 + x^2)\left[1 + \int_0^x \frac{f^2(t)}{1+t^2}\, dt\right]$ then the value of $f(-2)$ is :

(a) 0 (b) $\frac{17}{15}$ (c) $\frac{-17}{15}$ (d) $\frac{15}{17}$

25. Let $f(x)\,(f(x) > 0)$ be a differentiable function satisfying $f^2(x) = \int_0^x \left(f^2(t) - f^4(t) + (f'(t))^2\right) dt + 100$, where $f^2(0) = 100$, then $\lim_{x\to\infty} f(x)$ can be :

(a) 0 (b) 1 (c) $\frac{10}{9}$ (d) 10

Let f be a function defined on the interval $[0, 2\pi]$ such that $\int_0^x (f'(t) - \sin 2t)\,dt = \int_x^0 f(t)\tan t\,dt$ and $f(0) = 1$.

10 **10.** The maximum value of $f(x)$ is :

(a) $\frac{5}{4}$ (b) $\frac{9}{8}$ (c) 1 (d) $\frac{1}{4}$

11 **11.** The number of solution of $f(x) = 1$ in interval $[0, 2\pi]$ is :

(a) 2 (b) 3 (c) 4 (d) 6

Let f be a differentiable function satisfying

$$\int_0^{f(x)} f^{-1}(t)\,dt - \int_0^x (\cos t - f(t))\,dt = 0 \text{ and } f(0) = 1$$

15. The number of solution(s) of the equation $\begin{vmatrix} f(2x) & f(x) \\ \sin x & 2 \end{vmatrix} = 0$ in $(0, 2\pi)$ is :

(a) 2 (b) 3 (c) 4 (d) 5

16 The value of $\int_0^{\pi/2} f(x)\,dx$ lies in the interval :

(a) $\left(\frac{2}{\pi}, 1\right)$ (b) $\left(1, \frac{\pi}{2}\right)$ (c) $\left(\frac{3}{2}, \frac{\pi}{2}\right)$ (d) $\left(0, \frac{2}{\pi}\right)$

17. The value of $\lim_{x \to 0} \left(\left[\frac{\cos x}{f(x)}\right] + \left[\frac{\cos 2x}{f(2x)}\right] + \left[\frac{\cos 3x}{f(3x)}\right] + \ldots + \left[\frac{\cos(100x)}{f(100x)}\right] \right)$ is equal to :

[Note : where $[k]$ denotes greatest integer less than or equal to k.]

(a) 0 (b) 4950 (c) 5049 (d) 5050

38 Which of the following pair(s) is/are orthogonal?

(a) $16x^2 + y^2 = c$ and $y^{16} = kx$

(b) $y = x + ce^{-x}$ and $x + 2 = y + ke^{-y}$

(c) $y = cx^2$ and $x^2 + 2y^2 = k$

(d) $x^2 - y^2 = c$ and $xy = k$

where c and k arbitrary constant.

4(1) Let $y = f(x)$ be a curve C_1 passing through $(2, 2)$ and $\left(8, \frac{1}{2}\right)$ and satisfying a differential equation $y\left(\frac{d^2y}{dx^2}\right) = 2\left(\frac{dy}{dx}\right)^2$. Curve C_2 is the director circle of the circle $x^2 + y^2 = 2$. If the shortest distance between the curves C_1 and C_2 is $(\sqrt{p} - q)$ where $p, q \in N$, then find the value of $(p^2 - q)$. 62

6 Let $y = f(x)$ defined in $[0, 2]$ satisfies the differential equation $y^3 y'' + 1 = 0$ where $f(x) \geq 0 \ \forall x \in D_f$ and $f'(1) = 0, f(1) = 1$ then find the maximum value of $f(x)$. 1
[Note : D_f denotes the domain of the function and y'' denotes the 2^{nd} derivative of y w.r.t. x.]

$$(x^3y^3 + x^2y^2 + xy + 1)y\,dx + (x^3y^3 - x^2y^2 - xy + 1)x\,dy = 0.$$

$xy - 2\ln y - \frac{1}{xy} = -2\ln x + c$

$$\frac{dy}{dx} = \frac{(x+y)^2}{(x+2)(y-2)}.$$

$(x+2)^4\left(1 + \frac{2(y-2)}{(x+2)}\right) = c\,e^{\frac{2y-2}{x+2}}$

Show that if y_1 and y_2 be solutions of the equation $\frac{dy}{dx} + Py = Q$. where P and Q are functions of x alone, and $y_2 = y_1 z$, then $z = 1 + ae^{\int -Q/y_1\,dx}$, where a is an arbitrary constant.

Solve

$$\sec^2\theta\,d\theta + \tan\theta\,(1 - r\tan\theta)dr = 0.$$

$\cot\theta = r - 1 + ce^{r}$.

Solve $\frac{dy}{dx} = 1 - x(y-x) - x^3(y-x)^3$.

$(y-x)^{-2} = ce^{x^2} - (1+x^2)$

Solve $(y + x\sqrt{xy}\,(x + y))\,dx + (y\sqrt{xy}\,(x+y) - x)\,dy = 0.$

$\frac{x^2+y^2}{2} + 2\tan^{-1}\sqrt{\frac{x}{y}} = c$

Solve $\left[\frac{1}{x} - \frac{y^2}{(x-y)^2}\right] dx + \left[\frac{x^2}{(x-y)^2} - \frac{1}{y}\right] dy = 0.$

$\ln\left|\frac{x}{y}\right| + \frac{xy}{x-y} = c$

Note: $p = \frac{dy}{dx}$

Solve $p^2 + 2py \cot x = y^2$. Solve $y = yp^2 + 2px$.

Solve $pxy = y^2 \ln y - p^2$

$\ln y = cx + c^2$

Solve the equation $y = 2xy' + \ln y'$. $x = \frac{c}{p^2} - \frac{1}{p}$, $y = \ln p + \frac{2c}{p} - 2$

: set $y' = p$ th...

Solve the equation $y = xy' + \frac{a}{2y'}$ where a is a constant.

$y^2 = 2ax$

Solve $y = px + p - p^2$. $\sqrt{2}y + \sqrt{2y^2 + c_1} = \sqrt{2} + \ln c_2$

~~Example 9.~~ Reduce $xyp^2 - (x^2 + y^2 + 1)p + xy = 0$ to Clairaut's form and find its singular solution.

$y^2 = (x+1)^2$

Solve $\frac{d^2y}{dx^2} + a^2y = 0.$ or $y = c_1 \sin ax + c_2 \cos ax$

Solve $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} = a.$ $y = \frac{1}{2} a \ln^2|x| + c_1 \ln|x| + c_2$

Find the particular solution of the equation $xy'' + y' + x = 0$, that satisfies the conditions $y = 0$, $y' = 0$ when $x = 0$.

$y = -\frac{x^2}{4}$

4

Find the particular solution of the equation $yy'' - y'^2 = y^4$ provided that $y = 1, y' = 0$ when $x = 0$. $y = \sec x$

Let $f : R^+ \to R$ be a differentiable function where $f(x) = e - (x-1)(\ln x - 1) + \int_1^x f(x)dx$.
Find $f(x)$. $f(x) = \ln x + e^x$

Let $f : (0, \infty) \to (0, \infty)$ be a differentiable function satisfying,

$$x\int_0^x (1-t)f(t)\,dt = \int_0^x tf(t)\,dt \;\forall x \in R^+ \text{ and } f(1) = 1.$$

Determine $f(x)$. $f(x) = \frac{1}{x^3} e^{(1-\frac{1}{x})}$

If a continuous function $f(x)$ satisfies the relation, $\int_0^x t f(x-t)dt = \int_0^x f(t)dt + \sin x + \cos x - x - 1$, for all real numbers x, then find $f(x)$. $f(x) = e^x - \cos x.$

Let $f(x)$ is periodic function such that

$$\int_1^x (f(t))^3\,dt = \frac{1}{x^2}\left(\int_1^x (f(t)dt)\right)^3 \;\forall\; x \in R - \{0\}.$$ Find the function $f(x)$ if $f(1) = 1$. $\boxed{f(x) = 1}$ ±.

Find the curves orthogonal to the circles $x^2 + y^2 + 2\mu y - k^2 = 0$, where μ is the variable parameter. $x^2 + y^2 - 2\lambda x + k^2 = 0$

The concentration of the potassium in a kidney is 0.0025 mg/cm³. The kidney is placed in a vessel in which the potassium concentration is 0.0040 mg/cm³. In 2 hours, the potassium concentration in the kidney is found to be 0.0030 mg/cm³. What would be the concentration of potassium in the kidney 4 hours after it was placed in the vessel? How long does it take for the concentration to reach 0.0035 mg/cm³? Assume that the vessel is sufficiently large and that the vessel concentration a = 0.0040 mg/cm³ remains constant. t = 5.42 hrs (approx)

NLC

A body whose temperature T is initially 200° C is immersed in a liquid when temperature T_0 is constantly 100°C. If the temperature of the body is 150°C at t = 1 minute, what is its temperature at t = 2 minutes? T = 125°C

A body at an unknown temperature is placed in a room which is held at a constant temperature of 30° F. If after 10 minutes the temperature of the body is 0° F and after 20 minutes the temperature of the body is 15° F, find the unknown initial temperature. −30°F

A tank contains 100 litres brine in which 10 kg of salt are dissolved. Brine containing 2 kg salt per litre flows into the tank at 5 litre/min. If the well-stirred mixture is drawn off at 4 litre/min., find : (i) the amount of the salt in the tank at time t, and (ii) the amount of the salt in the tank at t = 10 min. 90.2 kg

Suppose that a sky diver falls from rest towards the earth and the parachute opens at an instant, call it $t = 0$, when the sky diver's speed is $v(0) = v_0 = 10.0$ m/s. Find $v(t)$ of the sky diver at any later time t. Does $v(t)$ increase indefinitely? Suppose that the weight of the man plus the equipment is $W = 712$ N, the air resistance R is proportional to v^2, say $R = bv^2$ N, where b is the constant of proportionality and depends mainly upon the parachute. We also assume that $b = 30.0\ Ns^2/m^2 = 30.0$ kg/m. $v(t) = 4.87\,\frac{1+0.345\,e^{-4.02t}}{1-0.345\,e^{-4.02t}}$.

A boat rowed with a velocity u directly across a stream of width a. If the velocity of the current is directly proportional to the product of the distances from the two banks, find the path of the boat and the distance down stream to the point where it lands. $x = \frac{k}{6u}y^2(3a-2y)$ $\quad d = \frac{ka^3}{6u}$

Solve $x^2\frac{dy}{dx} + y^2 e^{\frac{x(y-x)}{y}} = 2y(x-y)$ $\quad x(x-y) = y\ln(ce^x-1)$

The function $y(x)$ satisfies the equation $y(x) + 2x\int_0^x \frac{y(u)}{1+u^2}du = 3x^2 + 2x + 1$. Show that the substitution $z(x) = \int_0^x \frac{y(u)}{1+u^2}du$ converts the equation into a first order linear differential equation for $z(x)$ solve for $z(x)$. Hence solve the original equation for $y(x)$.

$y(x) = \frac{(1+x^2)^2 + 2x}{(1+x^2)}$

Given a function g which has a derivative $g'(x)$ for every real x and which satisfy $g'(0) = 2$ and $g(x+y) = e^y . g(x) + e^x . g(y)$ for all x and y, find $g(x)$ and determine the area bounded by the graph of the function, ordinate of its minima and the co-ordinate axes. $g(x) = 2xe^x$

$A = (2 - 4/e)$ sq. unit

A differentiable function f satisfies $(x-y)\, f(x+y) - (x+y)\, f(x-y) = 4xy(x^2 - y^2)\ \forall\ x, y \in R$, where $f(1) = 1$. Find $f(x)$. $f(x) = x^3$.

Prove the identity

$$\int_0^x e^{zx-z^2}\,dz = e^{x^2/4}\int_0^x e^{-z^2/4}\,dz$$ deriving for the function

$$I(x) = \int_0^x e^{zx-z^2}\,dz$$ a differential equation and solving it.

Let the function $ln(f(x))$ is defined where $f'(x)$ exists for $x \geq 2$ and k is fixed positive real number, prove that if $\frac{d}{dx}(x\, f(x)) \leq -k\, f(x)$ then $f(x) \leq A x^{-1+k}$ where A is independent of x.