Let $a_1, a_2, \ldots\ldots, a_n, a_{n+1}, \ldots\ldots$ be an A.P.

Let $S_1 = a_1 + a_2 + a_3 + \ldots\ldots + a_n$

$S_2 = a_{n+1} + a_{n+2} + \ldots\ldots + a_{2n}$

$S_3 = a_{2n+1} + a_{2n+2} + \ldots\ldots + a_{3n}$

....

....

Prove that the sequence $S_1, S_2, S_3, \ldots\ldots$ is an arithmetic progression whose common difference is n^2 times the common difference of the given progression.

(a) Let $a_1, a_2, a_3 \ldots\ldots a_n$ be an AP. Prove that :

$$\frac{1}{a_1 a_n} + \frac{1}{a_2 a_{n-1}} + \frac{1}{a_3 a_{n-2}} + \ldots\ldots + \frac{1}{a_n a_1} = \frac{2}{a_1 + a_n}\left[\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \ldots\ldots + \frac{1}{a_n}\right]$$

b) Show that in any arithmetic progression $a_1, a_2, a_3 \ldots\ldots$

$$a_1^2 - a_2^2 + a_3^2 - a_4^2 + \ldots\ldots + a^2_{2K-1} - a^2_{2K} = [K/(2K-1)](a_1^2 - a^2_{2K}).$$

If the roots of $10x^3 - cx^2 - 54x - 27 = 0$ are in harmonic progression, then find c & all the roots.

If the sum of m terms of an AP is equal to the sum of either the next n terms or the next p terms of the same AP prove that $(m+n)[(1/m) - (1/p)] = (m+p)[(1/m) - (1/n)]$ $(n \neq p)$

A computer solved several problems in succession. The time it took the computer to solve each successive problem was the same number of times smaller than the time it took to solve the preceding problem. How many problems were suggested to the computer if it spent 63.5 min to solve all the problems except for the first, 127 min to solve all the problems except for the last one, and 31.5 min to solve all the problems except for the first two?

A geometrical & harmonic progression have the same p^{th}, q^{th} & r^{th} terms a, b, c respectively. Show that $a(b-c)\log a + b(c-a)\log b + c(a-b)\log c = 0$.

If S_n represents the sum to n terms of a GP whose first term & common ratio are a & r respectively, then prove that $S_1 + S_3 + S_5 + \ldots\ldots + S_{2n-1} = \frac{an}{1-r} - \frac{ar(1-r^{2n})}{(1-r)^2(1+r)}$.

If there be m AP's beginning with unity whose common difference is 1, 2, 3 .... m. Show that the sum of their n^{th} terms is $(m/2)(mn - m + n + 1)$.

The sum of the squares of three distinct real numbers, which are in GP is S^2. If their sum is αS, show that $\alpha^2 \in (1/3, 1) \cup (1, 3)$.

The series of natural numbers is divided into groups (1), (2, 3, 4), (5, 6, 7, 8, 9), ... & so on. Show that the sum of the numbers in the n^{th} group is $(n-1)^3 + n^3$.

For or $0 < \phi < \pi/2$, if :

$x = \sum_{n=0}^{\infty} \cos^{2n}\phi$, $y = \sum_{n=0}^{\infty} \sin^{2n}\phi$, $z = \sum_{n=0}^{\infty} \cos^{2n}\phi \sin^{2n}\phi$ then : Prove that

(i) $xyz = xy + z$ (ii) $xyz = x + y + z$

Find the value of the sum

(a) $\sum_{r=1}^{n} \sum_{s=1}^{n} \delta_{rs} 2^r 3^s$ where δ_{rs} is zero if $r \neq s$ & δ_{rs} is one if $r = s$.

(b) $\sum_{i=1}^{n} \sum_{j=1}^{i} \sum_{k=1}^{j} 1$.

Sum the following series to n terms and to infinity :

(i) $\frac{1}{1.4.7} + \frac{1}{4.7.10} + \frac{1}{7.10.13} + \dots\dots$ (ii) $\sum_{r=1}^{n} r(r+1)(r+2)(r+3)$

(iii) $\sum_{r=1}^{n} \frac{1}{4r^2 - 1}$ (iv) $\frac{1}{4} + \frac{1.3}{4.6} + \frac{1.3.5}{4.6.8} + \dots\dots\dots$.

The harmonic mean of two numbers is 4. The airthmetic mean A & the geometric mean G satisfy the relation $2A + G^2 = 27$. Find the two numbers.

Find the nth term and the sum to n terms of the sequence :
(i) $1 + 5 + 13 + 29 + 61 + \dots\dots$ (ii) $6 + 13 + 22 + 33 + \dots\dots$

Find the sum of the first n terms of the sequence : $1 + 2\left(1+\frac{1}{n}\right) + 3\left(1+\frac{1}{n}\right)^2 + 4\left(1+\frac{1}{n}\right)^3 + \dots\dots\dots$

Given that $a^x = b^y = c^z = d^u$ & a, b, c, d are in GP, show that x, y, z, u are in HP.

Find three numbers a, b, c between 2 & 18 such that ;
(i) their sum is 25
(ii) the numbers 2, a, b are consecutive terms of an AP &
(iii) the numbers b, c, 18 are consecutive terms of a GP.

An AP & an HP have the same first term, the same last term & the same number of terms ; prove that the product of the r^{th} term from the beginning in one series & the r^{th} term from the end in the other is independent of r.

(a) The value of $x + y + z$ is 15 if a, x, y, z, b are in AP while the value of ; $(1/x)+(1/y)+(1/z)$ is 5/3 if a, x, y, z, b are in HP. Find a & b.
(b) The values of xyz is 15/2 or 18/5 according as the series a, x, y, z, b is an AP or HP. Find the values of a & b assuming them to be positive integer.

If n is a root of the equation $x^2(1 - ac) - x(a^2 + c^2) - (1 + ac) = 0$ & if n HM's are inserted between a & c, show that the difference between the first & the last mean is equal to $ac(a - c)$.

$x = 1 + 3a + 6a^2 + 10a^3 + \dots\dots \ |a| < 1$
$y = 1 + 4b + 10b^2 + 20b^3 + \dots\dots \ |b| < 1$, find $S = 1 + 3ab + 5(ab)^2 + \dots\dots$ in terms of x & y.

The real numbers x_1, x_2, x_3 satisfying the equation $x^3 - x^2 + \beta x + \gamma = 0$ are in A.P. Find the intervals in which β and γ lie.

For any odd integer $n \geq 1$, $n^3 - (n-1)^3 + \dots\dots + (-1)^{n-1} 1^3 =$ ______.

a, b, c are the first three terms of a geometric series. If the harmonic mean of a & b is 12 and that of b & c is 36, find the first five terms of the series.

Select the correct alternative(s).

Let T_r be the r^{th} term of an AP, for r = 1, 2, 3, ... If for some positive integers m, n we have $T_m = \frac{1}{n}$ & $T_n = \frac{1}{m}$, then T_{mn} equals :

(A) $\frac{1}{mn}$ (B) $\frac{1}{m} + \frac{1}{n}$ (C) 1 (D) 0

Given a three digit number whose digits are three successive terms of a G.P. If we subtract 792 from it, we get a number written by the same digits in the reverse order . Now if we subtract four from the hundred's digit of the initial number and leave the other digits unchanged, we get a number whose digits are successive terms of an A.P. Find the number.

Sum to n terms : (i) $\frac{1}{x+1} + \frac{2x}{(x+1)(x+2)} + \frac{3x^2}{(x+1)(x+2)(x+3)} + \dots\dots$

(ii) $\frac{a_1}{1+a_1} + \frac{a_2}{(1+a_1)(1+a_2)} + \frac{a_3}{(1+a_1)(1+a_2)(1+a_3)} + \dots\dots$

If a , b , c be in GP & $\log_c a$, $\log_b c$, $\log_a b$ be in AP , then show that the common difference of the AP must be 3/2.

If $a_1 = 1$ & for $n > 1$, $a_n = a_{n-1} + \frac{1}{a_{n-1}}$, then show that $12 < a_{75} < 15$.

In a GP the ratio of the sum of the first eleven terms to the sum of the last eleven terms is 1/8 and the ratio of the sum of all the terms without the first nine to the sum of all the terms without the last nine is 2. Find the number of terms in the GP.

Let p & q be roots of the equation $x^2 - 2x + A = 0$, and let r & s be the roots of the equation $x^2 - 18x + B = 0$. If $p < q < r < s$ are in arithmatic progression, then A = ______, and B = ______.

If there are n quantities in GP with common ratio r & S_m denotes the sum of the first m terms, show that the sum of the products of these m terms taken two & two together is $[r/(r+1)]\,[S_m]\,[S_{m-1}]$.

Find the condition that the roots of the equation $x^3 - px^2 + qx - r = 0$ may be in A.P. and hence solve the equation $x^3 - 12x^2 + 39x - 28 = 0$.

If $ax^2 + 2bx + c = 0$ & $a_1x^2 + 2b_1x + c_1 = 0$ have a common root & a/a_1 , b/b_1 , c/c_1 are in AP, show that a_1 , b_1 & c_1 are in GP.

If a, b, c, d, e be 5 numbers such that a, b, c are in AP ; b, c, d are in GP & c, d, e are in HP then:

Prove that a, c, e are in GP .

Prove that $e = (2b - a)^2/a$.

If a = 2 & e = 18 , find all possible values of b , c , d .

Prove that the sum of the infinite series $\frac{1.3}{2} + \frac{3.5}{2^2} + \frac{5.7}{2^3} + \frac{7.9}{2^4} + \dots\dots\dots \infty = 23$.

(a) The interior angles of a polygon are in AP. The smallest angle is $120°$ & the common difference is $5°$. Find the number of sides of the polygon.

b) The interior angles of a convex polygon form an arithmetic progression with a common difference of $4°$. Determine the number of sides of the polygon if its largest interior angle is $172°$.

There are n AM's between 1 & 31 such that 7th mean : $(n-1)^{th}$ mean $= 5 : 9$, then find the value of n.

Find the sum of the series , $7 + 77 + 777 + \ldots$ to n terms.

Express the recurring decimal $0.1\overline{576}$ as a rational number using concept of infinite geometric series.

Find the sum of the n terms of the sequence $\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}+\ldots\ldots$

If the p^{th}, q^{th} & r^{th} terms of an AP are in GP . Show that the common ratio of the GP is $\frac{q-r}{p-q}$.

If one AM 'a' & two GM's p & q be inserted between any two given numbers then show that $p^3 + q^3 = 2\,apq$.

The sum of n terms of two arithmetic series are in the ratio of $(7n+1):(4n+27)$. Find the ratio of their n^{th} term.

If S be the sum , P the product & R the sum of the reciprocals of a GP , find the value of $P^2\left(\frac{R}{S}\right)^n$.

The first and last terms of an A.P. are a and b. There are altogether $(2n + 1)$ terms. A new series is formed by multiplying each of the first 2n terms by the next term. Show that the sum of the new series is $\frac{(4n^2-1)(a^2+b^2)+(4n^2+2)ab}{6n}$.

In an AP of which 'a' is the Ist term, if the sum of the Ist p terms is equal to zero , show that the sum of the next q terms is $-a(p+q)q/(p-1)$.

The sequence $a_1, a_2, a_3, \ldots\ldots a_{98}$ satisfies the relation $a_{n+1} = a_n + 1$ for $n = 1, 2, 3, \ldots\ldots 97$ and has the sum equal to 4949. Evaluate $\sum_{k=1}^{49} a_{2k}$.

An AP , a GP & a HP have 'a' & 'b' for their first two terms . Show that their $(n+2)^{th}$ terms will be in GP if $\frac{b^{2n+2}-a^{2n+2}}{ba\left(b^{2n}-a^{2n}\right)}=\frac{n+1}{n}$.

If $x = 1, y > 1, z > 1$ are in GP, then $\frac{1}{1+\ln x}, \frac{1}{1+\ln y}, \frac{1}{1+\ln z}$ are in :

(A) AP (B) HP (C) GP (D) none of the above

Prove that a triangle ABC is equilateral if & only if $\tan A + \tan B + \tan C = 3\sqrt{3}$.

The harmonic mean of the roots of the equation $\left(5+\sqrt{2}\right)x^2 - \left(4+\sqrt{5}\right)x + 8 + 2\sqrt{5} = 0$ is

(A) 2 (B) 4 (C) 6 (D) 8

Let $a_1, a_2, \ldots, a_{10}$, be in A.P. & $h_1, h_2, \ldots, h_{10}$ be in H.P. If $a_1 = h_1 = 2$ & $a_{10} = h_{10} = 3$ then $a_4 h_7$ is:
(A) 2 (B) 3 (C) 5 (D) 6

The sum of an infinite geometric series is 162 and the sum of its first n terms is 160. If the inverse of its common ratio is an integer, find all possible values of the common ratio, n and the first terms of the series.

Consider an infinite geometric series with first term 'a' and common ratio r . If the sum is 4 and the second term is 3/4, then :

(A) $a = \frac{7}{4}$, $r = \frac{3}{7}$ (B) $a = 2$, $r = \frac{3}{8}$

(C) $a = \frac{3}{2}$, $r = \frac{1}{2}$ (D) $a = 3$, $r = \frac{1}{4}$

If a, b, c, d are positive real numbers such that $a + b + c + d = 2$, then $M = (a + b)(c + d)$ satisfies the relation :
(A) $0 \le M \le 1$ (B) $1 \le M \le 2$
(C) $2 \le M \le 3$ (D) $3 \le M \le 4$

The fourth power of the common difference of an arithmetic progression with integer entries added to the product of any four consecutive terms of it . Prove that the resulting sum is the square of an integer.

Given that α, γ are roots of the equation, $Ax^2 - 4x + 1 = 0$ and β, δ the roots of the equation, $Bx^2 - 6x + 1 = 0$, find values of A and B, such that α, β, γ & δ are in H.P.

The sum of roots of the equation $ax^2 + bx + c = 0$ is equal to the sum of squares of their reciprocals. Find whether bc^2, ca^2 and ab^2 in A.P., G.P. or H.P.?

Solve the following equations for x and y

$$\log_2 x + \log_4 x + \log_{16} x + \ldots\ldots = y$$

$$\frac{5 + 9 + 13 + \ldots\ldots + (4y + 1)}{1 + 3 + 5 + \ldots\ldots + (2y - 1)} = 4\log_4 x$$

Let α, β be the roots of $x^2 - x + p = 0$ and γ, δ be the roots of $x^2 - 4x + q = 0$. If $\alpha, \beta, \gamma, \delta$ are in G.P. then the integral values of p and q respectively, are
(A) −2, −32 (B) −2, 3 (C) −6, 3 (D) −6, −32

If the sum of the first 2n terms of the A.P. 2, 5, 8, ... is equal to the sum of the first n terms of the A.P. 57, 59, 61, ..., then n equals
(A) 10 (B) 12 (C) 11 (D) 13

Let the positive numbers a, b, c, d be in A.P. Then abc, abd, acd, bcd are
(A) NOT in A.P./G.P./H.P. (B) in A.P.
(C) in G.P. (D) H.P.

Let a_1, a_2 .... be positive real numbers in G.P. For each n, let A_n, G_n, H_n, be respectively, the arithmetic mean, geometric mean and harmonic mean of $a_1, a_2, a_3, \ldots\ldots a_n$. Find an expression for the G.M. of $G_1, G_2, \ldots\ldots G_n$ in terms of $A_1, A_2 \ldots\ldots A_n, H_1, H_2, \ldots\ldots H_n$.

[JEE 2004 (Mains), 5]

a) Suppose a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. If $a < b < c$ and $a + b + c = \frac{3}{2}$, then the value of a is

(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{2\sqrt{3}}$ (C) $\frac{1}{2} - \frac{1}{\sqrt{3}}$ (D) $\frac{1}{2} - \frac{1}{\sqrt{2}}$

Let a, b be positive real numbers. If a , A_1 , A_2 , b are in A.P. ; a , a_1 , a_2 , b are in G.P. and a , H_1 , H_2 , b are in H.P. , show that

$$\frac{G_1 G_2}{H_1 H_2} = \frac{A_1 + A_2}{H_1 + H_2} = \frac{(2a+b)(a+2b)}{9ab}$$

If a, b, c are in A.P., a^2 , b^2 , c^2 are in H.P. , then prove that either $a = b = c$ or a, b, $-\frac{c}{2}$ form a G.P.

The first term of an infinite geometric progression is x and its sum is 5. Then

(A) $0 \le x \le 10$ (B) $0 < x < 10$ (C) $-10 < x < 0$ (D) $x > 10$

If a, b, c are positive real numbers, then prove that $[(1 + a)(1 + b)(1 + c)]^7 > 7^7 a^4 b^4 c^4$.

In the quadratic equation $ax^2 + bx + c = 0$, if $\Delta = b^2 - 4ac$ and $\alpha + \beta$, $\alpha^2 + \beta^2$, $\alpha^3 + \beta^3$ are in GP. where α, β are the roots of $ax^2 + bx + c = 0$, then

(A) $\Delta \neq 0$ (B) $b\Delta = 0$ (C) $c\Delta = 0$ (D) $\Delta = 0$

If total number of runs scored in n matches is $\left(\frac{n+1}{4}\right)(2^{n+1} - n - 2)$ where $n > 1$, and the runs scored in the k^{th} match are given by $k \cdot 2^{n+1-k}$, where $1 \le k \le n$. Find n. [JEE 2005 (Mains), 2]

) Let T_r be the r^{th} term of an A.P whose first term is $-\frac{1}{2}$ and common difference is 1, then

$\sum_{r=1}^{n} \sqrt{1 + T_r T_{r+1} T_{r+2} T_{r+3}} =$

(a) $\frac{n(n+1)(2n+1)}{6} - \frac{5n}{4}$

(b) $\frac{n(n+1)(2n+1)}{6} - \frac{5n}{4} + \frac{1}{4}$

(c) $\frac{n(n+1)(2n+1)}{6} - \frac{5n}{4} + \frac{1}{2}$

(d) $\frac{n(n+1)(2n+1)}{12} - \frac{5n}{8} + 1$

The coefficient of x^8 in the polynomial $(x-1)(x-2)(x-3)\ldots\ldots(x-10)$ is :

(a) 2640 (b) 1320 (c) 1370 (d) 2740

Find the value of $\frac{2}{1^3} + \frac{6}{1^3 + 2^3} + \frac{12}{1^3 + 2^3 + 3^3} + \frac{20}{1^3 + 2^3 + 3^3 + 4^3} + \ldots$ upto ∞ terms :

(a) 2 (b) $\frac{1}{2}$ (c) 4 (d) $\frac{1}{4}$

Sum of first 10 terms of the series, $S = \frac{7}{2^2 \cdot 5^2} + \frac{13}{5^2 \cdot 8^2} + \frac{19}{8^2 \cdot 11^2} + \ldots\ldots$ is :

(a) $\frac{255}{1024}$ (b) $\frac{88}{1024}$ (c) $\frac{264}{1024}$ (d) $\frac{85}{1024}$

1024 1024 (d) 85/1024

$\sum_{r=1}^{10} \frac{r}{1-3r^2+r^4} =$

(a) $-\frac{50}{109}$ (b) $-\frac{54}{109}$ (c) $-\frac{55}{111}$ (d) $-\frac{55}{109}$

Let r^{th} term t_r of a series is given by $t_r = \frac{r}{\ldots}$

The value of $\sum_{n=3}^{\infty} \frac{1}{n^5 - 5n^3 + 4n} =$

(a) $\frac{1}{120}$ (b) $\frac{1}{96}$ (c) $\frac{1}{24}$ (d) $\frac{1}{144}$

Find the value of $\frac{2}{1^3} + \frac{6}{1^3+2^3} + \frac{12}{1^3+2^3+3^3} + \frac{20}{1^3+2^3+3^3+4^3} + \ldots\ldots$ up to infinite terms:

(a) 2 (b) $\frac{1}{2}$ (c) 4 (d) $\frac{1}{4}$

The value of $\sum_{r=1}^{\infty} \frac{(4r+5)5^{-r}}{r(5r+5)}$ is :

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{1}{25}$ (d) $\frac{2}{25}$

If a, b, c are in 3 distinct numbers in H.P., $a, b, c > 0$, then :

(a) $\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ are in A.P.

(b) $\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in A.P.

(c) $a^5 + c^5 \geq 2b^5$

(d) $\frac{a-b}{b-c} = \frac{a}{c}$

Given a, b, c are in A.P., b, c, d are in G.P. and c, d, e are in H.P. If $a = 2$ and $e = 18$, then the possible value of 'c' can be :

(a) 9 (b) -6 (c) 6 (d) -9

If $(x^2+x+1)+(x^2+2x+3)+(x^2+3x+5)+\ldots\ldots+(x^2+20x+39) = 4500$, then x is equal to :

(a) 10 (b) -10 (c) 20.5 (d) -20.5

For ΔABC, if $81 + 144a^4 + 16b^4 + 9c^4 = 144abc$, (where notations have their usual meaning), then :

(a) $a > b > c$

(b) $A < B < C$

(c) Area of $\Delta ABC = \frac{3\sqrt{3}}{8}$

(d) Triangle ABC is right angled

If $S_r = \sqrt{r + \sqrt{r + \sqrt{r + \sqrt{r + \sqrt{\ldots\infty}}}}}$, $r > 0$, then which of the following is/are correct.

(a) S_2, S_6, S_{12}, S_{20} are in A.P.

(b) S_4, S_9, S_{16} are irrational

(c) $(2S_3 - 1)^2, (2S_4 - 1)^2, (2S_5 - 1)^2$ are in A.P.

(d) S_2, S_{12}, S_{56} are in G.P.

Consider the A.P. 50, 48, 46, 44, ...

Given the sequence of number $a_1, a_2, a_3, \ldots\ldots, a_{1005}$ which satisfy $\frac{a_1}{a_1 + 1} = \frac{a_2}{a_2 + 3} = \frac{a_3}{a_3 + 5} = \ldots\ldots = \frac{a_{1005}}{a_{1005} + 2009}$

Also $a_1 + a_2 + a_3 + \ldots\ldots + a_{1005} = 2010$

Nature of the sequence is :

(a) A.P. (b) G.P. (c) A.G.P. (d) H.P.

21st term of the sequence is equal to :

(a) $\frac{86}{1005}$ (b) $\frac{83}{1005}$ (c) $\frac{82}{1005}$ (d) $\frac{79}{1005}$

Let $f(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots\ldots + \frac{1}{n}$ such that $P(n)\,f(n+2) = P(n)\,f(n) + q(n)$. Where $P(n), Q(n)$ are polynomials of least possible degree and $P(n)$ has leading coefficient unity. Then match the following Column-I with Column-II.

	Column-I		Column-II
(A)	$\sum_{n=1}^{m} \frac{p(n) - 2}{n}$	(P)	$\frac{m(m+1)}{2}$
(B)	$\sum_{n=1}^{m} \frac{q(n) - 3}{2}$	(Q)	$\frac{5m(m+7)}{2}$
(C)	$\sum_{n=1}^{m} \frac{p(n) + q^2(n) - 11}{n}$	(R)	$\frac{3m(m+7)}{2}$
(D)	$\sum_{n=1}^{m} \frac{q^2(n) - p(n) - 7}{n}$	(S)	$\frac{m(m+7)}{2}$

If $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{r+2}{2^{r+1} r(r+1)} = \frac{1}{k}$, then $k =$

If $\sqrt{\underbrace{(1111......1)}_{2n\text{ times}}-\underbrace{(222......2)}_{n\text{ times}}}=\underbrace{PPP......P}_{n\text{ times}}$ then $P=$

. Let $a_1, a_2, a_3, ..., a_n$ be real numbers in arithmatic progression such that $a_1 = 15$ and a_2 is an integer. Given $\sum_{r=1}^{10}(a_r)^2 = 1185$. If $S_n = \sum_{r=1}^{n} a_r$ and maximum value of n is N for which $S_n \geq S_{(n-1)}$, then find $N - 10$.

. Find the sum of series $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}+\frac{1}{9}+\frac{1}{12}+......\infty$, where the terms are the reciprocals of the positive integers whose only prime factors are two's and three's :

Paragraph for Question Nos. 11 to 13

The sequence $\{a_n\}$ is defined by formula $a_0 = 4$ and $a_{n+1} = a_n^2 - 2a_n + 2$ for $n \geq 0$. Let the sequence $\{b_n\}$ is defined by formula $b_0 = \frac{1}{2}$ and $b_n = \frac{2a_0 a_1 a_2 ...... a_{n-1}}{a_n}\ \forall\ n \geq 1$.

11. The value of a_{10} is equal to :
(a) $1+2^{1024}$ (b) 4^{1024} (c) $1+3^{1024}$ (d) 6^{1024}

12. The value of n for which $b_n = \frac{3280}{3281}$ is :
(a) 2 (b) 3 (c) 4 (d) 5

13. The sequence $\{b_n\}$ satisfies the recurrence formula :
(a) $b_{n+1} = \frac{2b_n}{1-b_n^2}$ (b) $b_{n+1} = \frac{2b_n}{1+b_n^2}$
(c) $\frac{b_n}{1+2b_n^2}$ (d) $\frac{b_n}{1-2b_n^2}$

If x, y, z are real and $4x^2+9y^2+16z^2-6xy-12yz-8zx=0$, then x, y, z are in
1) A.P. *2) G.P.* *3) H.P.* *4) A.G.P.*

If a_1, a_2, a_3 - - - are in A.P. with common difference d, $b_k = a_k + a_{k+1} . . . + a_{k+n-1}$ $(k=1, 2, 3 . . .$

a) $\sum_{k=1}^{n} b_k = n^2 a_n$
b) $\sum_{k=1}^{n} b_k = (n+1)^2 a_n$
c) $b_k = \frac{n}{2}\left[a_n + a_1 + 2d(k-1)\right]$
d) $\sum_{k=1}^{n} b_k = n(n+1)a_n$

Three numbers in A.P. with common difference 'd' are removed from first n natural numbers and average of remaining number is found to be $\frac{43}{4}$ then ordered pair (n, d) can be

a) (19, 5) b) (19, 2) c) (23, 5) d) (19, 8)

If $\sum_{j=1}^{21} a_j = 693$, where $a_1, a_2, \ldots, a_{21}$ are in A.P. and $\sum_{i=0}^{10} a_{2i+1} = 5k + l$ then $\frac{k+l}{25} =$

Define n_m for n and m positive integers as $n_m = n(n-m)(n2m)\ldots..(n-km)$ where k is the greatest integer for which $n > km$ then $\frac{72_8}{18_2}$ is

a) 4^9 b) 4^{18} c) 256 d) 36

$n > 20; a = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\ldots + \frac{1}{2n-1} - \frac{1}{2n}$, $b = \frac{1}{n+1} + \frac{1}{n+2} + \ldots. + \frac{1}{2n}$ then $\frac{19a+8b}{2a+b} =$

a) 21 b) 4 c) 9 d) 36

Let $a_1 \neq 0$ and for $n > 1$ $a_n = \frac{n}{a_{n-1}}$ then $a_1 . a_2 . a_3 \ldots\ldots . a_{10}$ is ......

Given that $x + y + z = 15$ when a, x, y, z, b are in A. P. and $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{5}{3}$ when a, x, y, z, b are in H. P. Then

a) G. M. of a and b is 3

b) one possible value of $(a + 2b)$ is 11

c) A. M. of a and b is 5

d) H. M. of a and b is $\frac{9}{5}$

If r^{th} term of a series can be written as $a_r = f(r) - f(r-1)$, then $S_n = \sum_{r=1}^{n} a_r = f(n) - f(0)$ and $S_\infty = \lim_{n \to \infty} S_n$, then

Value of $\sum_{r=1}^{\infty} \frac{(4r-1)5^r}{r^2+r}$ is

a) 5 b) – 5 c) 10 d) none of these

Value of $(\cos 1 - 1)\sum_{1}^{50} \sin n + \sin 1 \sum_{1}^{50} \cos n$ is equal to

a) sin50° b) sin50° – sin1° c) sin51° – sin 1° d) $\frac{1}{2}$

If 3(3!) + 4(4!) + 5(5!) + ...+ 50 terms = $a! - b$ then $a - b$ is equal to

a) 45 b) 46 c) 47 d) 48

Balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row two balls and so on. If 669 more ball are added, then all the balls can be arranged in the shape of a square and each of its sides then contains 8 balls less than each side of the triangle did.

COLUMN - I	COLUMN - II
a) The initial number of balls are	p) 55
b) The number of rows forming the equilateral triangle	q) 1540
c) The number of balls in the side of equilateral triangle	r) 20
d) The number of balls in the side of the square	s) 47

Given a, b, c are positive integers forming an increasing G.P., $b - a$ is a prefect square of a natur number, and $\log_6 a + \log_6 b + \log_6 c = 6$. Find the value of $(a + b + c - 105)$

If a_1, a_2, a_3 ...... a_n are in H.P and $f(k) = \left(\sum_{r=1}^{n} a_r\right) - a_k$ then $\frac{a_1}{f(1)}, \frac{a_2}{f(2)}, \frac{a_3}{f(3)}, \ldots, \frac{a_n}{f(n)}$ are in

a) A.P b) G.P c) H.P d) A.G.P

If a, b, c are in H.P then

a) $\frac{a}{b+c-a}, \frac{b}{c+a-b}, \frac{c}{a+b-c}$ are in H.P

b) $\frac{2}{b} = \frac{1}{b-a} + \frac{1}{b-c}$

c) $a - \frac{b}{2}, \frac{b}{2}, c - \frac{b}{2}$ are in G.P

d) $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in H.P

Linked comprehension type ...

Four different integers form an increasing A.P. One of these numbers is equal to the sum of the squares of the other three numbers. Then

. The product of all number is

a) –2 b) 1 c) 0 d) 2

. The sum of all the four numbers is

a) 3 b) 0 c) 4 d) 2

). The common difference of the four numbers is

a) 1 b) 3 c) 2 d) –2

ssage - III

The numbers a,b, and c are between 2 and 18, such that

(i) their sum is 25

(ii) the numbers 2,a, and b are consecutive terms of an A.P.

(iii) the number b,c, 18 are consecutive terms of a G.P.

. The value of abc is

a) 500 b) 450 c) 720 d) 480

. Roots of the equation $ax^2 + bx + c = 0$ are

a) real and positive

b) real and negative

c) imaginary

d) real and of opposite sign

. If a,b and c are roots of the equation $x^3 + qx^2 + rx + s = 0$, then the value of r is

a) 184 b) 196 c) 224 d) none of these

Natural numbers are divided into sets {1}, {2, 3}, {4, 5, 6}, {7, 8, 9, 10} ....
Now answer the following :

Sum of elements in 50th set

a) 61064 b) 62525 c) 57625 d) 60525

Last element in 2007th set

a) 2015028 b) 2107028 c) 2510208 d) 2370128

Average of middle two elements of 2008th set is λ. Then $[\lambda]$ .... where [.] is G.I.F

a) 2106132 b) 2016012 c) 2016032 d) 2160322

If $S_{(n)} = 1+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{n}, (n \in N)$, then $S_{(1)}+S_{(2)}+.......+S_{(n-1)}$ is equal to

a) $nS_{(n)}-n$ b) $nS_{(n)}-1$ c) $(n-1)S_{(n-1)}-n$ d) $nS_{(n-1)}-n+1$

) If the equation $x^4-4x^3+ax^2+bx+1=0$, has four positive roots, then $a =$ ________

Let $a = \frac{1^2}{1}+\frac{2^2}{3}+\frac{3^2}{5}+---+\frac{1001^2}{2001}$, $b=\frac{1^2}{3}+\frac{2^2}{5}+\frac{3^2}{7}+---+\frac{1001^2}{2003}$

a) $[a-b]=500$ where $[x]$ is greatest integer less than or equal to x

b) integer closest to $a-b$ is 501

c) $a-b = 500.74$

d) $a-b = 500.94$

$T_n = \sum_{r=2n}^{3'n-1} \frac{m}{r^2+n^2}, S_n = \sum_{r=2n+1}^{3n} \frac{m}{r^2+n^2}$, then $\forall n \in \{1,2,3,.....\}$

a) $T_n > \frac{1}{2}\ln 2$ b) $S_n < \frac{1}{2}\ln 2$ c) $T_n < \frac{1}{2}\ln 2$ d) $S_n > \frac{1}{2}\ln 2$

Let $a_1, a_2, ..., a_n$ be sequence of real numbers, with $a_{n+1} = a_n + \sqrt{1+a_n^2}$ and $a_0 = 0$ if $\lim_{n\to\infty} \frac{a_n}{2^{n-1}}$ is $\frac{k}{\pi}$, then k is equal to ________

There is a series of terms such that $T_1 = 1$, $T_2 = 5 + 5^2$, $T_3 = 5^3 + 5^4 + 5^5$ and so on. Find T_{10}.

1) $\frac{5^{54}(5^{10}-1)}{4}$ 2) $\frac{5^{45}(5^{9}-1)}{4}$ 3) $\frac{5^{45}(5^{10}-1)}{4}$ 4) $\frac{5^{44}(5^{10}-1)}{4}$

* The infinite sum $1 + \frac{4}{7}+\frac{9}{7^2}+\frac{16}{7^3}+\frac{25}{7^4} ...$ equals

1) $\frac{27}{14}$ 2) $\frac{21}{13}$ 3) $\frac{49}{27}$ 4) $\frac{256}{147}$

Sum of the series: $1 + 2 + \frac{7}{4} + \frac{5}{4} + \frac{13}{16} + \frac{16}{32} + \frac{19}{64} + ...$ is:

1) 8 2) 10 3) 9 4) ∞

The k^{th} term of an AP is given by formula $T_k = 2016 - 23k$. Find the smallest value of n for which S_n, the sum of first n terms, is negative.

A child was asked to add the first few natural numbers (i.e. 1 + 2 + 3 + ...) so long as his patience permitted. When he stopped, he gave the sum as 575. When the teacher declared the result wrong, the child discovered that he had missed one number in the sequence during addition. The number he missed was

1) 10 2) 18 3) 20 4) None of the above

The inverse of the sum of the following series up to n terms $\frac{3}{4}+\frac{5}{36}+\frac{7}{144}+....$ can be written as

1) $\frac{(n-1)^2}{n^2+2n}$ 2) $\frac{n^2+2n}{(n-1)^2}$ 3) $\frac{n^2+2n}{(n+1)^2}$ 4) $\frac{(n+1)^2}{n^2+2n}$

The value of the sum $\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\frac{k}{2^{n+k}}$ is equal to :

(a) 5 **(b)** 4 **(c)** 3 **(d)** 2

) Find the sum of the infinite series $\frac{1}{9}+\frac{1}{18}+\frac{1}{30}+\frac{1}{45}+\frac{1}{63}+......$

(a) $\frac{1}{3}$ **(b)** $\frac{1}{4}$ **(c)** $\frac{1}{5}$

If $S_1, S_2, S_3, ..., S_n$ are the sum of infinite geometric series whose first terms are $1, 3, 5, ..., (2n-1)$ and whose common ratios are $\frac{2}{3}, \frac{2}{5}, ..., \frac{2}{2n+1}$ respectively, then

$$\left\{\frac{1}{S_1S_2S_3}+\frac{1}{S_2S_3S_4}+\frac{1}{S_3S_4S_5}+\text{...... upto infinite terms}\right\}=$$

(a) $\frac{1}{15}$ (b) $\frac{1}{60}$ (c) $\frac{1}{12}$ (d) $\frac{1}{3}$

If $a_1, a_2, a_3, ..., a_{2n+1}$ are in AP then

$$\frac{a_{2n+1}-a_1}{a_{2n+1}+a_1}+\frac{a_{2n}-a_2}{a_{2n}+a_2}+....+\frac{a_{n+2}-a_n}{a_{n+2}+a_n}$$

is equal to

(a) $\frac{n(n+1)}{2}.\frac{a_2-a_1}{a_{n+1}}$ (b) $\frac{n(n+1)}{2}$ (c) $(n+1)(a_2-a_1)$ (d) none of these

If a,b,c,d are nonzero real numbers such that

$$(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) \leq (ab + bc + cd)^2$$

Then a, b, c, d are in

(a) AP (b) GP (c) HP (d) none of these

If $4a^2 + 9b^2 + 16c^2 = 2(3ab + 6bc + 4ca)$, where a, b, c are nonzero numbers, then a, b, c are in

(a) AP (b) GP (c) HP (d) none of these

If $x^2 + 9y^2 + 25z^2 = xyz\left(\frac{15}{x} + \frac{5}{y} + \frac{3}{z}\right)$ then x, y, z are in

(a) AP (b) GP (c) HP (d) none of these

If a, b, c, d and p are distinct real numbers such that

$$(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) \leq 0$$

then a, b, c, d are in

(a) AP (b) GP (c) HP (d) none of these

The largest term common to the sequences 1, 11, 21, 31, ... to 100 terms and 31, 36, 41, 46, ... to 100 terms is

(a) 381 (b) 471 (c) 281 (d) none of these

In the value of 100! the number of zeros at the end is

(a) 11 (b) 22 (c) 23 (d) 24

The sum of all the proper divisors of 9900 is

(a) 33851 (b) 23952 (c) 23951 (d) none of these

The sum of all odd proper divisors of 360 is

(a) 77 (b) 78 (c) 81 (d) none of these

In the sequence 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, ..., where n consecutive terms have the value n, the 150^{th} term is

(a) 17 (b) 16 (c) 18 (d) none of these

In the sequence 1, 2, 2, 4, 4, 4, 4, 8, 8, 8, 8, 8, 8, 8, 8, ..., where n consecutive terms have the value n, the 1025^{th} term I s

(a) 2^9 (b) 2^{10} (c) 2^{11} (d) 2^8

The number of terms common between the series 1 + 2 + 4 + 8 + ... to 100 terms and 1 + 4 + 7 + 10 + ... to 100 terms is

(a) 6 (b) 4 (c) 5 (d) none of these

The 10^{th} common term between the series 3 + 7 + 11 + ... and 1 + 6 + 11 + ... is

(a) 191 (b) 193 (c) 211 (d) none of these

If a_1, a_2, a_3 are in AP, a_2, a_3, a_4 are in GP and a_3, a_4, a_5 are in HP then a_1, a_3, a_5 are in

(a) AP (b) GP (c) HP (d) none of these

If a, b, c, d are four numbers such that the first three are in AP while the last three are in HP then

(a) bc = ad (b) ac = bd (c) ab = cd (d) none of these

If a, a_1, a_2, a_3,. ..... a_{2n-1}, b are in AP, a, b_1, b_2, b_3,....,b_{2n-1}, b are in GP and a, c_1, c_2, c_3,....., c_{2n-1}, b are in HP, where a, b are positive, then the equation $a_n x^2 - b_n x + c_n = 0$ has its roots

(a) real and unequal (b) real and equal (c) imaginary (d) none of these

If a, x, b are in AP, a, y, b are in GP and a, z, b are in HP such that x = 9z and a > 0, b > 0 then

(a) $|y| = 3z$ (b) $x = 3|y|$ (c) $2y = x + z$ (d) none of these

a, b, c, d, e are five numbers in which the first three are in AP and the last three are in HP. If the three numbers in the middle are in GP then the numbers in the odd places are in

(a) AP (b) GP (c) HP (d) none of these

In an AP, $S_p = q$, $S_q = p$ and S_r denote the sum of the first r terms. Then S_{p+q} is equal to

(a) 0 (b) $-(p + q)$ (c) p + q (d) pq

The coefficient of x^{49} in the product $(x - 1)(x - 3) \dots (x - 99)$ is

(a) -99^2 (b) 1 (c) -2 500 (d) none of these

Let $a_1 = 0$ and $a_1, a_2, a_3, \dots, a_n$ be real numbers such that $|a_i| = |a_{i-1} + 1|$ for all i then the AM of the numbers $a_1, a_2, a_3, \dots, a_n$ has the value A where

(a) $A < -\frac{1}{2}$ (b) $A < -1$ (c) $A \geq -\frac{1}{2}$ (d) $A = -\frac{1}{2}$

$\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the HM between a and b if n is

(a) 0 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) 1

a, b, c are three positive numbers and abc^2 has the greatest value $\frac{1}{64}$. Then

(a) $a = b = \frac{1}{2}, c = \frac{1}{4}$ (b) $a = b = \frac{1}{4}, c = \frac{1}{2}$ (c) $a = b = c = \frac{1}{3}$ (d) none of these

If a > 0, b > 0, c > 0 and the minimum value of

$$a(b^2 + c^2) + b(c^2 + a^2) + c(a^2 + b^2)$$

is λabc then λ is

(a) 2 (b) 1 (c) 6 (d) 3

$\sum_{r=1}^{n} r^2 - \sum_{m=1}^{n}\sum_{r=1}^{m} r$ is equal to

(a) 0 (b) $\frac{1}{2}\left(\sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r\right)$ (c) $\frac{1}{2}\left(\sum_{r=1}^{n} r^2 - \sum_{r=1}^{n} r\right)$ (d) none of these

If $(1 + x)(1 + x^2)(1 + x^4)\dots(1 + x^{128}) = \sum_{r=0}^{n} x^r$ then n is

(a) 255 (b) 127 (c) 63 (d) none of these

The sum of the products of the ten numbers $\pm1, \pm2, \pm3, \pm4, \pm5$ taking two at a time is

(a) 165 (b) -55 (c) 55 (d) none of these

It is known that $\sum_{r=1}^{\infty}\frac{1}{(2r-1)^2}=\frac{\pi^2}{8}$. Then $\sum_{r=1}^{\infty}\frac{1}{r^2}$ is equal to

(a) $\frac{\pi^2}{24}$ (b) $\frac{\pi^2}{3}$ (c) $\frac{\pi^2}{6}$ (d) none of these

It is given that $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+.....\text{to }\infty=\frac{\pi^4}{90}$. Then $\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+.....\text{to }\infty$ is equal to

(a) $\frac{\pi^4}{96}$ (b) $\frac{\pi^4}{45}$ (c) $\frac{89\pi^4}{90}$ (d) none of these

If in a series $t_n=\frac{n}{(n+1)!}$ then $\sum_{n=1}^{20}t_n$ is equal to

(a) $\frac{20!-1}{20!}$ (b) $\frac{21!-1}{21!}$ (c) $\frac{1}{2(n-1)!}$ (d) none of these

Let $f(n)=\left[\frac{1}{2}+\frac{n}{100}\right]$ where [x] denotes the integral part of x. Then the value of $\sum_{n=1}^{100}f(n)$ is

(a) 50 (b) 51 (c) 1 (d) none of these

The sum of infinite terms of a decreasing GP is equal to the greatest value of the function $f(x) = x^3 + 3x - 9$ in the interval [-2, 3] and the difference between the first two terms is $f'(0)$. Then the common ratio of the GP is

(a) $-\frac{2}{3}$ (b) $\frac{4}{3}$ (c) $\frac{2}{3}$ (d) $-\frac{4}{3}$

ABC is a right-angled triangle in which $\angle B = 90^\circ$ and BC = a. If n points $L_1, L_2, \ldots, L_n$ on AB are such that AB is divided in n + 1 equal parts and $L_1M_1, L_2M_2, \ldots, L_nM_n$ are line segments parallel to BC and $M_1, M_2, \ldots, M_n$ are on AC then the sum of the lengths of $L_1M_1, L_2M_2, \ldots, L_nM_n$ is

(a) $\frac{a(n+1)}{2}$ (b) $\frac{a(n-1)}{2}$ (c) $\frac{an}{2}$

(d) impossible to find from the given data

The value of $\sum_{r=1}^{n} \frac{1}{\sqrt{a+rx}+\sqrt{a+(r-1)x}}$ is

(a) $\frac{n}{\sqrt{a}+\sqrt{a+nx}}$ (b) $\frac{\sqrt{a+nx}-\sqrt{a}}{x}$ (c) $\frac{n(\sqrt{a+nx}-a)}{x}$ (d) none of these

Let $\sum_{n=1}^{n} r^4 = f(n)$. Then $\sum_{r=1}^{n} (2r-1)^4$ is equal to

(a) $f(2n) - 16f(n)$ for all $n \in N$

(b) $f(n) - 16f\left(\frac{n-1}{2}\right)$ when n is odd

(c) $f(n)-16f\left(\frac{n}{2}\right)$ when n is even

(d) none of these

Let $f(x) = \frac{1-x^{n+1}}{1-x}$ and $g(x) = 1-\frac{2}{x}+\frac{3}{x^2}-....+(-1)^n\frac{n+1}{x^n}$. Then the constant term in $f'(x) \times g(x)$ is equal to

(a) $\frac{n(n^2-1)}{6}$ when n is even

(b) $\frac{n(n+1)}{2}$ when n is odd

(c) $-\frac{n}{2}(n+1)$ when n is even

(d) $-\frac{n(n-1)}{2}$ when n is odd

If a, b, c are in GP and a, p, q are in AP such that 2a, b + p, c + q are in GP then the common difference of the AP is

(a) $\sqrt{2}a$ (b) $(\sqrt{2}+1)(a-b)$ (c) $\sqrt{2}(a+b)$ (d) $(\sqrt{2}-1)(b-a)$

Sum of first n terms of an AP (having positive terms) is given by $S_n = (1+2T_n)(1-T_n)$ (where T_n is the nth term of the series). Then the value of $2T_1^2$ is

(A) 2
(B) 4
(C) 1
(D) 6

In a ΔABC, A, B, C are in AP and a, b, c are in GP then value of $a^3 + b^3 + c^3 - a^2b - b^2c - c^2a$ is

(A) 0
(B) 1
(C) 3
(D) 4

b and c are arithmetic means between a and d (a > d > 0) and h and k are the geometric mean between a and d then

(A) bc is always greater than hk
(B) bc is always less than hk
(C) bc may be equal to hk
(D) none of these

If $H_1, H_2, H_3, \ldots\ldots, H_{2n+1}$ are in H.P., then $\sum_{i=1}^{2n}(-1)^i\left(\frac{H_i + H_{i+1}}{H_i - H_{i+1}}\right)$ is equal to

(A) $2n - 1$
(B) $2n + 1$
(C) $2n$
(D) $2n + 2$

If $f(x) = \frac{2^x}{2^x + \sqrt{2}}$, then find the value of $\sum_{r=1}^{2n-1} 2f\left(\frac{r}{2n}\right)$

Let $a_1, a_2, \ldots.. a_n$ be the terms of a G.P. whose common ratio is r. Set S_k denotes the sum of first k terms of the G.P. then the value of $\sum_{1 \le i < j \le m} a_i a_j$ in terms of S_{m-1} and S_m is

(A) $r\, S_{m-1}\, S_m$
(B) $\frac{r}{1+r} S_{m-1} S_m$
(C) $\frac{S_{m-1} S_m}{1+r}$
(D) $S_{m-1}\, S_m$

Consider the sequence of natural numbers. S_0, S_1, S_2 ..... such that $S_0 = 3$, $S_1 = 3$ a
($n > 1$), then

(A) S_{1545} is odd and S_{1546} is even
(B) S_{1545} is even and S_{1546} i
(C) both S_{1545} and S_{1546} are even
(D) both S_{1545} and S_{1546} are

In a H.P., $T_p = q(p+q)$ and $T_q = p(p+q)$, then p and q are roots of the equation

(A) $x^2 - T_{p+q}x + T_{pq} = 0$
(B) $x^2 - T_{pq}x + T_{p+q} = 0$
(C) $x^2 - 2T_{p+q}x + T_{pq} = 0$
(D) $x^2 - T_{pq}x + 2T_{p+q} = 0$

Consider the sequence of natural numbers. S_0, S_1, S_2 ..... such that $S_0 = 3$, $S_1 = 3$ and $S_n = 3 + S_{n-1}S_{n-2}$ ($n > 1$), then

(A) S_{1545} is odd and S_{1546} is even
(B) S_{1545} is even and S_{1546} is odd
(C) both S_{1545} and S_{1546} are even
(D) both S_{1545} and S_{1546} are odd

If a, b, c are in H.P., then the value of $\frac{a^3b^3 + b^3c^3 + c^3a^3}{a^2c^2}$ is

(A) $9ac - 6b^2$
(B) $3ac - 2b^2$
(C) $9ac - 4b^2$
(D) $9ac - 2b^2$

Consider straight line $ax + by = c$ where $abc \in R^+$ and a, b, c are distinct. This line meets the coordinate axes at P and Q respectively. If area of ΔOPQ, 'O' being origin does not depend upon a, b and c then

(A) a, b, c are in GP
(B) a, b, c are in HP
(C) a, b, c are in AP
(D) none of these

For the series 21, 22, 23, ...,k −1, k; the A.M. and G.M. of the first and last numbers exist in the given series. If 'k' is a three digits number, then 'k' can attain

(A) 5 values
(B) 6 values
(C) 2 values
(D) 4 values

If $x > 0$, then the minimum value of $x^{1000} + x^{900} + x^{90} + x^{6} + \frac{1996}{x}$ is

(A) 1000
(B) 2000
(C) 1996
(D) 3000

$\sum_{r=1}^{\infty} \frac{r^3 + (r^2+1)^2}{(r^4+r^2+1)(r^2+r)}$ is equal to

(A) $\frac{3}{2}$
(B) 1
(C) 2
(D) infinite

The value of $\sum\sum\sum\sum_{1\le i\le j<k\le w\le n} 1$ is

(A) ${}^{n}C_4 + {}^{n}C_3$
(B) ${}^{n}C_4 + {}^{n}C_3 + {}^{n}C_2$
(C) ${}^{n}C_4 + 2\,{}^{n}C_3 + {}^{n}C_2$
(D) ${}^{n}C_4 + {}^{n}C_3 + 2\,{}^{n}C_2$

The coeff of x^{39} in the expansion of $(x-1)\left(x-\frac{1}{2}\right)\left(x-\frac{1}{2^2}\right)\ldots\ldots\left(x-\frac{1}{2^{39}}\right)$ is

(A) $-2\left(1-\frac{1}{2^{40}}\right)$
(B) $2\left(1-\frac{1}{2^{40}}\right)$
(C) $-2\left(1-\frac{1}{2^{39}}\right)$
(D) $2\left(1-\frac{1}{2^{39}}\right)$

(D) f(x) = c if x = cos a

If $\sin^2\frac{A}{2}, \sin^2\frac{B}{2}, \sin^2\frac{C}{2}$ are in H.P. then

(A) a, b, c are in A.P.
(B) a, b, c are in H.P.
(C) a(b + c), b(c + a), c(a + b) are in A.P.
(D) a(b + c), b(c + a), c(a + b) are in H.P.

(C) a(b + c), b(c + a), c(a + b) are in A.P.

Sides of ΔABC are in AP. If a < minimum {b, c}, then cos A may be equal to

(A) $\frac{4b-3c}{2b}$

(B) $\frac{3c-4b}{2c}$

(C) $\frac{4c-3b}{2b}$

(D) $\frac{4c-3b}{2c}$

... exactly 24 small circles, each of radius 2, can be drawn

A straight line through a point $P(\alpha, 2)$, $(\alpha \neq 0)$ meets the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ at A and D & meets axes at B and C such that PA, PB, PC and PD are in GP; then 'α' can lie in interval

(A) $[7, \infty)$

(B) $(-12, -8)$

(C) $(-5, 0)$

(D) $(10, \infty)$

Let $X = \sum_{r=0}^{n}\left(\frac{(-1)^r}{r+1}\right)$, then $|X - \ln 2|$ will be

(A) greater than $\frac{1}{2(n+1)}$

(B) greater than $\frac{1}{2(n+2)}$

(C) less than $\frac{1}{2(n+1)}$

(D) less than $\frac{1}{2(n+2)}$

Read the paragraph carefully and answer the following questions:

If a, b, c are in A.P. and x, y, z are in G.P where a, b, c $\in R - \{0\}$, and x, y, z $\in R - \{0\}$.

329. If $\frac{a}{x}, \frac{b}{y}, \frac{c}{z}$ are in H.P, then

(A) $|c| = |a|$

(B) $3|a| = 2|c|$

(C) $|z| = |x|$

(D) $|y| = |c|$

330. If $\frac{x}{a}, \frac{y}{b}, \frac{z}{c}$ are in H.P., then

(A) $\frac{a}{c} = \sqrt{\frac{x}{z}}$, $x \neq z$

(B) $\sqrt{\frac{a}{c}} = \frac{x}{z}$, $x \neq z$

(C) $\frac{a}{c} = \frac{x}{z}$, $x \neq z$

(D) $ax = cz$, $x \neq z$

331. If ax, by, cz are in H.P, then

(A) $|z| \neq |x|$

(B) $|c| \neq |a|$

(C) $|a| = |c|$ & $|x| = |z|$

(D) $|a|=|c|$ or $|x| = |z|$

Read the paragraph carefully and answer the following questions:

If r^{th} term of a series can be written as $a_r = f(r) - f(r-1)$, then $S_n = \sum_{r=1}^{n} a_r = f(n) - f(0)$ and $S_\infty = \lim_{n \to \infty} S_n$, then

332. Value of $\sum_{r=1}^{\infty} \frac{(4r-1)5^r}{r^2+r}$ is

(A) 5
(B) – 5
(C) 10
(D) none of these

333. Value of $(\cos 1 - 1)\sum_{1}^{50} \sin n + \sin 1 \sum_{1}^{50} \cos n$ is equal to

(A) $\sin 50°$
(B) $\sin 50° - \sin 1°$
(C) $\sin 51° - \sin 1°$
(D) $\frac{1}{2}$

COMPREHENSION–XVII

Read the following passages and answer the following questions

Let f(n) denotes the number of different ways in which the positive integer n can be expressed as the sum of 1's and 2's. For example, f(4) = 5, since 4 = 2 + 2 = 2 + 1 + 1 = 1 + 2 + 1 = 1 + 1 + 2 = 1 + 1 + 1 + 1. Note that order of 1's and 2's are important.

334. The value of f(6) is

(A) 12
(B) 13
(C) 18
(D) 21

335. The value of f(f(6)) is

(A) 400
(B) 350
(C) 377
(D) none of these

336. $f : N \to N$ is

(A) one–one and onto
(B) one–one and into
(C) many–one and onto
(D) many–one and into

triangle is λ then [7λ] _____ **Note:** (If interior angles are $\theta_1 < \theta_2 < \theta_3$ then middle angle is θ_2)

N is the set of all natural number and R is the set of all real numbers. A function f: N → R is given by $f(n) = \frac{4n + \sqrt{4n^2 - 1}}{\sqrt{2n-1} + \sqrt{2n+1}}$, then the value of $\left[\frac{\sum_{r=1}^{40} f(r)}{40}\right]$ is equal to _____ (where [.] denotes the greatest integer function)

If $\frac{\left(1^4 + \frac{1}{4}\right)\left(3^4 + \frac{1}{4}\right)\ldots\left\{(2n-1)^4 + \frac{1}{4}\right\}}{\left(2^4 + \frac{1}{4}\right)\left(4^4 + \frac{1}{4}\right)\ldots\left\{(2n)^4 + \frac{1}{4}\right\}} = \frac{1}{k_1 n^2 + k_2 n + k_3}$, then $k_1 - k_2 + k_3$ is equal to _____

*49. The value of $\cot\left(\sum_{n=1}^{23}\cot^{-1}\left(1+\sum_{k=1}^{n}2k\right)\right)$ is

(A) $\frac{23}{25}$ (B) $\frac{25}{23}$

(C) $\frac{23}{24}$ (D) $\frac{24}{23}$

Sol. **(B)**

*53. Let $S_n = \sum_{k=1}^{4n}(-1)^{\frac{k(k+1)}{2}}k^2$. Then S_n can take value(s)

(A) 1056 (B) 1088

(C) 1120 (D) 1332

Sol. **(A, D)**

A pack contains n cards numbered from 1 to n. Two consecutive numbered cards are removed from the pack and the sum of the numbers on the remaining cards is 1224. If the smaller of the numbers on the removed cards is k, then k − 20 = ________

(5)

For a $\in \mathbb{R}$ (the set of all real numbers), a ≠ −1, $\lim_{n\to\infty}\frac{\left(1^a+2^a+...+n^a\right)}{(n+1)^{a-1}\left[(na+1)+(na+2)+...+(na+n)\right]}=\frac{1}{60}$.

Then a =

(A) 5 (B) 7

(C) $\frac{-15}{2}$ (D) $\frac{-17}{2}$

(B, D)

Required limit = $\frac{\int_0^1 x^a\,dx}{\int_0^1 (a+x)\,dx} = \frac{2}{(2a+1)(a+1)} = \frac{2}{120}$

$\Rightarrow$ a = 7 or $-\frac{17}{2}$.

Let a, b, c be positive integers such that $\frac{b}{a}$ is an integer. If a, b, c are in geometric progression and the arithmetic mean of a, b, c is $b + 2$, then the value of $\frac{a^2+a-14}{a+1}$ is ________

Let the harmonic mean of two positive real numbers a and b be 4. If q is a positive real number such that a, 5, q, b is an arithmetic progression, then the value(s) of $|q - a|$ is (are)

Suppose that all the terms of an arithmetic progression (A.P.) are natural numbers. If the ratio of the sum of the first seven terms to the sum of the first eleven terms is 6 : 11 and the seventh term lies in between 130 and 140, then the common difference of this A.P. is

The value of $\sum_{k=1}^{13} \frac{1}{\sin\left(\frac{\pi}{4}+\frac{(k-1)\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)}$ is equal to

(A) $3-\sqrt{3}$
(B) $2\left(3-\sqrt{3}\right)$
(C) $2\left(\sqrt{3}-1\right)$
(D) $2\left(2+\sqrt{3}\right)$

(C)

Let $b_i > 1$ for $i = 1, 2, \ldots, 101$. Suppose $\log_e b_1, \log_e b_2, \ldots, \log_e b_{101}$ are in Arithmetic Progression (A. P.) with the common difference $\log_e 2$. Suppose $a_1, a_2, \ldots, a_{101}$ are in A.P. such that $a_1 = b_1$ and $a_{51} = b_{51}$. If $t = b_1 + b_2 + \ldots + b_{51}$ and $s = a_1 + a_2 + \ldots + a_{51}$, then

(A) $s > t$ and $a_{101} > b_{101}$
(B) $s > t$ and $a_{101} < b_{101}$
(C) $s < t$ and $a_{101} > b_{101}$
(D) $s < t$ and $a_{101} < b_{101}$

(B)

The sides of a right angled triangle are in arithmetic progression. If the triangle has area 24, then what is the length of its smallest side ?

6

The number of real solutions of the equation

$$\sin^{-1}\left(\sum_{i=1}^{\infty} x^{i+1} - x\sum_{i=1}^{\infty}\left(\frac{x}{2}\right)^i\right) = \frac{\pi}{2} - \cos^{-1}\left(\sum_{i=1}^{\infty}\left(-\frac{x}{2}\right)^i - \sum_{i=1}^{\infty}(-x)^i\right)$$

lying in the interval $\left(-\frac{1}{2}, \frac{1}{2}\right)$ is _____ .

(Here, the inverse trigonometric functions $\sin^{-1}x$ and $\cos^{-1}x$ assume values in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $[0, \pi]$, respectively.)

2

For each positive integer n, let

$$y_n = \frac{1}{n}\left((n+1)(n+2)\ldots.(n+n)\right)^{1/n}.$$

For $x \in \mathbb{R}$, let $[x]$ be the greatest integer less than or equal to x. If $\lim_{n\to\infty} y_n = L$, then the value of $[L]$ is _____ .

For any positive integer n, define $f_n : (0, \infty) \to R$ as

$$f_n(x) = \sum_{j=1}^{n} \tan^{-1}\left(\frac{1}{1+(x+j)(x+j-1)}\right) \text{ for all } x \in (0, \infty)$$

$\left(\text{Here, the inverse trigonometric function } \tan^{-1}x \text{ assumes values in } \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right)$

Then, which of the following statement(s) is (are) TRUE ?

(A) $\sum_{j=1}^{5} \tan^2(f_j(0)) = 55$

(B) $\sum_{j=1}^{10} (1 + f_j'(0))\sec^2(f_j(0)) = 10$

(C) For any fixed positive integer n, $\lim_{x\to\infty} \tan(f_n(x)) = \frac{1}{n}$

(D) For any fixed positive integer n, $\lim_{x\to\infty} \sec^2(f_n(x)) = 1$

D

*Q. 1 Let α and β be the roots of $x^2 - x - 1 = 0$, with α > β. For all positive integers n, define $a_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}$, $n \geq 1$, $b_1 = 1$ and $b_n = a_{n-1} + a_{n+1}$, $n \geq 2$. Then which of the following options is/are correct?

A. $\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \frac{8}{89}$

B. $b_n = \alpha^n + \beta^n$ for all $n \geq 1$

C. $a_1 + a_2 + a_3 + \ldots.. + a_n = a_{n+2} - 1$ for all $n \geq 1$

D. $\sum_{n=1}^{\infty} \frac{a_n}{10^n} = \frac{10}{89}$

Sol. **B, C, D**

Let AP(a; d) denote the set of all the terms of an infinite arithmetic progression with first term a and common difference d > 0. If AP(1; 3) ∩ AP(2; 5) ∩ AP(3; 7) = AP (a; d) then a + d equals_____

157.00

For non–negative integers n, let

$$f(n) = \frac{\sum_{k=0}^{n} \sin\left(\frac{k+1}{n+2}\pi\right)\sin\left(\frac{k+2}{n+2}\pi\right)}{\sum_{k=0}^{n} \sin^2\left(\frac{k+1}{n+2}\pi\right)}$$

Assuming $\cos^{-1}x$ takes values in [0, π], which of the following options is/are correct?

A. $f(4) = \frac{\sqrt{3}}{2}$

B. If $\alpha = \tan(\cos^{-1}f(6))$, then $\alpha^2 + 2\alpha - 1 = 0$

C. $\sin(7\cos^{-1}f(5)) = 0$

D. $\lim_{n\to\infty} f(n) = \frac{1}{2}$

A, B, C

For $a \in \mathbb{R}\ |a| > 1$, let

$$\lim_{n\to\infty}\left(\frac{1+\sqrt[3]{2}+....+\sqrt[3]{n}}{n^{7/3}\left(\frac{1}{(an+1)^2}+\frac{1}{(an+2)^2}+...+\frac{1}{(an+n)^2}\right)}\right)=54$$

Then the possible value(s) of a is/are

A. 8

B. –6

C. 7

D. –9

A, D

Suppose $\det\begin{bmatrix}\sum_{k=0}^{n}k & \sum_{k=0}^{n}{}^{n}C_k k^2 \\ \sum_{k=0}^{n}{}^{n}C_k k & \sum_{k=0}^{n}{}^{n}C_k 3^k\end{bmatrix}=0$ holds for some positive integer n. Then $\sum_{k=0}^{n}\frac{{}^{n}C_k}{k+1}$ equals. ____

6.20

The value of $\sec^{-1}\left(\frac{1}{4}\sum_{k=0}^{10}\sec\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right)\sec\left(\frac{7\pi}{12}+\frac{(k+1)\pi}{2}\right)\right)$ in the interval $\left[-\frac{\pi}{4},\frac{3\pi}{4}\right]$ equals ____

0.00

Let a_1, a_2, a_3, ... be a sequence of positive integers in arithmetic progression with common difference 2. Also, let b_1, b_2, b_3, ... be a sequence of positive integers in geometric progression with common ratio 2. If $a_1 = b_1 = c$, then the number of all possible values of c, for which the equality

$$2(a_1 + a_2 + ... + a_n) = b_1 + b_2 + ... + b_n$$

holds for some positive integer n, is _____

1

Let the function $f : [0, 1] \to \mathbb{R}$ be defined by

$$f(x)=\frac{4^x}{4^x+2}.$$

Then the value of

$$f\left(\frac{1}{40}\right)+f\left(\frac{2}{40}\right)+f\left(\frac{3}{40}\right)+...+f\left(\frac{39}{40}\right)-f\left(\frac{1}{2}\right)$$

is ______

For any positive integer n, let $S_n : (0, \infty) \to R$ be defined by

$$S_n(x) = \sum_{k=1}^{n} \cot^{-1}\left(\frac{1+k(k+1)x^2}{x}\right),$$

where for any $x \in R$, $\cot^{-1}(x) \in (0, \pi)$ and $\tan^{-1}(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then which of the following statements is(are) **TRUE**?

(A) $S_{10}(x) = \frac{\pi}{2} - \tan^{-1}\left(\frac{1+11x^2}{10x}\right)$, for all $x > 0$

(B) $\lim_{n\to\infty} \cot(S_n(x)) = x$, for all $x > 0$

(C) The equation $S_3(x) = \frac{\pi}{4}$ has a root in $(0, \infty)$

(D) $\tan(S_n(x)) \le \frac{1}{2}$, for all $n \ge 1$ and $x > 0$

A, B

Let $l_1, l_2, \ldots, l_{100}$ be consecutive terms of an arithmetic with common difference d_1, and let $w_1, w_2, \ldots, w_{100}$ be consecutive terms of another arithmetic progression with common difference d_2, where $d_1 d_2 = 10$. For each i = 1, 2, ..., 100, let R_i be a rectangle with length l_i, width w_i and area A_i. If $A_{51} - A_{50} = 1000$, then the value of $A_{100} - A_{90}$ is __________.

Let $a_1, a_2, a_3, \ldots$ be an arithmetic progression with $a_1 = 7$ and common difference 8. Let $T_1, T_2, T_3, \ldots$ be such that $T_1 = 3$ and $T_{n+1} - T_n = a_n$ for $n \ge 1$. Then, which of the following is/are TRUE?

(A) $T_{20} = 1604$

(B) $\sum_{k=1}^{20} T_k = 10510$

(C) $T_{30} = 3454$

(D) $\sum_{k=1}^{30} T_k = 35610$

For positive integer n, define

$$f(n) = n + \frac{16+5n-3n^2}{4n+3n^2} + \frac{32+n-3n^2}{8n+3n^2} + \frac{48-3n-3n^2}{12n+3n^2} + \ldots + \frac{25n-7n^2}{7n^2}.$$

Then, the value of $\lim_{n\to\infty} f(n)$ is equal to

(A) $3 + \frac{4}{3}\log_e 7$

(B) $4 - \frac{3}{4}\log_e\left(\frac{7}{3}\right)$

(C) $4 - \frac{4}{3}\log_e\left(\frac{7}{3}\right)$

(D) $3 + \frac{3}{4}\log_e 7$.

Q.10

Let $7\overbrace{5\cdots5}^{r}7$ denote the $(r+2)$ digit number where the first and the last digits are 7 and the remaining r digits are 5. Consider the sum $S = 77 + 757 + 7557 + \cdots + 7\overbrace{5\cdots5}^{98}7$. If $S = \frac{7\overbrace{5\cdots5}^{99}7 + m}{n}$, where m and n are natural numbers less than 3000, then the value of $m + n$ is

Q. **A person is to count 4500 currency notes. Let a denote the number of notes he counts in the $\mathrm{n^{th}}$ minute. If $\mathrm{a_1 = a_2 = \ldots = a_{10} = 150}$ and $\mathrm{a_{10}a_{11}, \ldots}$ are in an AP with common difference - 2, then the time taken by him to count all notes is :-** *(1) 24 minutes (2) 34 minutes (3) 125 minutes (4) 135 minutes [AIEEE-2010]*

Q. **A man saves Rs. 200 in each of the first three months of his service. In each of the subsequent months his saving increases by Rs. 40 more the saving of immediately previous month. His total saving from the start of service will be Rs. 11040 after :-** *(1) 20 months (2) 21 months (3) 18 months (4) 19 months [AIEEE-2011]*

Q. **Let a_n be the n^{th} term of an A.P. If $\sum_{r=1}^{100} a_{2r} = \alpha$ and $\sum_{r=1}^{100} a_{2r-1} = \beta$, then the common difference of the A.P. is :** *(1) $\frac{\alpha-\beta}{200}$ (2) $\alpha - \beta$ (3) $\frac{\alpha-\beta}{100}$ (4) $\beta - \alpha$ [AIEEE-2011]*

Q. **Statement-1: The sum of the series $1 + (1+2+4) + (4+6+9) + (9+12+16)$ $+ \ldots\ldots + (361+380+400)$** *is* 8000. *Statement-2: $\sum_{\mathrm{k}=1}^{\mathrm{n}} \left(\mathrm{k}^3 - (\mathrm{k}-1)^3\right) = \mathrm{n}^3$, for any natural number* n *(1) Statement-1 is true, Statement-2 is false. (2) Statement- 1 is false, Statement- 2 is true. (3) Statement- 1 is true, Statement-2 is true ; Statement-2 is a correct explanation for Statement-1. (4) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-* - 1. *[AIEEE-2012]*

Ans. (4) *Statement-1:* $\left(1^3 - 0^3\right) + \left(2^3 - 1^3\right) + \left(3^3 - 2^3\right) + \ldots + \left(20^3 - 19^3\right) = 20^3 = 8000$ *Statement-1 is true. Statement-2:* $\sum_{k=1}^{n} k^3 - (k-1)^3 = \left(1^3 - 0^3\right) + \left(2^3 - 1^3\right) + \left(3^3 - 2^3\right) + \ldots . n^3 + (n-1)^3 = n^3$

Q. **If 100 times the 100th term of an A.P. with non-zero common difference equals the** *50 times its 50th term, then the 150th term of this A.P. is : (1) zero (2) –150 (3) 150 times its 50th term (4) 150 [AIEEE-2012]*

Q. **Let α and β be the roots of equation $\mathrm{px^2 + qx + r = 0, p \neq 0}$. If $\mathrm{p, q, r}$ are in A.P. and $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is:** *(1) $\frac{\sqrt{61}}{9}$ (2) $\frac{2\sqrt{17}}{9}$ (3) $\frac{\sqrt{34}}{9}$ (4) $\frac{2\sqrt{13}}{9}$ [JEE(Main)-2014]*

Q. **If $(10)^9 + 2(11)^1(10)^8 + 3(11)^2(10)^7 + \ldots\ldots + 10(11)^9 = \mathrm{k}(10)^9$, then k is equal to :** *(1) $\frac{121}{10}$ (2) $\frac{441}{100}$ (3) 100 (4) 110 [JEE(Main)-2014]*

Q. **If the sum of the first ten terms of the series** $\left(1\frac{3}{5}\right)^2+\left(2\frac{2}{5}\right)^2+\left(3\frac{1}{5}\right)^2+4^2+\left(4\frac{4}{5}\right)^2+\ldots$, **is** $\frac{16}{5}\mathrm{m}$, **then** m **is equal to :-** *(1) 99 (2) 102 (3) 101 (4) 100 [JEE(Main)-2016]*

Q. **If, for a positive integer n, the quadratic equation,** $\mathrm{x}(\mathrm{x}+1)+(\mathrm{x}+1)(\mathrm{x}+2)+\ldots\ldots+(\mathrm{x}+\overline{\mathrm{n}-1})(\mathrm{x}+\mathrm{n})=$ *10n has two consecutive integral solutions, then n is equal to : (1) 11 (2) 12 (3) 9 (4) 10 [JEE(Main)-2017]*

Q. **For any three positive real numbers a, b and** $c, 9\left(25a^2+b^2\right)+25\left(c^2-3ac\right)=15b(3a+c)$ **Then :** *(1) a, b and c are in G.P. (2) b, c and a are in G.P. (3) b, c and a are in A.P. (4) a, b and c are in A.P. [JEE(Main)-2017]*

Q. **Let** $a, b, c \in R$. **If** $f(x)=ax^2+bx+c$ **is such that** $a+b+c=3$ **and** $f(x+y)=f(x)+f(y)+\mathrm{xy}, \forall \mathrm{x}, \mathrm{y} \in \mathrm{R}$, **then** $\sum_{\mathrm{n}=1}^{10} \mathrm{f}(\mathrm{n})$ **is equal to :** *(1) 255 (2) 330 (3) 165 (4) 190 [JEE(Main)-2017]*

Q. **Let** $a_1, a_2, a_3, \ldots, a_{49}$ **be in A.P. such that** $\sum_{\mathrm{k}=0}^{12} \mathrm{a}_{4\mathrm{k}+1}=416$ **and** $a_9+a_{43}=66$. **If** $a_1^2+a_2^2+\ldots\ldots+a_{17}^2$ **= 140m, then m is equal to-** *(1) 68 (2) 34 (3) 33 (4) 66 [JEE(Main)-2018]*

Q. **Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series** $1^2+2\cdot 2^2+3^2+2\cdot 4^2+5^2+2\cdot 6^2+\ldots\ldots\ldots$ **If** $\mathrm{B}-2\mathrm{A}=100\lambda$, **then** λ **is equal to :** *(1) 248 (2) 464 (3) 496 (4) 232 [JEE(Main)-2018]*

Let A_1 and A_2 be two arithmetic means and G_1, G_2, G_3 be three geometric

means of two distinct positive numbers. Then $G_1^4+G_2^4+G_3^4+G_1^2G_3^2$ is equal to :

Let $s_1, s_2, s_3, \ldots, s_{10}$ respectively be the sum to 12 terms of 10 A.P. s whose first terms are $1, 2, 3, \ldots .10$ and the common differences are $1, 3, 5, \ldots\ldots, 19$ respectively. Then $\sum_{i=1}^{10} s_i$ is equal to :

Let $< a_n >$ be a sequence such that $a_1+a_2+\ldots+a_n=\frac{n^2+3n}{(n+1)(n+2)}$. If $28\sum_{k=1}^{10}\frac{1}{a_k}=p_1p_2p_3\ldots p_m$, where $\mathrm{p}_1, \mathrm{p}_2, \ldots, \mathrm{p}_{\mathrm{m}}$ are the first m prime numbers, then m is equal to

Let a, b, c and d be positive real numbers such that $a+b+c+d=11$. If the maximum value of $a^5b^3c^2d$ is 3750β, then the value of β is

Let $x_1, x_2, \ldots, x_{100}$ be in an arithmetic progression, with $x_1 = 2$ and their mean equal to 200 . If $y_i = i(x_i - i), 1 \leq i \leq 100$, then the mean of $y_1, y_2, \ldots, y_{100}$ is :

If $\mathrm{S}_n = 4 + 11 + 21 + 34 + 50 + \ldots$ to n terms, then $\frac{1}{60}(\mathrm{S}_{29} - \mathrm{S}_9)$ is equal to :

Let the first term α and the common ratio r of a geometric progression be positive integers. If the sum of squares of its first three terms is 33033, then the sum of these three terms is equal to

Let $S_K = \frac{1+2+\ldots+K}{K}$ and $\sum_{j=1}^{n} S_j^2 = \frac{n}{A}\left(Bn^2 + Cn + D\right)$, where $A, B, C, D \in \mathbb{N}$ and A has least value. Then

If $\gcd(\mathrm{m}, \mathrm{n}) = 1$ and
$1^2 - 2^2 + 3^2 - 4^2 + \ldots\ldots + (2021)^2 - (2022)^2 + (2023)^2 = 1012\, m^2 n$ then $m^2 - n^2$ is equal to :

The sum $\sum_{n=1}^{\infty} \frac{2n^2+3n+4}{(2n)!}$ is equal to :

Let $a_1, a_2, a_3, \ldots$ be an A.P. If $a_7 = 3$, the product $a_1 a_4$ is minimum and the sum of its first n terms is zero, then $n! - 4a_{n(n+2)}$ is equal to :

If the sum and product of four positive consecutive terms of a G.P., are 126 and 1296 , respectively, then the sum of common ratios of all such GPs is

Let $a, b, c > 1, a^3, b^3$ and c^3 be in A.P., and $\log_a b, \log_c a$ and $\log_b c$ be in G.P. If the sum of first 20 terms of an A.P., whose first term is $\frac{a+4b+c}{3}$ and the common difference is $\frac{a-8b+c}{10}$ is -444, then abc is equal to :

If $a_n = \frac{-2}{4n^2-16n+15}$, then $a_1 + a_2 + \ldots. + a_{25}$ is equal to :

For three positive integers p, q, r, $x^{pq^2} = y^{qr} = z^{p^2r}$ and r = pq + 1 such that 3, 3 $\log_y x$, 3 $\log_z y$, 7 $\log_x z$ are in A.P. with common difference $\frac{1}{2}$. Then r-p-q is equal to

Let $\{a_n\}_{n=0}^{\infty}$ be a sequence such that $a_0 = a_1 = 0$ and

$a_{n+2} = 3a_{n+1} - 2a_n + 1, \forall n \geq 0.$

Then $a_{25}a_{23} - 2a_{25}a_{22} - 2a_{23}a_{24} + 4a_{22}a_{24}$ is equal to

Consider the sequence $a_1, a_2, a_3, \ldots$ such that $a_1 = 1, a_2 = 2$ and $a_{n+2} = \frac{2}{a_{n+1}} + a_n$ for $\mathrm{n} = 1, 2, 3, \ldots$. If $\left(\frac{a_1+\frac{1}{a_2}}{a_3}\right) \cdot \left(\frac{a_2+\frac{1}{a_3}}{a_4}\right) \cdot \left(\frac{a_3+\frac{1}{a_4}}{a_5}\right) \cdots \left(\frac{a_{30}+\frac{1}{a_{31}}}{a_{32}}\right) = 2^{\alpha}\left({}^{61}C_{31}\right)$, then α is equal to :

Let the sum of an infinite G.P., whose first term is a and the common ratio is r, be 5 . Let the sum of its first five terms be $\frac{98}{25}$. Then the sum of the first 21 terms of an AP, whose first term is $10\mathrm{ar}, \mathrm{n}^{\text{th}}$ term is $\mathrm{a_n}$ and the common difference is $10\mathrm{ar}^2$, is equal to :

Suppose $a_1, a_2, \ldots, a_n, ..$ be an arithmetic progression of natural numbers. If the ratio of the sum of first five terms to the sum of first nine terms of the progression is $5 : 17$ and , $110 < a_{15} < 120$, then the sum of the first ten terms of the progression is equal to

Consider two G.Ps. 2, 2^2, 2^3, ... and 4, 4^2, 4^3, ... of 60 and n terms respectively. If the geometric mean of all the 60 + n terms is $(2)^{\frac{225}{8}}$, then $\sum_{k=1}^{n} k(n-k)$ is equal to :

The sum of the infinite series $1 + \frac{5}{6} + \frac{12}{6^2} + \frac{22}{6^3} + \frac{35}{6^4} + \frac{51}{6^5} + \frac{70}{6^6} + \ldots\ldots$ is equal to :

Let $\{a_n\}_{n=0}^{\infty}$ be a sequence such that $a_0 = a_1 = 0$ and $a_{n+2} = 2a_{n+1} - a_n + 1$ for all $n \geq 0$. Then, $\sum_{n=2}^{\infty} \frac{a_n}{7^n}$ is equal to:

If $A = \sum_{n=1}^{\infty} \frac{1}{(3+(-1)^n)^n}$ and $B = \sum_{n=1}^{\infty} \frac{(-1)^n}{(3+(-1)^n)^n}$, then $\frac{A}{B}$ is equal to :

If $\{a_i\}_{i=1}^{n}$, where n is an even integer, is an arithmetic progression with common difference 1, and $\sum_{i=1}^{n} a_i = 192$, $\sum_{i=1}^{n/2} a_{2i} = 120$, then n is equal to :

Let $S_n = 1 . (n-1) + 2 . (n-2) + 3 . (n-3) + \ldots\ldots + (n-1) . 1, n \geq 4$.

The sum $\sum_{n=4}^{\infty}\left(\frac{2S_n}{n!} - \frac{1}{(n-2)!}\right)$ is equal to :

Let $a_1, a_2, \ldots\ldots\ldots, a_{21}$ be an AP such that $\sum_{n=1}^{20} \frac{1}{a_n a_{n+1}} = \frac{4}{9}$. If the sum of this AP is 189, then $a_6 a_{16}$ is equal to :

If 0 < x < 1, then $\frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{7}{4}x^4 + \ldots\ldots$, is equal to :

If for x, y $\in$ R, x > 0, y = $\log_{10}x + \log_{10}x^{1/3} + \log_{10}x^{1/9} + \ldots\ldots$ upto ∞ terms

and $\frac{2+4+6+\ldots.+2y}{3+6+9+\ldots..+3y} = \frac{4}{\log_{10} x}$, then the ordered pair (x, y) is equal to :

The sum of the series

$\frac{1}{x+1} + \frac{2}{x^2+1} + \frac{2^2}{x^4+1} + \ldots\ldots + \frac{2^{100}}{x^{2^{100}}+1}$ when x = 2 is :

Let S_n be the sum of the first n terms of an arithmetic progression. If $S_{3n} = 3S_{2n}$, then the value of $\frac{S_{4n}}{S_{2n}}$ is :

Let S_1 be the sum of first 2n terms of an arithmetic progression. Let S_2 be the sum of first 4n terms of the same arithmetic progression. If $(S_2 - S_1)$ is 1000, then the sum of the first 6n terms of the arithmetic progression is equal to :

If α, β are natural numbers such that
$100^\alpha - 199\beta$ = (100)(100) + (99)(101) + (98)(102) + ... + (1)(199), then the slope of the line passing through (α, β) and origin is :

$\frac{1}{3^2-1} + \frac{1}{5^2-1} + \frac{1}{7^2-1} + \ldots. + \frac{1}{(201)^2-1}$ is equal to

The sum of the series

$\sum_{n=1}^{\infty} \frac{n^2+6n+10}{(2n+1)!}$ is equal to :

If the sum of the first 20 terms of the series
$\log_{(7^{1/2})} x + \log_{(7^{1/3})} x + \log_{(7^{1/4})} x + \ldots$ is 460,
then x is equal to :

If $2^{10} + 2^9.3^1 + 2^8.3^2 + \ldots.. + 2.3^9 + 3^{10} = S - 2^{11}$, then S is equal to :

If $3^{2\sin 2\alpha - 1}$, 14 and $3^{4-2\sin 2\alpha}$ are the first three terms of an A.P. for some α, then the sixth terms of this A.P. is:

Let $a_1, a_2, \ldots, a_n$ be a given A.P. whose common difference is an integer and $S_n = a_1 + a_2 + \ldots + a_n$. If $a_1 = 1$, $a_n = 300$ and $15 \leq n \leq 50$, then the ordered pair (S_{n-4}, a_{n-4}) is equal to:

If $1+(1-2^2.1)+(1-4^2.3)+(1-6^2.5)+\ldots\ldots+(1-20^2.19) = \alpha - 220\beta$, then an ordered pair (α, β) is equal to:

Let S be the sum of the first 9 terms of the series : $\{x + ka\} + \{x^2 + (k+2)a\} + \{x^3 + (k+4)a\} + \{x^4 + (k+6)a\} + \ldots$ where $a \neq 0$ and $x \neq 1$.

If $S = \frac{x^{10} - x + 45a(x-1)}{x-1}$, then k is equal to :

If $|x| < 1$, $|y| < 1$ and $x \neq y$, then the sum to infinity of the following series

$(x + y) + (x^2+xy+y^2) + (x^3+x^2y + xy^2+y^3) + \ldots$

Let a_n be the n^{th} term of a G.P. of positive terms.

$\sum_{n=1}^{100} a_{2n+1} = 200$ and $\sum_{n=1}^{100} a_{2n} = 100$,

then $\sum_{n=1}^{200} a_n$ is equal to :

If the sum of the first 40 terms of the series, $3 + 4 + 8 + 9 + 13 + 14 + 18 + 19 + \ldots$ is (102)m, then m is equal to :

For x ε R, let [x] denote the greatest integer $\leq$ x, then the sum of the series $\left[-\frac{1}{3}\right]+\left[-\frac{1}{3}-\frac{1}{100}\right]+\left[-\frac{1}{3}-\frac{2}{100}\right]+\ldots+\left[-\frac{1}{3}-\frac{99}{100}\right]$ is :

The sum $\frac{3\times1^3}{1^3}+\frac{5\times(1^3+2^3)}{1^2+2^2}+\frac{7\times(1^3+2^3+3^3)}{1^2+2^2+3^2}+\ldots$ upto 10 terms is:

If $a_1, a_2, a_3, \ldots\ldots\ldots\ldots\ldots a_n$ are in A.P. and $a_1 + a_4 + a_7 + \ldots\ldots\ldots\ldots + a_{16} = 114$, then $a_1 + a_6 + a_{11} + a_{16}$ is equal to :

Some identical balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row consists of two balls and so on. If 99 more identical balls are addded to the total number of balls used in forming the equilaterial triangle, then all these balls can be arranged in a square whose each side contains exactly 2 balls less than the number of balls each side of the triangle contains. Then the number of balls used to form the equilateral triangle is :-

Let the sum of the first n terms of a non-constant A.P., $a_1, a_2, a_3, \ldots$ be $50n+\frac{n(n-7)}{2}A$, where A is a constant. If d is the common difference of this A.P., then the ordered pair (d, a_{50}) is equal to

If ${}^nC_4$, ${}^nC_5$ and ${}^nC_6$ are in A.P., then n can be :

Let $S_k = \frac{1+2+3+\ldots+k}{k}$. If $S_1^2+S_2^2+\ldots+S_{10}^2=\frac{5}{12}A$, then A is equal to :

The sum of an infinite geometric series with positive terms is 3 and the sum of the cubes of its terms is $\frac{27}{19}$.Then the common ratio of this series is :

$$1+6+\frac{9(1^2+2^2+3^2)}{7}+\frac{12(1^2+2^2+3^2+4^2)}{9}$$

$$+\frac{15(1^2+2^2+\ldots+5^2)}{11}+\ldots \text{ up to 15 terms, is :}$$

Let $a_1, a_2, \ldots\ldots, a_{30}$ be an A.P.,

$$S = \sum_{i=1}^{30} a_i \text{ and } T = \sum_{i=1}^{15} a_{(2i-1)}.$$

If a_5 = 27 and S - 2T = 75, then a_{10} is equal to :

For any three positive real numbers a, b and c

$9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$.
Then

Statement-1: The sum of the series 1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) +.....+ (361 + 380 + 400) is 8000.

Statement-2: $\sum_{k=1}^{n} \left(k^3 - (k-1)^3\right) = n^3$, for any natural number n.

If the sides a, b, c of a ΔABC are in G.P. and $\log a - \log 2b$, $\log 2b - \log 3c$, $\log 3c - \log a$ are in A.P., then ΔABC is ...... triangle.

(A) a right angled (B) an acute angled (C) an obtuse angled (D) an equilateral

The sum of first n terms of $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + \frac{31}{32} + \ldots$ is .... (IIT :

(A) $2^{-n} + n - 1$ (B) 2^{-n} (C) $2^n \cdot n - 2^n + 1$ (D) $2^n - n + 1$

In a G.P. of positive real numbers, nth term is a_n. If $\sum_{n=1}^{100} a_{2n} = \alpha$ and $\sum_{n=1}^{100} a_{2n-1} = \beta$, where $\alpha \neq \beta$, then the common ratio of the G.P. .... (IIT : 1992)

(A) $\frac{\alpha}{\beta}$ (B) $\frac{\beta}{\alpha}$ (C) $\sqrt{\frac{\alpha}{\beta}}$ (D) $\sqrt{\frac{\beta}{\alpha}}$

In Δ ABC, if $a \cos^2 \frac{C}{2} + c \cos^2 \frac{A}{2} = \frac{3b}{2}$, then the sides of the triangle ......

(AIEEE : 2003)

(A) are in A.P. (B) are in G.P. (C) are in H.P. (D) satisfy $a + b = c$

Let $a_1, a_2, a_3, \ldots\ldots$ be a harmonic progression with $a_1 = 5$ and $a_{20} = 25$. The least positive integer n for which $a_n < 0$ is IIT 2012

(A) 22 (B) 23 (C) 24 (D) 25

Let $a_1, a_2, a_3, \ldots, a_{100}$ be an arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1} a_i$, $1 \le p \le 100$.

For any integer n with $1 \le n \le 20$, let $m = 5n$. If $\frac{S_m}{S_n}$ does not depend on n, then a_2 is

IIT 2011

ANSWER : 3, 9, 3 & 9 BOTH

If P is the point $(-1, 2)$, a variable line through P cuts the x & y axes at A & B respectively Q is the point on AB such that PA, PQ, PB are H.P., then the locus of Q is

(A*) $y = 2x$ (B) $y = -2x$ (C) $y = 2x^2$ (D) $y = -2x^2$

The point $A(x_1, y_1)$; $B(x_2, y_2)$ and $C(x_3, y_3)$ lie on the parabola $y = 3x^2$. If x_1, x_2, x_3 are in A.P. and y_1, y_2, y_3 are in G.P. then one of the values of the common ratio of the G.P. is

(A*) $3 + 2\sqrt{2}$ (B) $3 + \sqrt{2}$ (C) $3 - \sqrt{2}$ (D) none of these

$S_2 = 1 - 3x + 5x^2 - 7x^3 + \cdots \infty$ For $|x| < 1$

Find the sum of the following series:

$$\frac{1^4}{1.3} + \frac{2^4}{3\cdot 5} + \frac{3^4}{5\cdot 7} + \cdots + \frac{n^4}{(2n-1)(2n+1)}$$

Consider the sum S given by

$$S = \frac{1}{1001} + \frac{1}{1002} + \frac{1}{1003} + \cdots + \frac{1}{3001}$$

(a) Is it true that $S > 1$?

(b) Is it true that $S < \frac{3}{2}$?

(c) Is it true that $S < \frac{4}{3}$?

Evaluate $\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{m^2 n}{3^m (n 3^m + m 3^n)}$

Sum the series

$$\frac{a_1}{1+a_1} + \frac{a_2}{(1+a_1)(1+a_2)} + \cdots + \frac{a_n}{(1+a_1)(1+a_2)\ldots(1+a_n)}$$

Let $a_1, a_2, a_3, \ldots, a_{11}$ be real numbers satisfying $a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for $k = 3, 4, \ldots, 11$. If $\frac{a_1^2 + a_2^2 + \ldots + a_{11}^2}{11} = 90$, then the value of $\frac{a_1 + a_2 + \ldots + a_{11}}{11}$ is...... **(2010)**

Suppose that all the terms of an arithmetic progression are natural numbers. If the ratio of the sum of the first seven terms to the sum of the first eleven terms is 6 : 11 and the seventh term lies in between 130 and 140, then the common difference of this AP is **(2015 Adv.)**

Let $S_n = 1 + q + q^2 + \ldots + q^n$ and

$$T_n = 1 + \left(\frac{q+1}{2}\right) + \left(\frac{q+1}{2}\right)^2 + \ldots + \left(\frac{q+1}{2}\right)^n,$$ where q is a real number and $q \neq 1$. If ${}^{101}C_1 + {}^{101}C_2 \cdot S_1 + \ldots + {}^{101}C_{101} \cdot S_{100} = \alpha T_{100}$, then α is equal to **(2019 Main, 11 Jan II)**

(a) 2^{100} (b) 202

(c) 200 (d) 2^{99}

Let S_k, where $k = 1, 2, , \ldots, 100$, denotes the sum of the infinite geometric series whose first term is $\frac{k-1}{k!}$ and the common ratio is $\frac{1}{k}$. Then, the value of $\frac{100^2}{100!} + \sum_{k=1}^{100} |(k^2 - 3k + 1) S_k|$ is ...... **(2010)**

The sum of the following series

$$1 + 6 + \frac{9(1^2 + 2^2 + 3^2)}{7} + \frac{12(1^2 + 2^2 + 3^2 + 4^2)}{9} + \frac{15(1^2 + 2^2 + \ldots + 5^2)}{11} + \ldots$$ up to 15 terms is

(2019 Ma

(a) 7510 (b) 7820

(c) 7830 (d) 7520

Let a, b, c be positive integers such that b/a is an integer. If a, b, c are in geometric progression and the arithmetic mean of a, b, c is $b + 2$, then the value of $\frac{a^2 + a - 14}{a + 1}$ is **(2014 Adv.)**

The minimum value of the sum of real numbers $a^{-5}, a^{-4}, 3a^{-3}, 1, a^{8}$ and a^{10} with $a > 0$ is ……

$$\sum_{k=0}^{n-1} \cos(a + k \cdot d) = \frac{\sin(n \times \frac{d}{2})}{\sin(\frac{d}{2})} \times \cos\left(\frac{2a + (n-1) \cdot d}{2}\right)$$

ıt difference in case of sin, which is:

$$\sum_{k=0}^{n-1} \sin(a + k \cdot d) = \frac{\sin(n \times \frac{d}{2})}{\sin(\frac{d}{2})} \times \sin\left(\frac{2a + (n-1) \cdot d}{2}\right)$$

re spot for your organization's questions and answers.

All terms of the arithmetic progression are natural numbers. The sum of its nine consecutive terms, beginning with the first, is larger than 200 and smaller than 220. Find the progression if its second term is equal to 12.

The product of the third and the sixth term of the arithmetic progression is 406. The quotient of the division of the ninth term by the fourth term of the progression is equal to 2 and the remainder is —6. Find the first term and the common difference of the progression.

The sum of the first seven consecutive terms of the arithmetic progression is zero and the sum of their squares is a^2. Find the progression.

Suppose S_n is the sum of the first n terms of a geometric progression. Prove that $S_n (S_{3n} - S_{2n}) = (S_{2n} - S_n)^2$.

The sum of the terms of an infinitely decreasing geometric progression is equal to the greatest value of the function $f(x) = x^3 + 3x - 9$ on the interval $[-2; 3]$, and the difference between the first and the second term is $f'(0)$. Find the common ratio of the progression.

The first and the third term of an arithmetic progression are equal, respectively, to the first and the third term of a geometric progression, and the second term of the arithmetic progression exceeds the second term of the geometric progression by 0.25. Calculate the sum of the first five terms of the arithmetic progression if its first term is equal to 2.

The product of the arithmetic mean of the lengths of the sides of a triangle and harmonic mean of the lengths of the altitudes of the triangle is equal to :

(A) Δ (B*) 2Δ (C) 3Δ (D) 4Δ

[where Δ is the area of the triangle ABC]

If in a triangle $\sin A : \sin C = \sin(A - B) : \sin(B - C)$ then $a^2 : b^2 : c^2$

(A*) are in A.P. (B) are in G.P.

(C) are in H.P. (D) none of these

3/s&p If the roots of the cubic $x^3 - px^2 + qx - r = 0$ are in G.P. then

(A*) $q^3 = p^3r$ (B) $p^3 = q^3r$ (C) $pq = r$ (D) $pr = q$

4/s&p Along a road lies an odd number of stones placed at intervals of 10 m. These stones have to be assembled around the middle stone. A person can carry only one stone at a time. A man carried out the job starting with the stone in the middle, carrying stones in succession, thereby covering a distance of 4.8 km. Then the number of stones is

(A) 15 (B) 29 (C*) 31 (D) 35

9/s&p If $\log_{(5.2^x+1)} 2$; $\log_{(2^{1-x}+1)} 4$ and 1 are in Harmonical Progression then

(A) x is a positive real (B*) x is a negative real

(C) x is rational which is not integral (D) x is an integer

4/s&p If a, b, c are in G.P., then the equations, $ax^2 + 2bx + c = 0$ & $dx^2 + 2ex + f = 0$ have a common root, if $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in :

(A*) A.P. (B) G.P. (C) H.P. (D) none

15/s&p If the sum of the roots of the quadratic equation, $ax^2 + bx + c = 0$ is equal to sum of the squares of their reciprocals, then $\frac{a}{c}, \frac{b}{a}, \frac{c}{b}$ are in :

(A) A.P. (B) G.P. (C*) H.P. (D) none

)/s&p Consider an A.P. with first term 'a' and the common difference d. Let S_k denote the sum of the first K terms. Let $\frac{S_{kx}}{S_x}$ is independent of x, then

(A*) $a = d/2$ (B) $a = d$ (C) $a = 2d$ (D) none

18/s&p Concentric circles of radii 1, 2, 3......100 cms are drawn. The interior of the smallest circle is coloured red and the angular regions are coloured alternately green and red, so that no two adjacent regions are of the same colour. The total area of the green regions in sq. cm is equal to

(A) 1000π (B*) 5050π (C) 4950π (D) 5151π

5/s&p Consider a decreasing G.P. : $g_1, g_2, g_3, ... g_n$ .... such that $g_1 + g_2 + g_3 = 13$ and $g_1^2 + g_2^2 + g_3^2 = 91$ then which of the following does not hold?

(A) The greatest term of the G.P. is 9. (B) $3g_4 = g_3$

(C*) $g_1 = 1$ (D) $g_2 = 3$

i8/s&p If p , q, r in H.P. and p & r be different having same sign then the roots of the equation $px^2 + qx + r = 0$ are

(A) real & equal (B) real & distinct (C) irrational (D*) imaginary

119/s&p The point $A(x_1, y_1)$; $B(x_2, y_2)$ and $C(x_3, y_3)$ lie on the parabola $y = 3x^2$. If x_1, x_2, x_3 are in A.P. and y_1, y_2, y_3 are in G.P. then the common ratio of the G.P. is

1/s&p If a, b, c are in A.P., then $a^2 (b + c) + b^2 (c + a) + c^2 (a + b)$ is equal to :

(A) $\frac{(a+b+c)^3}{8}$ (B*) $\frac{2}{9}(a+b+c)^3$ (C) $\frac{3}{10}(a+b+c)^3$ (D) $\frac{1}{9}(a+b+c)^3$

'4/s&p If $S_n = \frac{1}{1^3} + \frac{1+2}{1^3+2^3} + ... + \frac{1+2+3+......+n}{1^3+2^3+3^3+......+n^3}$, n = 1, 2, 3,...... Then S_n is not greater than:

(A) 1/2 (B) 1 (C*) 2 (D) 4

0/s&p If S_n denotes the sum of the first n terms of a G.P. , with the first term and the common ratio both positive, then

(A) S_n , S_{2n} , S_{3n} form a G.P.

(B*) S_n , S_{2n} , $- S_n$, S_{3n} , $-S_{2n}$ form a G.P.

(C) $S_{2n} - S_n$, $S_{3n} - S_{2n}$, $S_{3n} - S_n$ form a G.P.

(D) $S_{2n}-S_n$, $S_{3n}-S_{2n}$, $S_{3n}-S_n$ form a G.P.

5/s&p $\frac{1}{2.4} + \frac{1.3}{2.4.6} + \frac{1.3.5}{2.4.6.8} + \frac{1.3.5.7}{2.4.6.8.10} + ...\infty$ is equal to

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C*) $\frac{1}{2}$ (D) 1

i9/s&p A circle of radius r is inscribed in a square. The mid points of sides of the square have been connected by line segment and a new square resulted. The sides of the resulting square were also connected by segments so that a new square was obtained and so on, then the radius of the circle inscribed in the n^{th} square is

(A*) $\left[2^{\frac{1-n}{2}}\right]r$ (B) $\left[2^{\frac{3-3n}{2}}\right]r$ (C) $\left[2^{-\frac{n}{2}}\right]r$ (D) $\left[2^{-\frac{5-3n}{2}}\right]r$

06/s&p Given $a_{m+n} = A$; $a_{m-n} = B$ as the terms of the G.P. a_1 , a_2 , a_3 ,............. then for $A \neq 0$ which of the following holds?

(A*) $a_m = \sqrt{AB}$ (B) $a_n = \sqrt[2n]{A^n B^n}$

(C) $a_m = a_1\left(\frac{A}{B}\right)^{\frac{m^2-m-n-mn}{m+n}}$ (D) $a_n = a_1\left(\frac{A}{B}\right)^{\frac{m^2-m-n-n^2}{m+n}}$

.14/s&p The sum of the infinite series, $1^2 - \frac{2^2}{5} + \frac{3^2}{5^2} - \frac{4^2}{5^3} + \frac{5^2}{5^4} - \frac{6^2}{5^5} + ...$ is :

(A) $\frac{1}{2}$ (B) $\frac{25}{24}$ (C*) $\frac{25}{54}$ (D) $\frac{125}{252}$

\1 A number sequence $a_1, a_2, a_3 .... a_n$ is such that
$a_1 = 0$; $|a_2| = |a_1 + 1|$; $|a_3| = |a_2 + 1|$ ...... $|a_n| = |a_{n-1} + 1|$.

Prove that the arithmetic mean of $a_1, a_2, ... a_n$ is not less than $-\frac{1}{2}$.

Prove that $\sqrt{2}, \sqrt{3}, \sqrt{5}$ can not be the terms of an A.P. (not necessarily adjacent)

If $S_1, S_2, S_3, ... S_p$ are the sums of infinite G.P. whose first terms are 1, 2, 3, ...p, and whose common ratios are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ..., \frac{1}{p+1}$ respectively.

Prove that $S_1 + S_2 + S_3 + ... S_p = \frac{p}{2}(p+3)$.

)E Find the value(s) of the positive integer n for which the quadratic equation, $\sum_{k=1}^{n}(x+k-1)(x+k) = 10n$ has solutions α and $\alpha + 1$ for some α. [Ans. 11]

$5 + 7 + 13 + 31 + 85 + ...$ up to n terms. [Ans. $4n + \frac{1}{2}(3^n + 1)$]

$2 + 5 + 14 + 41 + 122 + ...$ up to n terms. [Ans. $\frac{1}{4}(3^{n+1} + 2n - 3)$]

$1 + \left(1+\frac{1}{3}\right) + \left(1+\frac{1}{3}+\frac{1}{3^2}\right) + ... + \left(1+\frac{1}{3}+\frac{1}{3^2}+.....+\frac{1}{3^{n-1}}\right)$ [Ans. $\frac{3^n(2n-1)+1}{4\cdot 3^{n-1}}$]

$\frac{1}{1\cdot 2\cdot 3\cdot 4} + \frac{1}{2\cdot 3\cdot 4\cdot 5} + \frac{1}{3\cdot 4\cdot 5\cdot 6} + ...$[Ans. $S_n = \frac{1}{18} - \frac{1}{3(n+1)(n+2)(n+3)}$; $S_\infty = \frac{1}{18}$]

$\frac{1}{1\cdot 3\cdot 5} + \frac{1}{3\cdot 5\cdot 7} + ... + \frac{1}{5\cdot 7\cdot 9} + ...$

In case a factor is missing

e.g. $\frac{3}{1\cdot 2\cdot 4} + \frac{4}{2\cdot 3\cdot 5} + \frac{5}{3\cdot 4\cdot 6} + ...$ then split the n^{th} as given below

$\frac{1}{1\cdot3}+\frac{2}{1\cdot3\cdot5}+\frac{3}{1\cdot3\cdot5\cdot7}+\frac{4}{1\cdot3\cdot5\cdot7\cdot9}+\ldots\ldots\ldots$

Find S_n and S_∞ for $\frac{1}{2\cdot4}+\frac{1\cdot3}{2\cdot4\cdot6}+\frac{1\cdot3\cdot5}{2\cdot4\cdot6\cdot8}+\ldots\ldots\ldots$

If a^2, b^2, c^2 are in A.P. show that $b + c$, $c + a$, $a + b$ are in H.P.

If $\frac{a-x}{px}=\frac{a-y}{qy}=\frac{a-z}{rz}$ and p, q, r are in A.P. then prove that x, y, z are in H.P.

If a, b, c are three distinct positive reals in H.P. then prove that $a^n + c^n > 2b^n$.

If a, b, c are in H.P. p, q, r are in H.P. and ap, bq, cr are in G.P. then prove that

$$\frac{p}{r}+\frac{r}{p}=\frac{a}{c}+\frac{c}{a}$$

If a, b, c are in A.P., then show that:

(i) $a^2(b + c)$, $b^2(c + a)$, $c^2(a + b)$ are also in A.P.

(ii) $b + c - a$, $c + a - b$, $a + b - c$ are in A.P.

If a, b, c, d are in G.P., prove that :

(i) $(a^2 - b^2)$, $(b^2 - c^2)$, $(c^2 - d^2)$ are in G.P.

(ii) $\frac{1}{a^2+b^2}, \frac{1}{b^2+c^2}, \frac{1}{c^2+d^2}$ are in G.P.

If $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots$ upto $\infty = \frac{\pi^2}{6}$, then $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots = $

(A) $\pi^2/12$ (B) $\pi^2/24$ (C*) $\pi^2/8$ (D) none of these

If $a_1, a_2, \ldots a_n$ are in A.P. with common difference $d \neq 0$, then the sum of the series

$(\sin d)\,[\operatorname{cosec} a_1 \operatorname{cosec} a_2 + \operatorname{cosec} a_2 \operatorname{cosec} a_3 + \ldots + \operatorname{cosec} a_{n-1} \operatorname{cosec} a_n]$

(A) $\sec a_1 - \sec a_n$ (B) $\operatorname{cosec} a_1 - \operatorname{cosec} a_n$

(C*) $\cot a_1 - \cot a_n$ (D) $\tan a_1 - \tan a_n$

If $H_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots\ldots\ldots + \frac{1}{n}$, then value of $1 + \frac{3}{2} + \frac{5}{3} + \ldots\ldots\ldots + \frac{2n-1}{n}$ is

;fn $H_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots\ldots\ldots + \frac{1}{n}$ gks] rks $1 + \frac{3}{2} + \frac{5}{3} + \ldots\ldots\ldots + \frac{2n-1}{n}$ dk eku gSa &

(A*) $2n - H_n$ (B) $2n + H_n$ (C) $H_n - 2n$ (D) $H_n + n$

If S_1, S_2, S_3 are the sums of first n natural numbers, their squares, their cubes respectively, then $\frac{S_3(1+8S_1)}{S_2^2}$ is equal to

;fn S_1, S_2, S_3 Øe'k% izFke n izkd`r la[;kvksa dk] muds oxksZ dk vkSj muds ?kuksa dk ;ksxQy gks] rks $\frac{S_3(1+8S_1)}{S_2^2}$ dk eku gSa &

(A) 1 (B) 3 (C*) 9 (D) 10

The sides of a right triangle form a G.P. The tangent of the smallest angle is

fdlh ledks.k f=kHkqt dh Hkqtk,i xq-Js-esa gks] rks lclsNksVs dks.k dh Li'kZT;k gSa–

(A) $\sqrt{\frac{\sqrt{5}+1}{2}}$ (B*) $\sqrt{\frac{\sqrt{5}-1}{2}}$ (C*) $\sqrt{\frac{2}{\sqrt{5}+1}}$ (D) $\sqrt{\frac{2}{\sqrt{5}-1}}$

The sum of the first ten terms of an AP is 155 & the sum of first two terms of a GP is 9. The first term of the AP is equal to the common ratio of the GP & the first term of the GP is equal to the common difference of the AP. Find the two progressions.

,d l-Js-ds izFke 10 inksa dk ;ksxQy 155 gS vkSj ,d xq-Js- ds izFke 2 inksa dk ;ksx 9 gSA l-Js-dk izFke i
xq-Js-ds lkoZvuqikr ds cjkcj gS vkSj xq-Js-dk izFke in] l-Js-ds lkoZUrj ds cjkcj gSA nksuksa Js.f<+;ki Kkr dhf

Ans. (3 + 6 + 12 +......); (2/3 + 25/3 + 625/6 +......)

Find the sum of the series

$$\frac{5}{13} + \frac{55}{(13)^2} + \frac{555}{(13)^3} + \frac{5555}{(13)^4} + \ldots \text{ up to } \infty$$

Js.kh $\frac{5}{13} + \frac{55}{(13)^2} + \frac{555}{(13)^3} + \frac{5555}{(13)^4} + \ldots$ ds vuUr inksa dk ;ksxQy Kkr dhft,A

Ans. $\frac{65}{36}$

If $0 < x < \pi$ and the expression

$$\exp\{(1 + |\cos x| + \cos^2 x + |\cos^3 x| + \cos^4 x + \ldots\ldots \text{ upto } \infty) \log_e 4\}$$

satisfies the quadratic equation $y^2 - 20y + 64 = 0$ the find the value of x.

;fn $0 < x < \pi$ vkSj O;atd

$$\exp\{(1 + |\cos x| + \cos^2 x + |\cos^3 x| + \cos^4 x + \ldots\ldots \text{ upto } \infty) \log_e 4\}$$

f}?kkr lehdj.k $y^2 - 20y + 64 = 0$ dks larq"V djrk gks] rks x dk eku Kkr dhft,A

Ans. $\frac{\pi}{2}, \frac{2\pi}{3}, \frac{\pi}{3}$

$\frac{n}{1.2.3} + \frac{n-1}{2.3.4} + \ldots\ldots + \frac{1}{n(n+1)(n+2)}$. (i) $1^2 - \frac{2^2}{5} + \frac{3^2}{5^2} - \frac{4^2}{5^3} + \frac{5^2}{5^4} - \frac{6^2}{5^5} + \ldots\ldots\ \infty.$

Statement 1 : The series for which sum to n terms, S_n, is given by $S_n = 5n^2 + 6n$ is an A.P.

Statement 2 : The sum to n terms of an A.P. having non–zero common difference is a quadratic in n, i.e., $an^2 + bn$.

(A) Statement –1 is true, Statement - 2 is true ; Statement - 2 is correct explanation for statement–1.

(B*) Statement–1 is true, Statement-2 is true;Statement–2 is not a correct explanation for statement-1.

(C) Statement-1 is true, Statement-2 is false.

(D) Statement –1 is false, Statement – 2 is true.

Comprehension

In a sequence of (4n + 1) terms the first (2n + 1) terms are in AP whose common difference is 2, and the last (2n + 1) terms are in GP whose common ratio 0.5. If the middle terms of the AP and GP are equal, then

Middle term of the sequence is

(A*) $\frac{n\,.\,2^{n+1}}{2^n - 1}$ (B) $\frac{n\,.\,2^{n+1}}{2^{2n} - 1}$ (C) $n\,.\,2^n$ (D) None of these

First term of the sequence is

(A) $\frac{4n + 2n\,.\,2^n}{2^n - 1}$ (B*) $\frac{4n - 2n\,.\,2^n}{2^n - 1}$ (C) $\frac{2n - n\,.\,2^n}{2^n - 1}$ (D) $\frac{2n + n\,.\,2^n}{2^n - 1}$

Middle term of the GP is

(A) $\frac{2^n}{2^n - 1}$ (B) $\frac{n\,.\,2^n}{2^n - 1}$ (C) $\frac{n}{2^n - 1}$ (D*) $\frac{2n}{2^n - 1}$

If $a_n = \frac{3}{4} - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + \ldots\ldots (-1)^{n-1}\left(\frac{3}{4}\right)^n$ and $b_n = 1 - a_n$, then find the minimum natural number n_0 such that $b_n > a_n \ \forall \ n > n_0$ **[IIT – 2006, 6]**

Let a, b be positive real numbers. If a, A_1, A_2, b are in arithmetic progression, a, G_1, G_2, b are in geometric progression and a, H_1, H_2, b are in harmonic progression, show that

$$\frac{G_1\,G_2}{H_1\,H_2} = \frac{A_1 + A_2}{H_1 + H_2} = \frac{(2a + b)\,(a + 2b)}{9\,a\,b}.$$ **[IIT – 2002, 5]**

Let α, β be the roots of $x^2 - x + p = 0$ and γ, δ be the roots of $x^2 - 4x + q = 0$. If α, β, γ, δ are in G.P., then the integral values of p and q respectively, are **[IIT - 2001]**

;fn lehdj.k $x^2 - x + p = 0$ ds ewy α, β gS vkSj lehdj.k $x^2 - 4x + q = 0$ ds ewy γ, δ gSA ;fn α, β, γ, δ xq-Js-esa gks] rks p rFkk q ds iw.kkZad eku Øe'k% gSa & **[IIT-2001]**

(A*) –2, –32 (B) –2, 3 (C) –6, 3 (D) –6, –32

For a positive integer n, let $a(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots\ldots \frac{1}{(2^n) - 1}$. Then **[IIT - 1999, 3]**

fdlh /kukRed iw.kkZd n ds fy, ;fn $a(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots\ldots \frac{1}{(2^n) - 1}$ gks] rks **[IIT-1999, 3]**

(A*) a (100) ≤ 100 (B) a (100) > 100 (C) a (200) ≤ 100 (D*) a(200) > 100

If cos (x – y), cos x and cos (x + y) are in H.P., then cosx sec $\left(\frac{y}{2}\right)$ = ______

Ans. $\pm\sqrt{2}$

Let a_1, a_2, a_3,..... cannot be terms of an AP. If $\frac{a_1 + a_2 + ...+ a_p}{a_1 + a_2 + ...+ a_q} = \frac{p^2}{q^2}$, $p \neq q$, then $\frac{a_6}{a_{21}}$ equals :

(A) $\frac{7}{2}$ (B) $\frac{2}{7}$ (C*) $\frac{11}{41}$ (D) $\frac{41}{11}$

Let R_1 and R_2 respectively be the maximum ranges up and down an inclined plane and R be the maximum range on the horizontal plane. Then R_1, R_2, R_3 are in :

(A) arithmetrico-geometric progression (AGP) (B) AP (C) GP (D*) HP

The sum to n term of the series 1(1!) + 2(2!) + 3(3!) + ...

(A*) (n + 1) ! – 1 (B) (n – 1) ! – 1 (C) (n – 1) ! + 1 (D) (n + 1) ! – 1

The sum of all possible products of first n natural numbers taken two by two is

(A*) $\frac{1}{24}n(n+1)(n-1)(3n+2)$ (B) $\frac{n(n+1)(2n+1)}{6}$

(C) $\frac{n(n+1)(2n-1)(n+3)}{24}$ (D) None of these

mple 2 Let $a_n = \sum_{k=1}^{n} \frac{1}{k(n+1-k)}$, then for $n \geq 2$

(a) $a_{n+1} > a_n$ (b) $a_{n+1} < a_n$

(c) $a_{n+1} = a_n$ (d) $a_{n+1} - a_n = 1/n$

(b)

mple 9 If $1^2 + 2^2 + 3^2 + \ldots + 2003^2$

$= (2003)(4007)(334)$

$(1)(2003) + (2)(2002) + (3)(2001) + \ldots + (2003)(1)$

$= (2003)(334)(x)$,

ı x equals

(a) 2005 (b) 2004 (c) 2003 (d) 2001

. (a)

Example 11 If

$$(1 + 3 + 5 + \dots + p) + (1 + 3 + 5 + \dots + q)$$
$$= (1 + 3 + 5 + \dots + r)$$

where each set of parentheses contains the sum of consecutive odd integers as shown, the smallest possible value of $p + q + r$, (where $p > 6$) is

(a) 12 (b) 21 (c) 45 (d) 54

Ans. (b)

Example 23 S_n, the sum to n terms of the series

$(n^2 - 1^2) + 2(n^2 - 2^2) + 3(n^2 - 3^2) + \dots$

is

(a) $\frac{1}{4}n^2(n^2 - 1)$ (b) $\frac{1}{4}n(n + 1)^2$

(c) 0 (d) $2n(n^2 - 1)$

Ans. (a)

Example 34 The sum upto $(2n + 1)$ terms of the series

$a^2 - (a + d)^2 + (a + 2d)^2 - (a + 3d)^2 + \dots$ is

(a) $a^2 + 3nd^2$

(b) $a^2 + 2nad + n(n - 1)d^2$

(c) $a^2 + 3nad + n(n - 1)d^2$

(d) $a^2 + 2nad + n(2n + 1)d^2$

Ans. (d)

Example 44 If $H_n = 1+\frac{1}{2}+\ldots+\frac{1}{n}$, then value of $S_n = 1+\frac{3}{2}+\frac{5}{3}+\ldots+\frac{2n-1}{n}$ is

(a) $H_n + n$ (b) $2n - H_n$

(c) $(n - 1) + H_n$ (d) $H_n + 2n$

Ans. (b)

Example 46 If a, b, c are in H.P., then the expression

$$E = \left(\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right)\left(\frac{1}{c}+\frac{1}{a}-\frac{1}{b}\right) \text{ equals}$$

(a) $\frac{2}{bc}-\frac{1}{b^2}$ (b) $\frac{1}{4}\left(\frac{3}{c^2}+\frac{2}{ca}-\frac{1}{a^2}\right)$

(c) $\frac{3}{b^2}-\frac{2}{ab}$ (d) none of these.

Ans. (a), (b), (c)

Example 48 If a, b, c are in H.P., then

(a) $\frac{a}{b+c-a}, \frac{b}{c+a-b}, \frac{c}{a+b-c}$ are in H.P.

(b) $\frac{2}{b} = \frac{1}{b-a} + \frac{1}{b-c}$

(c) $a - \frac{b}{2}, \frac{b}{2}, c - \frac{b}{2}$ are in G.P.

(d) $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in H.P.

Ans. (a), (b), (c), (d)

Example 53 In the nth row of the triangle

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 2 & & 3 & & \\ & 4 & & 5 & & 6 & \\ 7 & & 8 & & 9 & & 10 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \end{array}$$

(a) Last term $= \frac{1}{2} n (n + 1)$

(b) First term $= \frac{1}{2} (n^2 - n + 2)$

(c) Sum $= \frac{1}{2} n (n^2 + 1)$

(d) Sum $= \frac{1}{2} n^2 (n + 1)$

Ans. (a), (b), (c)

Example 55 Let $a_n = \underbrace{(111 \dots 1)}_{n \text{ times}}$, then

(a) a_{912} is not prime. (b) a_{951} is not prime.

(c) a_{480} is not prime (d) a_{91} is not prime.

Ans. (a), (b), (c), (d)

Paragraph for Question Nos. 66 to 70

Sum of the following three series is given

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\ldots\ldots = \log 2 \quad (1)$$

$$1+\frac{1}{3}-\frac{1}{5}-\frac{1}{7}+\frac{1}{9}+\frac{1}{11}-\ldots\ldots = \frac{\pi}{2\sqrt{2}} \quad (2)$$

$$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}\ldots\ldots = \frac{\pi}{4} \quad (3)$$

Example 66 Sum of the series

$$1-2\left(\frac{1}{(3)(5)}+\frac{1}{(7)(9)}+\frac{1}{(11)(13)}+\ldots\ldots\right)$$

is

(a) $\pi/2$ (b) $\pi/2-1$ (c) $\pi/4$ (d) $\pi/4+1$

Example 67 Sum of the series

$$1-\frac{1}{7}+\frac{1}{9}-\frac{1}{15}+\frac{1}{17}\ldots\ldots \text{ upto } \infty$$

is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{8}(1+\sqrt{2})$

(c) $\frac{\pi}{4\sqrt{2}}-1$ (d) $\frac{\pi}{4\sqrt{2}}$

Example 68 Sum of the series

$$\frac{1}{1.2.3}+\frac{1}{5.6.7}+\frac{1}{9.10.11}+\ldots\ldots \text{ upto } \infty$$

is

(a) $\frac{1}{4}\log 2$ (b) $\frac{1}{6}\log 2$

(c) $\frac{1}{3}\log 2$ (d) $\log 4$

Example 69 Sum of the series

$$1-\frac{1}{2}-\frac{1}{3}+\frac{1}{4}+\frac{1}{5}-\frac{1}{6}-\frac{1}{7}+\ldots\ldots \text{ upto } \infty$$

is

(a) $\frac{\pi}{4}-\log\sqrt{2}$ (b) $\frac{\pi}{4}+\log\sqrt{2}$

(c) $\pi-\log 2$ (d) $\pi+\log 2$

Example 70 Sum of the series

$$\left(1-\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{1}{6}-\frac{1}{8}\right)+\left(\frac{1}{5}-\frac{1}{10}-\frac{1}{12}\right)+\ldots \text{ upto } \infty$$

is

(a) $\frac{1}{2}\log 2$ (b) $\frac{1}{3}\log 2$

(c) $\log 4$ (d) $\pi-\log 2$

Ans. 66. (c), 67. (b), 68. (a), 69. (a), 70. (a)

∴

Takin

$\sum$

69.

70.

∴

Para

Give

$f(r)$

then

Exa

$\sum_{r=1}^{\infty}$

Exa

Paragraph for Question Nos. 81 to 85

Let V_r denote the sum of first r terms of an arithmetic progression (A.P.) whose first term is r and the common difference is $(2r - 1)$. Let $T_r = V_{r+1} - V_r - 2$;

$Q_r = T_{r+1} - T_r$, $W_r = \frac{1}{T_r} + \frac{1}{4(r+1)}$ and

$X_r = 3^{Q_r}$ for $r = 1, 2, \ldots$

81. The sum $V_1 + V_2 + \ldots + V_n$ is:
   (a) $\frac{1}{12}n(n + 1)(3n^2 - n + 1)$
   (b) $\frac{1}{12}(n + 1)(3n^2 + n + 2)$
   (c) $\frac{1}{2}n(2n^2 - n + 1)$
   (d) $\frac{1}{3}(2n^3 - 2n + 3)$
82. T_r is always:
   (a) an odd number
   (b) an even number
   (c) a prime number
   (d) a composite number
83. Which one of the following is a correct statement?
   (a) $Q_1, Q_2, Q_3, \ldots$ are in A.P. with common difference 5.
   (b) $Q_1, Q_2, Q_3, \ldots$ are in A.P. with common difference 6.
   (c) $Q_1, Q_2, Q_3, \ldots$ are in A.P. with common difference 11.
   (d) $Q_1 = Q_2 = Q_3 = \ldots$
84. $W_1, W_2, W_3, \ldots$ are in
   (a) A.P. (b) G.P.
   (c) H.P. (d) none of these
85. $X_1, X_2, X_3, \ldots$ are in
   (a) A.P. (b) G.P.
   (c) H.P. (d) none of these.

Ans. 81. (b) 82. (d) 83. (b) 84. (c) 85. (b).

Let A_1, G_1, H_1 denote the arithmetic, geometric and harmonic means, respectively of two distinct positive numbers. For $n \geq 2$, let A_{n-1} and H_{n-1} has arithmetic, geometric and harmonic means as A_n, G_n, H_n respectively.

55. Which of the following statements is a correct statement?
    (a) $G_1 > G_2 > G_3 > \cdots$
    (b) $G_1 < G_2 < G_3 < \cdots$
    (c) $G_1 = G_2 = G_3 = \cdots$
    (d) $G_1 < G_3 < G_5 < \cdots$ and $G_2 > G_4 > G_6 > \cdots$
56. Which of the following statements is correct?
    (a) $A_1 > A_2 > \cdots$
    (b) $A_1 < A_2 < A_3 < \cdots$
    (c) $A_1 > A_3 > A_5 > \cdots$ and $A_2 < A_4 < A_6 < \cdots$
    (d) $A_1 < A_3 < A_5 < \cdots$ and $A_2 > A_4 > A_6 \cdots$
57. Which of the following statements is correct?
    (a) $H_1 > H_2 > H_3 > \cdots$
    (b) $H_1 < H_2 < H_3 < \cdots$
    (c) $H_1 > H_3 > H_5 > \cdots$ and $H_2 < H_4 < H_6 < \cdots$
    (d) $H_1 < H_3 < H_5 < \cdots$ and $H_2 > H_4 > H_6 > \cdots$

[2007]

Suppose four distinct positive numbers a_1, a_2, a_3, a_4 are in G.P. Let $b_1 = a_1$, $b_2 = b_1 + a_2$, $b_3 = b_2 + a_3$ and $b_4 = b_3 + a_4$.

STATEMENT-1: The numbers b_1, b_2, b_3, b_4 are neither in A.P. nor in G.P.

STATEMENT-2: The numbers b_1, b_2, b_3, b_4 are in H.P. [2008]

If sum of first n terms of an A.P. is cn^2, then sum of squares of these n terms is

(a) $\frac{1}{6}n(4n^2 - 1)c^2$

(b) $\frac{1}{3}n(4n^2 + 1)c^2$

(c) $\frac{1}{3}n(4n^2 - 1)c^2$

(d) $\frac{1}{6}n(4n^2 + 1)c^2$ [2009]

➤ **Example 13.** How many terms are identical in the two arithmetic progressions 2, 4, 6, 8,.... up to 100 terms and 3, 6, 9,... up to 80 terms.

Example 18. Sum to n terms

$$\left[\frac{1}{1.3}+\frac{2}{1.3.5}+\frac{3}{1.3.5.7}+\frac{4}{1.3.5.7.9}+....\right]$$

➤ **Problem 15.** If $f(t) = 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{t}$,

prove that $\sum_{k=1}^{n} (2k + 1) f(k) = (n + 1)^2 . f(n) - \frac{n(n+1)}{2}$

➤ **Problem16.** Let $H_{1n} = 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n}$ and

$$H_{2n} = \frac{n+1}{2} - \left[\frac{1}{n(n-1)} + \frac{2}{(n-1)(n-2)} + ... + \frac{n-2}{2.3}\right].$$

Show that $H_{1n} = H_{2n}$

Let V_r denote the sum of first r terms of an arithmetic progression (A.P.) whose first term is r and the common difference is $(2r - 1)$. Let $T_r = V_{r+1} - V_r - 2$ and $Q_t = T_{r+1} - T_r$ for $r = 1, 2, ...$

47. The sum $V_1 + V_2 + ... + V_n$ is **[IIT - 2007]**

(A) $\frac{1}{12}n(n+1)(3n^2 - n + 1)$

(B) $\frac{1}{12}n(n+1)(3n^2 + n + 2)$

(C) $1/2n(2n^2 - n + 1)$

(D) $1/3(2n^3 - 2n + 3)$

48. T_r is always **[IIT - 2007]**

(A) an odd number

(B) an even number

(C) a prime number

(D) a composite number

49. Which one of the following is a correct statement? **[IIT - 2007]**

(A) $Q_1, Q_2, Q_3, ...$ are in A.P., with common difference 5

(B) $Q_1, Q_2, Q_3, ...$ are in A.P., with common difference 6

(C) $Q_1, Q_2, Q_3, ...$ are in A.P., with common difference 11

(D) $Q_1 = Q_2 = Q_3 = ...$

If $(b + c - a)/a, (c + a - b)/b, (a + b - c)/c$ are in A.P., then prove that $1/a, 1/b, 1/c$ are also in A.P.

If a, b, c are in A.P., then prove that the following are also in A.P.

(i) $a^2(b + c), b^2(c + a), c^2(a + b)$

(ii) $\frac{1}{\sqrt{b}+\sqrt{c}}, \frac{1}{\sqrt{c}+\sqrt{a}}, \frac{1}{\sqrt{a}+\sqrt{b}}$

(iii) $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$

Find the sum to n terms of the series,

$$1+\left(1+\frac{1}{2}+\frac{1}{2^2}\right)+\left(1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}\right)+\cdots$$

Find the sum of the infinite series,

$$1+\left(1+\frac{1}{5}\right)\left(\frac{1}{2}\right)+\left(1+\frac{1}{5}+\frac{1}{5^2}\right)\left(\frac{1}{2^2}\right)+\ldots.$$

Find the sum $\sum_{0\le i<j\le n}\sum 1$.

Let the terms $a_1, a_2, a_3, \ldots, a_n$ be in G.P. with common ratio r. Let S_k denote the sum of first k terms of this G.P.. Prove that

$$S_{m-1}\times S_m=\frac{r+1}{r}\sum_{1\le i<j\le n}\sum a_i a_j.$$

Find the sum of the series $\sum_{r=1}^{99}\left(\frac{1}{r\sqrt{r+1}+(r+1)\sqrt{r}}\right)$.

Find the sum of firs 100 terms of the series whose general term is given by $T_r=(r^2+1)\,r!$.

4

Find the sum

$$\frac{3}{1!+2!+3!}+\frac{4}{2!+3!+4!}+\ldots+\frac{1000}{998!+999!+1000!}.$$

$$\text{Let } S=\frac{\sqrt{1}}{1+\sqrt{1}+\sqrt{2}}+\frac{\sqrt{2}}{1+\sqrt{2}+\sqrt{3}}+\frac{\sqrt{3}}{1+\sqrt{3}+\sqrt{4}}+\ldots+\frac{\sqrt{n}}{1+\sqrt{n}+\sqrt{n+1}}=10$$

Then find the value of n.

Find the value of $\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\frac{1}{3^i3^j3^k}$ $(i\ne j\ne k)$.

Find the sum $\sum_{j=1}^{10}\sum_{\substack{i=1\\ i\ne j}}^{10} i\times 2^j$.

Let $a_1, a_2, ..., a_n$ be real numbers such that

$$\sqrt{a_1} + \sqrt{a_2 - 1} + \sqrt{a_3 - 2} + ... + \sqrt{a_n - (n-1)} = \frac{1}{2}(a_1 + a_2 + ... + a_n) - \frac{n(n-3)}{4}.$$

Then the value of find the value of $\sum_{i=1}^{100} a_i$

A sequence of numbers A_n, $n = 1, 2, 3, \ldots$ is defined as follows:

$A_1 = \frac{1}{2}$ and for each $n \geq 2$, $A_n = \left(\frac{2n-3}{2n}\right) A_{n-1}$

Then prove that $\sum_{r=1}^{n} A_r < 1$, $n \geq 1$.

If $f : R \to R$ is continuous such that $f(x) - f\left(\frac{x}{2}\right) = \frac{4x^2}{3}$ for all $x \in R$ and $f(0) = 0$, find the value of $f\left(\frac{3}{2}\right)$.

Find the value of value of $\dfrac{\sum_{r=1}^{n} \frac{1}{r}}{\sum_{k=1}^{n} \frac{k}{(2n-2k+1)(2n-k+1)}}$.

Sol. Let

Find the sum $\sum_{n=1}^{\infty} \frac{6^n}{(3^n - 2^n)(3^{n+1} - 2^{n+1})}$

If $|a| < 1$ and $|b| < 1$, then the sum of the series $1 + (1 + a)b + (1 + a + a^2)b^2 + (1 + a + a^2 + a^3)b^3 + \cdots$ is

(1) $\frac{1}{(1-a)(1-b)}$ (2) $\frac{1}{(1-a)(1-ab)}$

(3) $\frac{1}{(1-b)(1-ab)}$ (4) $\frac{1}{(1-a)(1-b)(1-ab)}$

If ω is a complex nth root of unity, then $\sum_{r=1}^{n}(ar + b)\omega^{r-1}$ is equal to

(1) $\frac{n(n+1)a}{2}$ (2) $\frac{nb}{1-n}$

(3) $\frac{na}{\omega - 1}$ (4) none of these

$ABCD$ is a square of length a, $a \in N$, $a > 1$. Let $L_1, L_2, L_3, \ldots$ be points on BC such that $BL_1 = L_1L_2 = L_2L_3 = \cdots = 1$ and $M_1, M_2, M_3, \ldots$ be points on CD such that $CM_1 = M_1M_2 = M_2M_3 = \cdots = 1$. Then $\sum_{n=1}^{a-1}(AL_n^2 + L_nM_n^2)$ is equal to

(1) $\frac{1}{2}a(a-1)^2$ (2) $\frac{1}{2}(a-1)(2a-1)(4a-1)$

(3) $\frac{1}{2}a(a-1)(4a-1)$ (4) none of these

The value of $\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{j} 1 = 220$, then the value of n equals

(1) 11 (2) 12 (3) 10 (4) 9

Let $a_1, a_2, a_3, \ldots, a_{101}$ are in G.P. with $a_{101} = 25$ and $\sum_{i=1}^{201} a_i = 625$. Then the value of $\sum_{i=1}^{201}\frac{1}{a_i}$ equals ________.

$\sum_{r=1}^{50}\frac{r^2}{r^2 + (11-r)^2}$ is equal to ________.

If $\sum_{r=0}^{100} \frac{1}{2^r + 2^{50}} = \frac{n}{2^{50}}$, then the value of n is ______.

Let $<a_n>$ be an arithmetic sequence of 99 terms such that sum of its odd numbered terms is 1000 then the value of $\sum_{r=1}^{50} (-1)^{\frac{r(r+1)}{2}} \cdot a_{2r-1}$ is ______.

Let $S = \sum_{n=1}^{9999} \frac{1}{\left(\sqrt{n} + \sqrt{n+1}\right)\left(\sqrt[4]{n} + \sqrt[4]{n+1}\right)}$, then S equals ______.

Let S denote sum of the series $\frac{3}{2^3} + \frac{4}{2^4 \cdot 3} + \frac{5}{2^6 \cdot 3} + \frac{6}{2^7 \cdot 5} + \cdots \infty$

Then the value of S^{-1} is ______.

The sum $\frac{7}{2^2 \times 5^2} + \frac{13}{5^2 \times 8^2} + \frac{19}{8^2 \times 11^2} + \ldots 10 \text{ terms}$ is S, then the value of 10245 is ______.

Let $S_n = \sum_{k=1}^{4n} (-1)^{\frac{k(k+1)}{2}} k^2$. Then S_n can take value(s)

(1) 1056 (2) 1088 (3) 1120 (4) 1332

(JEE Advanced 2013)

The sides of a right angled triangle are in arithmetic progression. If the triangle has area 24, then what is the length of its smallest side? **(JEE Advanced 2017)**

Let X be the set consisting of the first 2018 terms of the arithmetic progression 1, 6, 11, ..., and Y be the set consisting of the first 2018 terms of the arithmetic progression 9, 16, 23, ... Then, the number of elements in the set $X \cup Y$ is ______. **(JEE Advanced 2018)**

Q.10

Let $7\overbrace{5\cdots5}^{r}7$ denote the $(r+2)$ digit number where the first and the last digits are 7 and the remaining r digits are 5. Consider the sum $S = 77 + 757 + 7557 + \cdots + 7\overbrace{5\cdots5}^{98}7$. If $S = \dfrac{7\overbrace{5\cdots5}^{99}7 + m}{n}$, where m and n are natural numbers less than 3000, then the value of $m + n$ is