MATHEMATICS JEE ADVANCED 2024 MOCK 2

Section 1

Eight questions; +3/0 Marks. The answer to each question is a numerical value. If the numerical value has more than two decimal places, round off to two decimal places.

1. Given $f(x) = x^{13} + 2x^{12} + 3x^{11} + 4x^{10} + ... + 13x + 14$ and denote $N = f(\alpha) \times f(\alpha^2) \times f(\alpha^3) \times ... \times f(\alpha^{14})$ where $\alpha = \cos\left(\frac{2\pi}{15}\right) + i\sin\left(\frac{2\pi}{15}\right)$. The value of M for which $N^{\frac{1}{M}} = 15$ is _____?

2. The number of ways in which 12 identical balls can be grouped into four marked non-empty boxes A, B, C, D such that n(A)<n(B) is _____?

3. Consider the parabolas $(y-k)^2 = 4(x-k)$ and $(x-k)^2 = 4(y-k)$. Let S be the largest circle touching the two parabolas internally in the bounded region. If r is the radius of the circle S, then r^2 is _____?

4. Three positive real numbers x,y,z satisfy

$$x^2 + y^2 = 3^2$$
$$y^2 + yz + z^2 = 4^2$$
$$x^2 + \sqrt{3}xz + z^2 = 5^2$$

Then the value of $2xy + xz + \sqrt{3}yz$ is _____?

5. If you break a pencil into three parts, the probability that the three parts can be arranged to form a triangle is _____?

6. The area enclosed by $\max\{|x|,|y|\} < 2$ and $||x| - |y|| > 1$ is _____?

7. Suppose that both the roots of the equation $x^2 + ax + 2016 = 0$ are positive even integers, then the number of possible values of a is _____?

8. Let $g(x) = \underset{n\to\infty}{Lim}\frac{x^n f(x) + h(x) + 1}{2x^n + 3x + 2}$, $x \neq 1$ and $g(1) = \underset{x\to 1}{Lim}\frac{\sin^2(\pi.2^x)}{\ln(\sec(\pi.2^x))}$ be a continuous function at x=1. Assuming f(x) and h(x) are continuous at x=1, the value of $4g(1) + 2f(1) - h(1)$ is _____?

Section 2

Six questions; +4/0 Marks. Each question has Four options. One or more than one of these four options is/are correct answer(s).

9. Let $f:[0,2] \to \mathbb{R}$ be a continuous function such that $\frac{1}{2}\int_0^2 f(x)dx < f(2)$. Then which of the following statement(s) is/are must be true?

   (a) f must be strictly increasing.

   (b) f must attain a maximum value at x=2.

   (c) f cannot have a minimum at x=2

   (d) f must be an odd function

10. Suppose $a_0, a_1, a_2, a_3\ldots$ is an arithmetic progression with a_0 and a_1 as positive integers. Let $g_0, g_1, g_2, g_3\ldots$ be the geometric progression such that $g_0= a_0$ and $g_1=a_1$. Then which of the following statement(s) is/are must be true?

    (a) We must have $a_5^2 \geq a_0 a_{10}$.

    (b) The sum $a_0+a_1+a_2+\ldots+ a_{10}$ must be a multiple of the integer a_0.

    (c) If $\sum_{i=0}^{\infty} a_i$ is $+\infty$ then $\sum_{i=0}^{\infty} g_i$ is also $+\infty$

    (d) If $\sum_{i=0}^{\infty} g_i$ is finite then $\sum_{i=0}^{\infty} a_i$ is $-\infty$

11. A line L passing through the origin is perpendicular to the lines

$$L_1:(3+t)\vec{i}+(-1+2t)\vec{j}+(4+2t)\vec{k}$$

$$L_2:(3+2s)\vec{i}+(3+2s)\vec{j}+(2+s)\vec{k}$$

where t and s are parameters. The coordinate(s) of the point(s) on L_2 at a distance of $\sqrt{17}$ from the point of intersection of L and L_1 is/are

(a) $\left(\frac{7}{3},\frac{7}{3},\frac{5}{3}\right)$ (b) $\left(\frac{7}{9},\frac{7}{9},\frac{8}{9}\right)$

(c) $(-1,-1,0)$ (d) $(1,1,1)$

12. Suppose A and B are two non-singular matrices such that $AB = BA^2$ and $B^5 = I$, then

(a) $A^{31} = I$ (b) $A^{32} = I$

(c) $AB^n = B^n A^{n+1}$ (d) $AB^n = B^n A^{2^n}$

13. If $I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x} dx$, n=0,1,2…. Then

(a) $I_n = I_{n+2}$ (b) $\sum_{k=1}^{10} I_{2k+1} = 10\pi$

(c) $\sum_{k=1}^{10} I_{2k} = 0$ (d) $I_n = I_{n+1}$

14. A function $y = f(x)$ satisfying the differential equation $\sin x\frac{dy}{dx} - y\cos x + \frac{\sin^2 x}{x^2} = 0$ is such that, $y \to 0$ as $x \to \infty$ then the statement(s) which is/are correct is/are

(b) $\lim_{x\to 0} f(x) = 1$ (b) $\int_0^{\frac{\pi}{2}} f(x)dx$ is less than $\frac{\pi}{2}$

(c) $\int_0^{\frac{\pi}{2}} f(x)dx$ is greater than unity (d) f(x) is an odd function

Section 3

Two Matching list question set; +3/-1 Marks. FOUR options are given in each Multiple Choice Question based on List–I and List–II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.

15. Match the following loci for the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ for a>b.

(I) Locus of point of intersection of two perpendicular tangents	(P) $(x^2+y^2)^2=a^2x^2+b^2y^2$
(II) Locus of foot of perpendicular from any focus upon any tangent	(Q) $4(x^2+y^2)^2=a^2x^2+b^2y^2$
(III)Locus of foot of the perpendicular from centre on any tangent	(R) $x^2+y^2=a^2$
(IV) Locus of mid points of segment OM where M is the foot of the perpendicular from centre O to any tangent	(S) $x^2+y^2=a^2+b^2$

(a) $I \to S$; $II \to R$; $III \to P$; $IV \to Q$

(b) $I \to R$; $II \to S$; $III \to P$; $IV \to Q$

(c) $I \to S$; $II \to R$; $III \to Q$; $IV \to P$

(d) $I \to R$; $II \to S$; $III \to Q$; $IV \to P$

16. $|x|$ represents modulus functions, $[x]$ represents greatest integer function, $\{x\}$ represents fractional function. Match the functions in List I with number of real solutions in List II.

(I) $\lvert 2x-1\rvert=3[x]+2\{x\}$	(P) 0
(II) $\log_{\frac{3}{4}}(\log_8(x^2+7))+\log_{\frac{1}{2}}(\log_{\frac{1}{4}}(x^2+7)^{-1})=-2$	(Q) 1
(III)The number of pairs of integers (x, y) satisfying the equation $xy(x+y+1)=5^{2018}+1$ is	(R) 2
(IV) $x^2=e^x$	(S) 3

(a) $1 \to Q$; $II \to R$; $III \to P$; $IV \to R$

(b) $I \to R$; $II \to S$; $III \to Q$; $IV \to Q$

(c) $I \to R$; $II \to S$; $III \to Q$; $IV \to R$

(d) $1 \to Q$; $II \to R$; $III \to P$; $IV \to Q$

Section 4

Two questions based on the given passage. ONLY ONE of these four options is correct in each question. +3/-1 marks.

Let $f(x) = 12x^2 \int_0^1 yf(y)dy + 20\int_0^1 xy^2 f(y)dy + 4x$

17. The maximum value of f(x) is

(a) 8 (b) 1/8

(c) 16 (d) 1/16

18. The range of $f(-2^x)$ is

(a) $(-\infty, 0)$ (b) $(0, \infty)$

(c) $\left(-\infty, \frac{1}{8}\right)$ (d) $\left(\frac{1}{8}, \infty\right)$