SECTION 1 (Maximum Marks: 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −2 In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
choosing ONLY (A), (B) and (D) will get +4 marks;
choosing ONLY (A) and (B) will get +2 marks;
choosing ONLY (A) and (D) will get +2 marks;
choosing ONLY (B) and (D) will get +2 marks;
choosing ONLY (A) will get +1 mark;
choosing ONLY (B) will get +1 mark;
choosing ONLY (D) will get +1 mark;
choosing no option (i.e. the question is unanswered) will get 0 marks; and
choosing any other combination of options will get −2 marks.

1. If $f(x) = 4x^3 - x^2 - 2x + 1$ and

$$g(x) = \begin{cases} Min\{f(t) : 0 \le t \le x\} & 0 \le x \le 1 \\ 3 - x & 1 < x \le 2 \end{cases}$$ then

(a) $g\left(\frac{1}{4}\right) + g\left(\frac{3}{4}\right) + g\left(\frac{5}{4}\right) = \frac{5}{2}$

(b) $g(x)$ is discontinuous at x=1

(c) $g(x)$ is continuous but not differentiable at x=1

(d) $g(x)$ has no local maxima or minima in $0 \le x \le 2$

2. Let the unit vectors $\vec{a}$ and $\vec{b}$ be perpendicular and the unit vector $\vec{c}$ be inclined at an angle θ to both $\vec{a}$ and $\vec{b}$. If $\vec{c} = \alpha\vec{a} + \beta\vec{b} + \gamma(\vec{a} \times \vec{b})$ then

(a) $\alpha = \beta$ (b) $\gamma^2 = 1 - 2\alpha^2$ (c) $\gamma^2 = -\cos 2\theta$ (d) $\beta^2 = \frac{1 + \cos 2\theta}{2}$

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3. If $\sin^2\frac{A}{2}$, $\sin^2\frac{B}{2}$, $\sin^2\frac{C}{2}$ are in HP, then

(a) a, b, c are in AP (b) a, b, c are in HP

(c) a(b+c), b(c+a), c(a+b) are in AP (d) a(b+c), b(c+a), c(a+b) are in HP

SECTION 2 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −1 In all other cases.

4. The number of monotonically increasing functions from the set {1,2,3,4,5,6} to itself with the property $f(x) \geq x$ for all x, is equal to $\frac{2}{k}\,{}^{11}C_5$, then the value of k is

(a) 5 (b) 6 (c) 7 (d) 8

5. Given that the mean, median, range and the only mode of 200 integers are also 200. If A is the largest integer among those 200 integers, the maximum value of A is

(a) 397 (b) 398 (c) 399 (d) 400

6. Inside a semi-circle of radius 1 unit, two circles of radii r_1 and r_2 are drawn, each touching the circumference and the diameter of the semi-circle also touches each other externally. Then the value of $\max(r_1 + r_2)$ is

(a) $\sqrt{2}-1$ (b) $\sqrt{3}-1$ (c) $2\left(\sqrt{2}-1\right)$ (d) $2\left(\sqrt{3}-1\right)$

7. The equations to the line of greatest slope through the point (7,2,-1) in the plane x-2y+3z=0 (assuming that the axes are so placed that the plane 2x+3y-4z=0 is horizontal) is

(a) $\frac{x-7}{22}=\frac{y-2}{5}=\frac{z+1}{-4}$ (b) $\frac{x-7}{2}=\frac{y-2}{3}=\frac{z-4}{-4}$

(c) $\frac{x-7}{-1}=\frac{y-2}{10}=\frac{z-4}{7}$ (d) $\frac{x-7}{-1}=\frac{y-2}{10}=\frac{z+1}{7}$

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SECTION 3 (Maximum Marks: 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If **ONLY** the correct integer is entered;
Zero Marks : 0 In all other cases.

8. Consider the function $f: R \to R$, defined as follows:

$$f(x) = \begin{cases} (x-1)\min\{x, x^2\} & \text{if } x \geq 0 \\ x\min\left\{x, \frac{1}{x}\right\} & \text{if } x < 0 \end{cases}$$

If $f(x)$ is not differentiable at exactly k points, then k is __________?

9. The maximum number of common normal of the parabolas $y^2 = 4ax$ and $x^2 = 4by$ is __________?

10. Let $A = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}^n = [a_{ij}(\text{n})]$. If $\lim\limits_{n\to\infty} \frac{a_{12}(\text{n})}{a_{22}(\text{n})} = a + \sqrt{b}$, $(a, b \in N)$ then the value of $a+b$ is__________?

11. If $\int_0^{10\pi} (\sin^{-1}|\sin x| + \cos^{-1}(\cos x))\, dx = k\pi^2$, then the value of $\frac{2}{3}k$ is__________?

12. Let $r_1, r_2, \ldots, r_9$ be the distinct complex roots of the polynomial $P(x) = x^9 - 9$. Let $k = \prod\limits_{1 \leq i < j \leq 9} (r_i + r_j)$, that is, product of all numbers of the form $r_i + r_j$, where i and j are integers for which $1 \leq i < j \leq 9$. The value of $\frac{k^2}{9^7}$ is __________?

13. If $k = \tan^2\frac{\pi}{7} + \tan^2\frac{2\pi}{7} + \tan^2\frac{3\pi}{7}$, the sum of digits of k is __________?

MATHSMERIZING

SECTION 4 (Maximum Marks: 12)

- This section contains **TWO (02)** paragraphs.
- Based on each paragraph, there are **TWO (02)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −1 In all other cases.

Passage I

There are 8 seats in the front row of a theatre in which 4 persons are to be seated.

14. The probability of seating them so that no two persons sit side by side, is

(a) $\frac{1}{14}$ (b) $\frac{3}{14}$ (c) $\frac{1}{7}$ (d) $\frac{3}{7}$

15. The probability of seating them so that each person has exactly one neighbour, is

(a) $\frac{1}{14}$ (b) $\frac{3}{14}$ (c) $\frac{1}{7}$ (d) $\frac{3}{7}$

Passage II

A conic pass through the point (2,4) and is such that the segment of any of its tangents at any point contained between the coordinate axes is bisected at the point of tangency.

16. The eccentricity of the conic is

(a) 2 (b) $\sqrt{2}$ (c) $\sqrt{3}$ (d) $\sqrt{\frac{3}{2}}$

17. The foci of the conic are

(b) $(\pm 2\sqrt{2}, 0)$ (b) $(\pm 2\sqrt{2}, \pm 2\sqrt{2})$ (c) $(\pm 4, \pm 4)$ (d) $(\pm 4\sqrt{2}, \pm 4\sqrt{2})$

MATHSMERIZING