1. The line $lx + my + n = 0$ intersects the curve $ax^2 + 2hxy + by^2 = 1$ at the point P and Q. The circle on PQ as diameter passes through the origin. Prove that $n^2(a^2 + b^2) = l^2 + m^2$.

2. Find the equation to the circle which is such that the length of the tangents to it from the points (1, 0), (2, 0) and (3, 2) are 1, $\sqrt{7}$, $\sqrt{2}$ respectively. $2(x^2+y^2)+6x-17y-6=0$

3. A (–a, 0) ; B (a, 0) are fixed points. C is a point which divides AB in a constant ratio $\tan\alpha$. If AC & CB subtend equal angles at P, prove that the equation of the locus of P is $x^2 + y^2 + 2ax\sec 2\alpha + a^2 = 0$.

4. A circle is drawn with its centre on the line $x + y = 2$ to touch the line $4x - 3y + 4 = 0$ and pass through the point (0, 1). Find its equation. $x^2+y^2-2x-2y+1=0$ or $x^2+y^2-42x+38y-39=0$

5. A point moving around circle $(x + 4)^2 + (y + 2)^2 = 25$ with centre C broke away from it either at the point A or point B on the circle and moved along a tangent to the circle passing through the point D (3, – 3). Find the following.
   - (i) Equation of the tangents at A and B. $4x+3y=3,\ 3x-4y=21$
   - (ii) Coordinates of the points A and B. A(0,1) B(–1,–6)
   - (iii) Angle ADB and the maximum and minimum distances of the point D from the circle. $90°,\ 5(\sqrt{2}\pm1)$
   - (iv) Area of quadrilateral ADBC and the ΔDAB. $25\sqrt{2}$, 12.5
   - (v) Equation of the circle circumscribing the ΔDAB and also the intercepts made by this circle on the coordinate axes. $x^2+y^2+x+5y-6=0$

6. Find the locus of the mid point of the chord of a circle $x^2 + y^2 = 4$ such that the segment intercepted by the chord on the curve $x^2 - 2x - 2y = 0$ subtends a right angle at the origin. $x^2+y^2-2x-2y=0$

7. Find the equation of a line with gradient 1 such that the two circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 10x - 14y + 65 = 0$ intercept equal length on it. $2x-2y-3=0$

8. Tangents are drawn to the concentric circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2$ at right angle to one another. Show that the locus of their point of intersection is a 3rd concentric circle. Find its radius. $\sqrt{a^2+b^2}$

9. Consider a circle S with centre at the origin and radius 4. Four circles A, B, C and D each with radius unity and centres (–3, 0), (–1, 0), (1, 0) and (3, 0) respectively are drawn. A chord PQ of the circle S touches the circle B and passes through the centre of the circle C. If the length of this chord can be expressed as $\sqrt{x}$, find x. = 63.

10. Consider a curve $ax^2 + 2hxy + by^2 = 1$ and a point P not on the curve. A line is drawn from the point P intersects the curve at points Q & R. If the product PQ. PR is independent of the slope of the line, then show that the curve is a circle.

11. The line $2x - 3y + 1 = 0$ is tangent to a circle $S = 0$ at (1, 1). If the radius of the circle is $\sqrt{13}$. Find the equation of the circle S. $x^2+y^2+2x-8y+4=0,\ x^2+y^2-6x+4y=0$

12. Find the equation of the circle which passes through the point (1, 1) & which touches the circle $x^2 + y^2 + 4x - 6y - 3 = 0$ at the point (2, 3) on it. $x^2+y^2+x-6y+3=0$

13. Let a circle be given by $2x(x - a) + y(2y - b) = 0$, $(a \neq 0,\ b \neq 0)$. Find the condition on a & b if two chords, each bisected by the x-axis, can be drawn to the circle from the point $\left(a, \frac{b}{2}\right)$. $a^2 > 2b^2$

(14) Show that the equation of a straight line meeting the circle $x^2 + y^2 = a^2$ in two points at equal distances 'd' from a point (x_1, y_1) on its circumference is $xx_1 + yy_1 - a^2 + \frac{d^2}{2} = 0$.

(15) The radical axis of the circles $x^2 + y^2 + 2gx + 2fy + c = 0$ and $2x^2 + 2y^2 + 3x + 8y + 2c = 0$ touches the circle $x^2 + y^2 + 2x - 2y + 1 = 0$. Show that either $g = 3/4$ or $f = 2$.

(16) Find the equation of the circle through the points of intersection of circles $x^2 + y^2 - 4x - 6y - 12=0$ and $x^2 + y^2 + 6x + 4y - 12 = 0$ & cutting the circle $x^2 + y^2 - 2x - 4 = 0$ orthogonally. $(-\frac{1}{2}, \frac{1}{2})$ $(-4, 4)$

(17) The centre of the circle $S = 0$ lie on the line $2x - 2y + 9 = 0$ & $S = 0$ cuts orthogonally the circle $x^2 + y^2 = 4$. Show that circle $S = 0$ passes through two fixed points & find their coordinates. $x^2+y^2+16x+14y-4=$

C.18 (18) Find the equation of a circle passing through the origin if the line pair, $xy - 3x + 2y - 6 = 0$ is orthogonal to it. If this circle is orthogonal to the circle $x^2 + y^2 - kx + 2ky - 8=0$ then find the value of k. $k=1$.

(19) Find the equation of the circle whose radius is 3 and which touches the circle $x^2 + y^2 - 4x - 6y - 12=0$ internally at the point $(-1, -1)$. $5x^2+5y^2-8x-14y-32=0$

(20) Show that the locus of the centres of a circle which cuts two given circles orthogonally is a straight line & hence deduce the locus of the centers of the circles which cut the circles $x^2 + y^2 + 4x - 6y + 9=0$ & $x^2 + y^2 - 5x + 4y + 2 = 0$ orthogonally. Interpret the locus. $9x-10y+7=0$, Radical axis

(21) A variable circle passes through the point A (a, b) & touches the x-axis; show that the locus of the other end of the diameter through A is $(x - a)^2 = 4by$.

(22) Consider a family of circles passing through two fixed points A (3, 7) & B(6, 5). Show that the chords in which the circle $x^2 + y^2 - 4x - 6y - 3 = 0$ cuts the members of the family are concurrent at a point. Find the coordinates of this point. $(2, \frac{23}{3})$

(23) Find the equation of circle passing through (1, 1) belonging to the system of co–axal circles that are tangent at (2, 2) to the locus of the point of intersection of mutually perpendicular tangent to the circle $x^2 + y^2 = 4$. $x^2+y^2-3x-3y+4=0$

(24) Find the locus of the mid point of all chords of the circle $x^2 + y^2 - 2x - 2y = 0$ such that the pair of lines joining (0, 0) & the point of intersection of the chords with the circles make equal angle with axis of x. $x+y=2$

(25) The circle $C : x^2 + y^2 + kx + (1 + k)y - (k + 1) = 0$ passes through the same two points for every real number k. Find

(i) the coordinates of these two points. $(\frac{1}{2}, \frac{1}{2})$, $(1, 0)$

(ii) the minimum value of the radius of a circle C. $r = \frac{1}{2\sqrt{2}}$

(26) Show that the locus of the point the tangents from which to the circle $x^2 + y^2 - a^2 = 0$ include a constant angle α is $(x^2 + y^2 - 2a^2)^2 \tan^2\alpha = 4a^2(x^2 + y^2 - a^2)$.

(27) A circle with center in the first quadrant is tangent to $y = x + 10$, $y = x - 6$, and the y-axis. Let (h, k) be the center of the circle. If the value of $(h + k) = a + b\sqrt{a}$ where $\sqrt{a}$ is a surd, find the value of a + b. $=10$

(28) A circle is described to pass through the origin and to touch the lines $x = 1$, $x + y = 2$. Prove that the radius of the circle is a root of the equation $\left(3 - 2\sqrt{2}\right)t^2 - 2\sqrt{2}\,t + 2 = 0$.

(29) Find the condition such that the four points in which the circle $x^2 + y^2 + ax + by + c = 0$ and $x^2 + y^2 + a'x + b'y + c' = 0$ are intercepted by the straight lines $Ax + By + C = 0$ & $A'x + B'y + C' = 0$ respectively, lie on another circle.

$\begin{vmatrix} a-a' & b-b' & c-c' \\ A & B & C \\ A' & B' & C' \end{vmatrix} = 0$

(30) A circle C is tangent to the x and y axis in the first quadrant at the points P and Q respectively. BC and AD are parallel tangents to the circle with slope -1. If the points A and B are on the y-axis while C and D are on the x-axis and the area of the figure ABCD is $900\sqrt{2}$ sq. units then find the radius of the circle. $r = 15$

(31) The circle $x^2 + y^2 - 4x - 4y + 4 = 0$ is inscribed in a triangle which has two of its sides along the coordinate axes. The locus of the circumcentre of the triangle is $x + y - xy + K\sqrt{x^2 + y^2} = 0$. Find K $= 1$

(32) An isosceles right angled triangle whose sides are 1, 1, $\sqrt{2}$ lies entirely in the first quadrant with the ends of the hypotenuse on the coordinate axes. If it slides prove that the locus of its centroid is $(3x - y)^2 + (x - 3y)^2 = \dfrac{32}{9}$.

(33) Find the equation of a circle which touches the lines $7x^2 - 18xy + 7y^2 = 0$ and the circle $x^2 + y^2 - 8x - 8y = 0$ and is contained in the given circle. $x^2+y^2-12x-12y+64=0$

(34) Let W_1 and W_2 denote the circles $x^2 + y^2 + 10x - 24y - 87 = 0$ and $x^2 + y^2 - 10x - 24y + 153 = 0$ respectively. Let m be the smallest possible value of 'a' for which the line $y = ax$ contains the centre of a circle that is externally tangent to W_2 and internally tangent to W_1. Given that $m^2 = \dfrac{p}{q}$ where p and q are relatively prime integers, find $(p + q)$. $= 169$

(35) Find the equation of the circle which passes through the origin, meets the x-axis orthogonally & cuts the circle $x^2 + y^2 = a^2$ at an angle of 45°. $x^2+y^2 \pm \sqrt{2}ax = 0$

(36) The ends A, B of a fixed straight line of length 'a' & ends A' & B' of another fixed straight line of length 'b' slide upon the axis of x & the axis of y (one end on axis of x & the other on axis of y). Find the locus of the centre of the circle passing through A, B, A' & B'. $(2ax-2by)^2+(2bx-2ay)^2=(a^2-b^2)^2$

(37) The angle between a pair of tangents drawn from a point P to the circle $x^2 + y^2 + 4x - 6y + 9\sin^2\alpha + 13\cos^2\alpha = 0$ is 2α. The equation of the locus of the point P is

(A) $x^2 + y^2 + 4x - 6y + 4 = 0$ (B) $x^2 + y^2 + 4x - 6y - 9 = 0$

(C) $x^2 + y^2 + 4x - 6y - 4 = 0$ (D) $x^2 + y^2 + 4x - 6y + 9 = 0$ ✓

(38) Find the intervals of values of a for which the line $y + x = 0$ bisects two chords drawn from a point $\left(\dfrac{1+\sqrt{2}a}{2}, \dfrac{1-\sqrt{2}a}{2}\right)$ to the circle; $2x^2 + 2y^2 - (1+\sqrt{2}a)x - (1 - \sqrt{2}a)y = 0$.

$a < -2$ or $a > 2$

39 (a) The chords of contact of the pair of tangents drawn from each point on the line $2x + y = 4$ to the circle $x^2 + y^2 = 1$ pass through the point ______. $\left(\frac{1}{2}, \frac{1}{4}\right)$

(b) Let C be any circle with centre $(0, \sqrt{2})$. Prove that at the most two rational point can be there on C. (A rational point is a point both of whose co-ordinate are rational numbers).

40 The number of common tangents to the circle $x^2 + y^2 = 4$ & $x^2 + y^2 - 6x - 8y = 24$ is :

(A) 0 (B) 1 (C) 3 (D) 4

40 C_1 & C_2 are two concentric circles, the radius of C_2 being twice that of C_1. From a point P on C_2, tangents PA & PB are drawn to C_1. Prove that the centroid of the triangle PAB lies on C_1.

41 If two distinct chords, drawn from the point (p, q) on the circle $x^2 + y^2 = px + qy$ (where $pq \neq q$) are bisected by the x – axis, then :

(A) $p^2 = q^2$ (B) $p^2 = 8q^2$ (C) $p^2 < 8q^2$ (D) $p^2 > 8q^2$

42 Let T_1, T_2 be two tangents drawn from $(-2, 0)$ onto the circle $C : x^2 + y^2 = 1$. Determine the circles touching C and having T_1, T_2 as their pair of tangents. Further, find the equations of all possible common tangents to these circles, when taken two at a time. $(x-4)^2 + y^2 = 9$, $\left(x + \frac{4}{3}\right)^2 + y^2 = \frac{1}{9}$

43 The triangle PQR is inscribed in the circle, $x^2 + y^2 = 25$. If Q and R have co-ordinates (3, 4) & (– 4, 3) respectively, then $\angle$ QPR is equal to :

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

44 Find the point on the straight line, $y = 2x + 11$ which is nearest to the circle, $16(x^2 + y^2) + 32x - 8y - 50 = 0$. $\left(-\frac{9}{2}, 2\right)$

45 Let PQ and RS be tangents at the extremities of the diameter PR of a circle of radius r. If PS and RQ intersect at a point X on the circumference of the circle then 2r equals

(A) $\sqrt{PQ \cdot RS}$ (B) $\frac{PQ + RS}{2}$ (C) $\frac{2PQ \cdot RS}{PQ + RS}$ (D) $\sqrt{\frac{(PQ)^2 + (RS)^2}{2}}$

46 A circle of radius 2 units rolls on the outerside of the circle, $x^2 + y^2 + 4x = 0$, touching it externally. Find the locus of the centre of this outer circle. Also find the equations of the common tangents of the two circles when the line joining the centres of the two circles makes on angle of 60° with x-axis. $x^2 + y^2 + 4x - 12 = 0$; $\sqrt{3}x \mp y = \pm 4 - 2\sqrt{3}$, $x + \sqrt{3}y - 2 = 0$.

47 Let $2x^2 + y^2 - 3xy = 0$ be the equation of a pair of tangents drawn from the origin 'O' to a circle of radius 3 with centre in the first quadrant. If A is one of the points of contact, find the length of OA. $= 3(3 + \sqrt{10})$

48 Tangents TP and TQ are drawn from a point T to the circle $x^2 + y^2 = a^2$. If the point T lies on the line $px + qy = r$, find the locus of centre of the circumcircle of triangle TPQ. $2(px + qy) = r$.

49 If the tangent at the point P on the circle $x^2 + y^2 + 6x + 6y = 2$ meets the straight line $5x - 2y + 6 = 0$ at a point Q on the y-axis, then the length of PQ is

(A) 4 (B) $2\sqrt{5}$ (C) 5 (D) $3\sqrt{5}$

50 If $a > 2b > 0$ then the positive value of m for which $y = mx - b\sqrt{1 + m^2}$ is a common tangent to $x^2 + y^2 = b^2$ and $(x - a)^2 + y^2 = b^2$ is

(A) $\frac{2b}{\sqrt{a^2 - 4b^2}}$ (B) $\frac{\sqrt{a^2 - 4b^2}}{2b}$ (C) $\frac{2b}{a - 2b}$ (D) $\frac{b}{a - 2b}$

51. The radius of the circle, having centre at (2, 1), whose one of the chord is a diameter of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$

(A) 1 (B) 2 (C) 3 (D) $\sqrt{3}$

52. A circle is given by $x^2 + (y-1)^2 = 1$, another circle C touches it externally and also the x-axis, then the locus of its centre is

(A) $\{(x, y) : x^2 = 4y\} \cup \{(x, y) : y \le 0\}$
(B) $\{(x, y) : x^2 + (y-1)^2 = 4\} \cup \{x, y) : y \le 0\}$
(C) $\{(x, y) : x^2 = y\} \cup \{(0, y) : y \le 0\}$.
(D) $\{(x, y) : x^2 = 4y\} \cup \{(0, y) : y \le 0\}$.

53. The length of the tangent drawn from any point on the circle $x^2 + y^2 + 2gx + 2fy + p = 0$ to the circle $x^2 + y^2 + 2gx + 2fy + q = 0$ is:

(A*) $\sqrt{q-p}$ (B) $\sqrt{p-q}$ (C) $\sqrt{q+p}$ (D) none

54. Equation of the circle cutting orthogonally the three circles $x^2 + y^2 - 2x + 3y - 7 = 0$, $x^2 + y^2 + 5x - 5y + 9 = 0$ and $x^2 + y^2 + 7x - 9y + 29 = 0$ is

(A*) $x^2 + y^2 - 16x - 18y - 4 = 0$
(B) $x^2 + y^2 - 7x + 11y + 6 = 0$
(C) $x^2 + y^2 + 2x - 8y + 9 = 0$
(D) none of these

55. The circumference of the circle $x^2 + y^2 - 2x + 8y - q = 0$ is bisected by the circle $x^2 + y^2 + 4x + 12y + p = 0$, then p + q is equal to:

(A) 25 (B) 100 (C*) 10 (D) 48

56. The chords of contact of the pair of tangents drawn from each point on the line $2x + y = 4$ to the circle $x^2 + y^2 = 1$ pass through the point

(A) (1, 2) (B*) $\left(\frac{1}{2}, \frac{1}{4}\right)$ (C) (2, 4) (D) none

57. Find the equation to the circle which touches the axis of x at a distance 3 from the origin and intercepts a distance 6 on the axis of y.

$x^2 + y^2 \pm 6x \pm 6\sqrt{2}y + 9 = 0$

58. Tangents are drawn from the point (h, k) to the circle $x^2 + y^2 = a^2$; prove that the area of the triangle formed by them and the straight line joining their points of contact is $\frac{a(h^2 + k^2 - a^2)^{3/2}}{h^2 + k^2}$

59. Find the equations to the common tangents of the circles $x^2 + y^2 - 2x - 6y + 9 = 0$ and $x^2 + y^2 + 6x - 2y + 1 = 0$

$3x + 4y = 10$, $x = 0$, $y = 4$, $4x - 3y = 0$

60. If $\left(a, \frac{1}{a}\right), \left(b, \frac{1}{b}\right), \left(c, \frac{1}{c}\right)$ & $\left(d, \frac{1}{d}\right)$ are four distinct points on a circle of radius 4 units then, abcd is equal to

(A) 4 (B) 16 (C) 1 (D) none

61. From the point A (0, 3) on the circle $x^2 + 4x + (y-3)^2 = 0$ a chord AB is drawn & extended to a point M such that AM = 2 AB. The equation of the locus of M is:

(A) $x^2 + 8x + y^2 = 0$
(B*) $x^2 + 8x + (y-3)^2 = 0$
(C) $(x-3)^2 + 8x + y^2 = 0$
(D) $x^2 + 8x + 8y^2 = 0$

62) Two thin rods AB & CD of lengths 2a & 2b move along OX & OY respectively, when 'O' is the origin. The equation of the locus of the centre of the circle passing through the extremities of the two rods is:

(A) $x^2 + y^2 = a^2 + b^2$ (B*) $x^2 - y^2 = a^2 - b^2$ (C) $x^2 + y^2 = a^2 - b^2$ (D) $x^2 - y^2 = a^2 + b^2$

63) Let x & y be the real numbers satisfying the equation $x^2 - 4x + y^2 + 3 = 0$. If the maximum and minimum values of $x^2 + y^2$ are M & m respectively, then the numerical value of M − m is:

(A) 2 (B*) 8 (C) 15 (D) none of these

64) The area of the triangle formed by the tangents from the point (4, 3) to the circle $x^2 + y^2 = 9$ and the line joining their point of contact is:

(A*) $\frac{192}{25}$ (B) 192 (C) 25 (D) 250

65) A point A(2, 1) is outside the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ & AP, AQ are tangents to the circle. The equation of the circle circumscribing the triangle APQ is:

(A*) $(x + g)(x - 2) + (y + f)(y - 1) = 0$ (B) $(x + g)(x - 2) - (y + f)(y - 1) = 0$
(C) $(x - g)(x + 2) + (y - f)(y + 1) = 0$ (D) none

66) The locus of the mid points of the chords of the circle $x^2 + y^2 + 4x - 6y - 12 = 0$ which subtend an angle of $\frac{\pi}{3}$ radians at its circumference is:

(A) $(x - 2)^2 + (y + 3)^2 = 6.25$ (B*) $(x + 2)^2 + (y - 3)^2 = 6.25$
(C) $(x + 2)^2 + (y - 3)^2 = 18.75$ (D) $(x + 2)^2 + (y + 3)^2 = 18.75$

67) If the circle C_1: $x^2 + y^2 = 16$ intersects another circle C_2 of radius 5 in such a manner that the common chord is of maximum length and has a slope equal to 3/4, then the co-ordinates of the centre of C_2 are:

(A) $\left(\pm\frac{9}{5}, \pm\frac{12}{5}\right)$ (B*) $\left(\pm\frac{9}{5}, \mp\frac{12}{5}\right)$ (C) $\left(\pm\frac{12}{5}, \pm\frac{9}{5}\right)$ (D) $\left(\pm\frac{12}{5}, \mp\frac{9}{5}\right)$

68) If from any point P on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, tangents are drawn to the circle $x^2 + y^2 + 2gx + 2fy + c\sin^2\alpha + (g^2 + f^2)\cos^2\alpha = 0$ then the angle between the tangents is:

(A) α (B*) 2α (C) $\frac{\alpha}{2}$ (D) none

69) If the length of a common internal tangent to two circles is 7, and that of a common external tangent is 11, then the product of the radii of the two circles is:

(A) 36 (B) 9 (C*) 18 (D) 4

70) Through a fixed point (h, k) secants are drawn to the circle $x^2 + y^2 = r^2$. Show that the locus of the midpoints of the portions of the secants intercepted by the circle is $x^2 + y^2 = hx + ky$.

71) Find the equation of the circle which cuts each of the circles, $x^2 + y^2 = 4$, $x^2 + y^2 - 6x - 8y + 10 = 0$ & $x^2 + y^2 + 2x - 4y - 2 = 0$ at the extremities of a diameter. $x^2+y^2-4x-6y-4=0$

72) Find the values of a for which the point (2a, a + 1) is an interior point of the larger segment of the circle $x^2 + y^2 - 2x - 2y - 8 = 0$ made by the chord whose equation is $x - y + 1 = 0$. $a \in (0, \frac{9}{5})$

73) Column – I

Column – II

(A)	Number of values of a for which the common chord of the circles $x^2 + y^2 = 8$ and $(x - a)^2 + y^2 = 8$ subtends a right angle at the origin is	(p)	4
(B)	A chord of the circle $(x - 1)^2 + y^2 = 4$ lies along the line $y = 22\sqrt{3}(x - 1)$. The length of the chord is equal to	(q)	2
(C)	The number of circles touching all the three lines $3x + 7y = 2$, $21x + 49y = 5$ and $9x + 21y = 0$ are	(r)	0
(D)	If radii of the smallest and largest circle passing through the point $(\sqrt{3}, \sqrt{2})$ and touching the circle $x^2 + y^2 - 2\sqrt{2}y - 2 = 0$ are r_1 and r_2 respectively, then the mean of r_1 and r_2 is	(s)	1

Ans. (A) → (q), (B) → (p), (C) → (r), (D) → (s)

74) Column – I

Column – II

(A)	Number of common tangents of the circles $x^2 + y^2 - 2x = 0$ and $x^2 + y^2 + 6x - 6y + 2 = 0$ is	(p)	1
(B)	Number of indirect common tangents of the circles $x^2 + y^2 - 4x - 10y + 4 = 0$ & $x^2 + y^2 - 6x - 12y - 55 = 0$ is	(q)	2
(C)	Number of common tangents of the circles $x^2 + y^2 - 2x - 4y = 0$ & $x^2 + y^2 - 8y - 4 = 0$ is	(r)	3
(D)	Number of direct common tangents of the circles $x^2 + y^2 + 2x - 8y + 13 = 0$ & $x^2 + y^2 - 6x - 2y + 6 = 0$ is	(s)	0

Ans. (A) → (r), (B) → (s), (C) → (p), (D) → (q)

75) **STATEMENT-1 :** If three circles which are such that their centres are non-collinear, then exactly one circle exists which cuts the three circles orthogonally.

STATEMENT-2 : Radical axis for two intersecting circles is the common chord.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B*) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

76) **STATEMENT - 1 :** If a line $L = 0$ is tangent to the circle $S = 0$, then it will also be a tangent to the circle $S + \lambda L = 0$.

STATEMENT - 2 : If a line touches a circle, then perpendicular distance of the line from the centre of the circle is equal to the radius of the circle.

(A*) Statement-1 is True, Statement-2 is True, Statement-2 is a correct explanation for Statement-1

For each natural number k, let C_k denote the circle with radius k centimetres and centre at the origin. On the circle C_k, α-particle moves k centrimetres in the counter - clockwise direction. After completing its motion on C_k, the particle moves to C_{k+1}, in the radial direction. The motion of the particle continues in this manner. The particle starts at (1, 0). If the particle crosses the positive direction of the x-axis for the first time on the circle C_n then n = ________.

A circle passes through three points A, B and C with the line segment AC as its diameter. A line passing through A intersects the chord BC at a point D inside the circle. If angle DAB and CAB are α and β respectively and the distance between the point A and the mid point of the line segment DC is d, prove that the area of the circle is $\dfrac{\pi d^2 \cos^2\alpha}{\cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta\cos(\beta-\alpha)}$

A circle having centre at C is made to pass through the point P(1, 2), touching the straight lines $7x - y = 5$ and $x + y + 13 = 0$ at A and B respectively, then
(A) area of quadrilateral ACBP is 100 sq. units
(B) radius of smaller circle is $\sqrt{50}$
(C) area of quadrilateral ACBP is 200 sq. units
(D) radius of smaller circle is 10

Given pair of lines $2x^2 + 5xy + 2y^2 + 4x + 5y + a = 0$ and line $L \equiv bx + y + 5 = 0$

	Column – I		Column – II
(A)	If there exist 4 circles which touch pair of lines and the line L simultaneously then the value of b can be	(p)	$\frac{1}{2}$
(B)	If there exist 2 circles which touch pair of lines and the line L simultaneously then the value of b can be	(q)	2
(C)	If there exist no circle which touches pair of lines and the line L simultaneously then the value of b can be	(r)	5
(D)	If there exist infinite circles which touch pair of lines and the line L simultaneously then the value of b can not be	(s)	4
		(t)	1

Read the following write up carefully and answer the following questions:
Tangents are drawn to the parabola $y^2 = 4x$ from the point P(6, 5) to touch the parabola at Q and R. C_1 is a circle which touches the parabola at Q and C_2 is a circle which touches the parabola at R. Both the circles C_1 and C_2 pass through the focus of the parabola.

16. Area of the ΔPQR equals
(A) 1/2
(B) 1
(C) 2
(D) 1/4

17. Radius of the circle C_2 is
(A) $5\sqrt{5}$
(B) $5\sqrt{10}$
(C) $10\sqrt{2}$
(D) $\sqrt{210}$

18. The common chord of the circles C_1 and C_2 passes through the
(A) incentre
(B) circumcentre
(C) centroid
(D) orthocentre of the ΔPQR

81. 8 spheres of radius 1 unit is kept on a table with their centres at the vertices of a regular octagon and each sphere touching its two neighbours. If a sphere is placed in the centre on the table touching all of the 8 sphers, then its radius is

(A) $\frac{1}{\sqrt{2}}$ (B) $1-\frac{1}{\sqrt{2}}$

(C) $\sqrt{2}$ (D) $1+\frac{1}{\sqrt{2}}$

82. Let ellipse $4x^2 + 16y^2 = 64$ and circle $x^2 + y^2 = r^2$ have common tangent touching at A and B respectively. Maximum length of AB can be

(A) 4 (B) 3

(C) 2 (D) 5

83. Let $S_1 = x^2 - 10x + y^2 + 16 = 0$, and S_2 be the image of S_1 w.r.t. the line $x - y = 0$ and S_3 be the image of S_2 w.r.t. $y = 0$. Let S' is the circle which cut all three circles orthogonally and S'' is the circle of minimum radius which contain all three circles then

(A) equation of circle S' is $x^2 + y^2 = 16$ (B) equation of circle S'' is $x^2 + y^2 = 64$

(C) the radius of S' is 4 (D) the radius of S'' is 8

84. A pair of perpendicular lines passing through P(1, 4) intersect x axis at Q and R, then locus of incentre of ΔPQR is

(A) $x^2 + y^2 - 2x - 8y + 17 = 0$ (B) $x^2 + y^2 + 2x - 8y + 17 = 0$

(C) $x^2 - y^2 - 2x - 8y + 17 = 0$ (D) $x^2 + y^2 - 2x + 8y + 17 = 0$

85. Triangle formed by the lines $x + y = 0$, $x - y = 0$ and $\ell x + my = 1$. If ℓ and m vary subject to the condition $\ell^2 + m^2 = 1$ then locus of its circumcentre is

(A) $(x^2 - y^2)^2 = x^2 + y^2$ (B) $(x^2 + y^2)^2 = x^2 - y^2$

(C) $(x^2 + y^2) = 4x^2y^2$ (D) $(x^2 - y^2)^2 = (x^2 + y^2)^2$

86. The line $lx + my + n = 0$ intersects the curve $ax^2 + 2hxy + by^2 = 1$ at the points P and Q. The circle on PQ as diameter passes through the origin then

(A) $n^2(a + b) = l^2 + m^2$ (B) $l^2(a + b) = n^2 + m^2$

(C) $m^2(a + b) = l^2 + n^2$ (D) none of these

87. Consider a circle, $x^2 + y^2 = 1$ and point $P(1, \sqrt{3})$. PAB is a secant drawn from P intersecting circle in A and B (distinct) then range of $|PA| + |PB|$ is

(A) $[2, 2\sqrt{3}]$ (B) $(2\sqrt{3}, 4]$

(C) $(\sqrt{3}, 4]$ (D) none of these

88. Let ABC be a triangle whose vertices are A(−5, 5) and B(7, −1). If vertex C lies on the circle whose director circle has equation $x^2 + y^2 = 100$, then the locus of orthocentre of ΔABC is

(A) $x^2 + y^2 - 4x - 8y - 30 = 0$ (B) $x^2 + y^2 + 4x - 8y - 30 = 0$

(C) $x^2 + y^2 + 4x + 8y - 30 = 0$ (D) $x^2 + y^2 - 4x + 8y + 30 = 0$

89. The radical centre of three circles described on the three sides $x + y = 5$, $2x + y - 9 = 0$ and $x - 2y + 3 = 0$ of a triangle as diameters is

(A) (4, 4) (B) (3, 3)

(C) (3, 4) (D) (4, 1)

(90) If θ is the angle of intersection of two circles $x^2 + y^2 = a^2$ and $(x-c)^2 + y^2 = b^2$, then the length of common chord of two circles is

(A) $\frac{ab}{\sqrt{a^2+b^2-2ab\cos\theta}}$

(B) $\frac{2ab}{\sqrt{a^2+b^2-2ab\cos\theta}}$

(C) $\frac{2ab\sin\theta}{\sqrt{a^2+b^2-2ab\cos\theta}}$

(D) $\frac{2ab\cos\theta}{\sqrt{a^2+b^2-2ab\sin\theta}}$

(91) Consider circles $S: x^2 + y^2 = 1$ and $S': (x-4)^2 + (y-0)^2 = 9$. Now a circle is drawn touching S, S' externally and also their direct common tangent then its radius is

(A) $3(4+2\sqrt{3})$

(B) $\frac{3}{4}(4-2\sqrt{3})$

(C) $4-2\sqrt{3}$

(D) none of these

(92) The point $([P+1], [P])$ (where [.] denote greatest integer function) lying inside the region bounded by the circle $x^2 + y^2 - 2x - 15 = 0$ and $x^2 + y^2 - 2x - 7 = 0$, then

(A) $P \in [-1, 0) \cup [0, 1) \cup [1, 2)$

(B) $P \in [-1, 2) - \{0, 1\}$

(C) $P \in (-1, 2)$

(D) none of these

(93) The locus of the centre of a variable circle touching two circles of radius r_1, r_2 externally, which also touch each other externaly, is a conic. The eccentricity of the conic if $\frac{r_1}{r_2} = 3 + 2\sqrt{2}$ is

(A) 1

(B) $\sqrt{2}$

(C) $\frac{1}{2}$

(D) $2\sqrt{2}$

(94) P(x, y) satisfies $x^2 + y^2 = 1$ and let maximum value of $x^2 + 4xy + y^2$ is λ, then number of tangent(s)/asymptote(s) drawn from point $(\lambda, 1)$ to the hyperbola $(x-2)^2 - y^2 = 1$ is

(A) 1

(B) 2

(C) 0

(D) 4

(95) The centre of the circle $S = 0$ lies on the line $2x - 2y + 9 = 0$ and $S = 0$ cut orthogonally the circle $x^2 + y^2 = 4$, then $S = 0$ passes through 2 fixed points and their coordinates are

(A) $(-4, 9), \left(\frac{1}{2}, -\frac{1}{2}\right)$

(B) $(-4, 4), \left(-\frac{1}{2}, \frac{1}{2}\right)$

(C) $(4, -4), \left(\frac{1}{2}, \frac{1}{2}\right)$

(D) $(-4, -4), \left(-\frac{1}{2}, -\frac{1}{2}\right)$

(C) $\frac{4c-3b}{2b}$ $\quad$ 2c

97. If inside a big circle exactly 24 small circles, each of radius 2, can be drawn in such a way that each small circle touches the big circle and also touches both its adjacent small circles, then radius of the big circle is

(A) $2\left(1+\operatorname{cosec}\frac{\pi}{24}\right)$

(B) $\left(\dfrac{1+\tan\frac{\pi}{24}}{\cos\frac{\pi}{24}}\right)$

(C) $2\left(1+\operatorname{cosec}\frac{\pi}{12}\right)$

(D) $\dfrac{2\left(\sin\frac{\pi}{48}+\cos\frac{\pi}{48}\right)^2}{\sin\frac{\pi}{24}}$

98. If the conics whose equations are

$S_1 : (\sin^2\theta)x^2 + (2h\tan\theta)xy + (\cos^2\theta)y^2 + 32x + 16y + 19 = 0$

$S_2 : (\cos^2\theta)x^2 - (2h'\cot\theta)xy + (\sin^2\theta)y^2 + 16x + 32y + 19 = 0$

intersect in four concyclic points, where $\theta \in \left[0, \frac{\pi}{2}\right]$, then the correct statement(s) can be

(A) $h + h' = 0$

(B) $h - h' = 0$

(C) $\theta = \frac{\pi}{4}$

(D) none of these

99. Tangent is drawn at any point (x_1, y_1) other than vertex on the parabola $y^2 = 4ax$. If tangents are drawn from any point on this tangent to the circle $x^2 + y^2 = a^2$ such that all the chords of contact pass through a fixed point (x_2, y_2) then

(A) x_1, a, x_2 are in G.P.

(B) $\frac{y_1}{2}$, a, y_2 are in G.P.

(C) -4, $\frac{y_1}{y_2}$, $\frac{x_1}{x_2}$ are in G.P.

(D) $x_1x_2 + y_1y_2 = a^2$.

100. A circle 'S' is described on the focal chord of the parabola $y^2 = 4x$ as diameter. If the focal chord is inclined at an angle of 45° with axis of x, then which of the following is/are true

(A) radius of the circle is 4

(B) centre of the circle is (3, 2)

(C) the line $x + 1 = 0$ touches the circle

(D) the circle $x^2 + y^2 + 2x - 6y + 3 = 0$ is orthogonal to 'S'

COMPREHENSION–X

Read the paragraph carefully and answer the following questions: real number k not equal to unity. Introduce a
Given two fixed points A and B in a plane and a positive direction of x-
rectangular Cartesian co-ordinate system by choosing mid-point of AB as origin and the positive direction of x-
axis is chosen from A to B. The locus of points M for which the equation $\frac{MA}{MB} = k$ holds is a circle C_k.

319. If the circles C_{k_1} and C_{k_2} $(0 < k_1 \neq 1,\ 0 < k_2 \neq 1)$, are symmetric to each other with respect to the perpendicular bisector of segment AB, then
(A) $k_1 = k_2$
(B) $k_1k_2 < 1$
(C) $k_1k_2 > 1$
(D) $k_1k_2 = 1$

An arbitrary circle passes through points A and B intersects the circle C_k at an angle
(A) $\frac{\pi}{4}$
(B) nothing can be said
(C) $\frac{\pi}{2}$
(D) π

Read the paragraph carefully and answer the following questions:
Let C_1 and C_2 be the parabola $x^2 = y - 2$ and $y^2 = x - 2$ respectively. Let P be any point on C_1 and Q be any point on C_2. Let P_1 and Q_1 be reflections of P and Q respectively with respect to the line $y = x$, and P_0 and Q_0 on the parabola C_1 and C_2 respectively. Such that $P_0Q_0 \leq PQ$ for all pairs of points (P, Q) with P on C_1 and Q on C_2.

321. Then reflection of point P and Q are
(A) P_1 and Q_1 lies on C_2 and C_1 respectively and $PQ \geq \max(PP_1, QQ_1)$
(B) P_1 and Q_1 lies on C_2 and C_1 respectively and $PQ \geq \min(PP_1, QQ_1)$
(C) P_1 and Q_1 lies on C_2 and C_1 respectively and $PQ \leq \min(PP_1, QQ_1)$
(D) P_1 and Q_1 lies on C_2 and C_1 respectively and $PQ \leq \max(PP_1, QQ_1)$

322. The coordinate of P_0 is
(A) $\left(\frac{1}{2}, \frac{4}{9}\right)$
(B) $\left(\frac{1}{2}, \frac{9}{4}\right)$
(C) $\left(\frac{9}{4}, \frac{1}{2}\right)$
(D) $\left(\frac{1}{3}, \frac{19}{9}\right)$

Six points (x_i, y_i), $i = 1, 2, \dots, 6$ are taken on the circle $x^2 + y^2 = 4$ such that $\sum_{i=1}^{6} x_i = 8$ and $\sum_{i=1}^{6} y_i = 4$. The line segment joining orthocentre of a triangle made by any three points and the centroid of the triangle made by other three points passes through a fixed points (h, k), then h + k is ______

Given a circle $(x + 4)^2 + (y - 2)^2 = 25$. Another circle is drawn passing through (–4, 2) and touching the given circle internally at the point A(–4, 7). AB is a chord of length 8 units of the larger circle intersecting the other circle at the point C. Then AC will be ______

A circle C is tangent to the x and y-axis in first quadrant at the points P and Q respectively. BC and AD are parallel tangents to the circle with slope −1. If the points A and B are on the y-axis while C and D are on x-axis and area of figure ABCD is $900\sqrt{2}$ sq. units. The radius of the circle is k then $\frac{k}{3}$ is equal to ____

If p_1, p_2, p_3 are the altitudes of a triangle which circumscribe a circle of diameter $\frac{4}{3}$ units, then the least value of $p_1 + p_2 + p_3$ is equal to ____

The centres of two circles C_1 and C_2 each of unit radius are at a distance of 6 unit from each other. Let P be the mid point of the line segment joining the centres of C_1 and C_2 and C be a circle touching C_1 and C_2 externally. If a common tangent to C_1 and C, passes through P is also a common tangent to C_2 and C_1, then the radius of circle C is ____

Match the following List–I with List–II.

	List – I		List – II
(A)	Diameter of the circle touching the line $(x-1)\cos\theta + (y-1)\sin\theta = 1$ for all values of θ	(p)	2
(B)	Radius of smallest circle which touches the circle $x^2 + y^2 = 1$ and $x^2 + y^2 - 6x - 8y + 21 = 0$	(q)	1
(C)	Number of values of m for which the line $(y-2) = m(x-1)$ cuts the circle $x^2 + y^2 = 5$ at two real points	(r)	4
(D)	Number of circle touching both the axes and the line $x + y = 4$	(s)	infinite
		(t)	3

A circle S passes through the point (0, 1) and is orthogonal to the circles $(x-1)^2 + y^2 = 16$ and $x^2 + y^2 = 1$. Then

(A) radius of S is 8
(B) radius of S is 7
(C) centre of S is (−7, 1)
(D) centre of S is (−8, 1)

The common tangents to the circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ touch the circle at the points P, Q and the parabola at the points R, S. Then the area of the quadrilateral PQRS is

(A) 3
(B) 6
(C) 9
(D) 15

If the normals of the parabola $y^2 = 4x$ drawn at the end points of its latus rectum are tangents to the circle $(x-3)^2 + (y+2)^2 = r^2$, then the value of r^2 is ____

(110) The circle $C_1 : x^2 + y^2 = 3$, with centre at O, intersects the parabola $x^2 = 2y$ at the point P in the first quadrant. Let the tangent to the circle C_1 at P touches other two circles C_2 and C_3 at R_2 and R_3, respectively. Suppose C_2 and C_3 have equal radii $2\sqrt{3}$ and centres Q_2 and Q_3, respectively. If Q_2 and Q_3 lie on the y-axis, then

(A) $Q_2Q_3 = 12$

(B) $R_2R_3 = 4\sqrt{6}$

(C) area of the triangle OR_2R_3 is $6\sqrt{2}$

(D) area of the triangle PQ_2Q_3 is $4\sqrt{2}$

(A, B, C)

y

(111) *49. Let RS be the diameter of the circle $x^2 + y^2 = 1$, where S is the point (1, 0). Let P be a variable point (other than R and S) on the circle and tangents to the circle at S and P meet at the point Q. The normal to the circle at P intersects a line drawn through Q parallel to RS at point E. Then the locus of E passes through the point(s)

(A) $\left(\frac{1}{3}, \frac{1}{\sqrt{3}}\right)$

(B) $\left(\frac{1}{4}, \frac{1}{2}\right)$

(C) $\left(\frac{1}{3}, -\frac{1}{\sqrt{3}}\right)$

(D) $\left(\frac{1}{4}, -\frac{1}{2}\right)$

Sol. **(A, C)**

(112) Let P be the point on the parabola $y^2 = 4x$ which is at the shortest distance from the center S of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$. Let Q be the point on the circle dividing the line segment SP internally. Then

(A) $SP = 2\sqrt{5}$

(B) $SQ : QP = (\sqrt{5} + 1) : 2$

(C) the x-intercept of the normal to the parabola at P is 6

(D) the slope of the tangent to the circle at Q is $\frac{1}{2}$

(113) For how many values of p, the circle $x^2 + y^2 + 2x + 4y - p = 0$ and the coordinate axes have exactly three common points ?

(2)

(114)

Columns 1, 2 and 3 contain conics, equations of tangents to the conics and points of contact, respectively.		
Column 1	**Column 2**	**Column 3**
(I) $x^2 + y^2 = a^2$	(i) $my = m^2x + a$	(P) $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$
(II) $x^2 + a^2y^2 = a^2$	(ii) $y = mx + a\sqrt{m^2 + 1}$	(Q) $\left(\frac{-ma}{\sqrt{m^2+1}}, \frac{a}{\sqrt{m^2+1}}\right)$
(III) $y^2 = 4ax$	(iii) $y = mx + \sqrt{a^2m^2 - 1}$	(R) $\left(\frac{-a^2m}{\sqrt{a^2m^2+1}}, \frac{1}{\sqrt{a^2m^2+1}}\right)$
(IV) $x^2 - a^2y^2 = a^2$	(iv) $y = mx + \sqrt{a^2m^2 + 1}$	(S) $\left(\frac{-a^2m}{\sqrt{a^2m^2-1}}, \frac{-1}{\sqrt{a^2m^2-1}}\right)$

The tangent to a suitable conic (Column 1) at $\left(\sqrt{3}, \frac{1}{2}\right)$ is found to be $\sqrt{3}x + 2y = 4$, then which of the following options is the only CORRECT combination ?

[A] (II) (iii) (R) [B] (IV) (iv) (S)

[C] (IV) (iii) (S) [D] (II) (iv) (R)

If a tangent to a suitable conic (Column 1) is found to be $y = x + 8$ and its point of contact is (8, 16), then which of the following options is the only CORRECT combination ?

[A] (III) (i) (P) [B] (III) (ii) (Q)

[C] (II) (iv) (R) [D] (I) (ii) (Q)

For $a = \sqrt{2}$, if a tangent is drawn to a suitable conic (Column 1) at the point of contact (−1, 1), then which of the following options is the only CORRECT combination for obtaining its equation ?

[A] (II) (ii) (Q) [B] (III) (i) (P)

[C] (I) (i) (P) [D] (I) (ii) (Q)

Let E_1E_2 and F_1F_2 be the chords of S passing through the point $P_0(1, 1)$ and parallel to the x-axis and the y-axis, respectively. Let G_1G_2 be the chord of S passing through P_0 and having slope −1. Let the tangents to S at E_1 and E_2 meet at E_3, the tangents to S at F_1 and F_2 meet at F_3, and the tangents to S at G_1 and G_2 meet at G_3. Then, the points E_3, F_3, and G_3 lie on the curve

(A) $x + y = 4$ (B) $(x-4)^2 + (y-4)^2 = 16$

(C) $(x-4)(y-4) = 4$ (D) $xy = 4$

A

Let P be a point on the circle S with both coordinates being positive. Let the tangent to S at P intersect the coordinate axes at the points M and N. Then, the mid-point of the line segment MN must lie on the curve

(A) $(x+y)^2 = 3xy$ (B) $x^{2/3} + y^{2/3} = 2^{4/3}$

(C) $x^2 + y^2 = 2xy$ (D) $x^2 + y^2 = x^2y^2$

D

Let T be the line passing through the points P(−2, 7) and Q(2, −5). Let F_1 be the set of all pairs of circles (S_1, S_2) such that T is tangent to S_1 at P and tangent to S_2 at Q, and also such that S_1 and S_2 touch each other at a point, say, M. Let E_1 be the set representing the locus of M as the pair (S_1, S_2) varies in F_1. Let the set of all straight line segments joining a pair of distinct points of E_1 and passing through the point R(1, 1) be F_2. Let E_2 be the set of the mid-points of the line segments in the set F_2. Then, which of the following statement(s) is (are) TRUE ?

(A) The point (−2, 7) lies in E_1 (B) The point $\left(\frac{4}{5}, \frac{7}{5}\right)$ does **NOT** lie in E_2

(C) The point $\left(\frac{1}{2}, 1\right)$ lies in E_2 (D) The point $\left(0, \frac{3}{2}\right)$ does **NOT** lie in E_1

B,D

Consider two straight lines, each of which is tangent to both the circle $x^2 + y^2 = \frac{1}{2}$ and the parabola $y^2 = 4x$. Let these lines intersect at the point Q. Consider the ellipse whose center is at the origin O(0, 0) and whose semi-major axis is OQ. If the length of the minor axis of this ellipse is $\sqrt{2}$, then which of the following statement(s) is (are) TRUE ?

(A) For the ellipse, the eccentricity is $\frac{1}{\sqrt{2}}$ and the length of the latus rectum is 1

(B) For the ellipse, the eccentricity is $\frac{1}{2}$ and the length of the latus rectum is $\frac{1}{2}$

(C) The area of the region bounded by the ellipse between the lines $x = \frac{1}{\sqrt{2}}$ and $x = 1$ is $\frac{1}{4\sqrt{2}}(\pi - 2)$

(D) The area of the region bounded by the ellipse between the lines $x = \frac{1}{\sqrt{2}}$ and $x = 1$ is $\frac{1}{16}(\pi - 2)$

A, C

In a non-right-angled triangle ΔPQR, let p, q, r denote the lengths of the sides opposite to the angles at P, Q, R respectively. The median from R meets the side PQ at S, the perpendicular from P meets the side QR at E, and RS and PE intersect at O. If $p = \sqrt{3}$, $q = 1$, and the radius of the circumcircle of the ΔPQR equals 1, then which of the following options is/are correct?

Let the point B be the reflection of the point A(2, 3) with respect to line $8x - 6y - 23 = 0$. Let Γ_A and Γ_B be circles of radii 2 and 1 with centres A and B respectively. Let T be a common tangent to the circles Γ_A and Γ_B such that both the circles are on the same side of T. If C is the point of intersection of T and the line passing through A and B, then the length of the line segment AC is

10.00

Answer the following by appropriately matching the lists based on the information given in the paragraph

Let the circles $C_1 : x^2 + y^2 = 9$ and $C_2 : (x - 3)^2 + (y - 4)^2 = 16$, intersect at the points X and Y. Suppose that another circle $C_3 : (x - h)^2 + (y - k)^2 = r^2$ satisfies and following conditions:

(i) centre of C_3 is collinear with the centres of C_1 and C_2

(ii) C_1 and C_2 both lie inside C_3, and

(iii) C_3 touches C_1 at M and C_2 at N.

Let the line through X and Y intersect C_3 at Z and W, and let a common tangent of C_1 and C_3 be a tangent to the parabola $x^2 = 8\alpha y$.

There are some expressions given in the List–I whose values are given in List–II below:

List – I		List – II	
(I)	$2h + k$	(P)	6
(II)	$\dfrac{\text{Length of ZW}}{\text{Length of XY}}$	(Q)	$\sqrt{6}$
(III)	$\dfrac{\text{Area of triangle MZN}}{\text{Area of triaangle ZMW}}$	(R)	$\dfrac{5}{4}$
(IV)	α	(S)	$\dfrac{21}{5}$
		(T)	$2\sqrt{6}$
		(U)	$\dfrac{10}{3}$

Which of the following is the only CORRECT combination?

A. (II), (T)
B. (II), (Q)
C. (I), (U)
D. (I), (S)

Which of the following is the only INCORRECT combination?

A. (IV), (S)
B. (III), (R)
C. (IV), (U)
D. (I), (P)

Let O be the centre of the circle $x^2 + y^2 = r^2$, where $r > \frac{\sqrt{5}}{2}$. Suppose PQ is a chord of this circle and the equation of the line passing through P and Q is $2x + 4y = 5$. If the centre of the circumcircle of the triangle OPQ lies on the line $x + 2y = 4$, then the value of r is ______

2

Consider a triangle Δ whose two sides lie on the x-axis and the line $x + y + 1 = 0$. If the orthocentre of Δ is (1, 1), then the equation of the circle passing through the vertices of the triangle Δ is

(A) $x^2 + y^2 - 3x + y = 0$
(B) $x^2 + y^2 + x + 3y = 0$
(C) $x^2 + y^2 + 2y - 1 = 0$
(D) $x^2 + y^2 + x + y = 0$

B

(124) Consider the region $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x \geq 0 \text{ and } y^2 \leq 4 - x\}$. Let F be the family of all circles that are contained in R and have centers on the x-axis. Let C be the circle that has largest radius among the circles in F. Let (α, β) be a point where the circle C meets the curve $y^2 = 4 - x$.

JA21 P2

The radius of the circle C is ____

1.5

The value of α is ____

2

(125) Let $M = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x^2 + y^2 \leq r^2\}$, where $r > 0$. Consider the geometric progression $a_n = \frac{1}{2^{n-1}}$, $n = 1, 2, 3, \ldots$. Let $S_0 = 0$ and for $n \geq 1$, let S_n denote the sum of the first n terms of this progression. For $n \geq 1$, let C_n denote the circle with center $(S_{n-1}, 0)$ and radius a_n, and D_n denote the circle with center (S_{n-1}, S_{n-1}) and radius a_n.

JA P2.

Consider M with $r = \frac{1025}{513}$. Let k be the number of all those circles C_n that are inside M. Let l be the maximum possible number of circles among these k circles such that no two circles intersect. Then

(A) $k + 2l = 22$ (B) $2k + l = 26$ (C) $2k + 3l = 34$ (D) $3k + 2l = 40$

D

Consider M with $r = \frac{(2^{199} - 1)\sqrt{2}}{2^{198}}$. The number of all those circles D_n that are inside M is

(A) 198 (B) 199 (C) 200 (D) 201

(126) Let ABC be the triangle with $AB = 1$, $AC = 3$ and $\angle BAC = \frac{\pi}{2}$. If a circle of radius $r > 0$ touches the sides AB, AC and also touches internally the Circumcircle of the triangle ABC, then the value of r is ________.

JA22 P1.

(127) Let G be a circle of radius $R > 0$. Let $G_1, G_2, \ldots, G_n$ be n circles of equal radius $r > 0$. Suppose each of the n circles $G_1, G_2, \ldots, G_n$ touches the circle G externally. Also, for $i = 1, 2, \ldots, n-1$, the circle G_i touches G_{i+1} externally, and G_n touches G_1 externally. Then, which of the following statements is/are TRUE?

(A) If $n = 4$, then $(\sqrt{2} - 1)r < R$

(B) If $n = 5$, then $r < R$

(C) If $n = 8$, then $(\sqrt{2} - 1)r < R$

(D) If $n = 12$, then $\sqrt{2}(\sqrt{3} + 1)r > R$.

JA22 P2

A circle of radius r passes through the origin O and cuts the axes at A and B. If the locus of the foot of the perpendicular from O to AB is

$$(x^2+y^2)^2\left(\frac{1}{x^2}+\frac{1}{y^2}\right)=\lambda r^2 \rho$$

the value of λ is

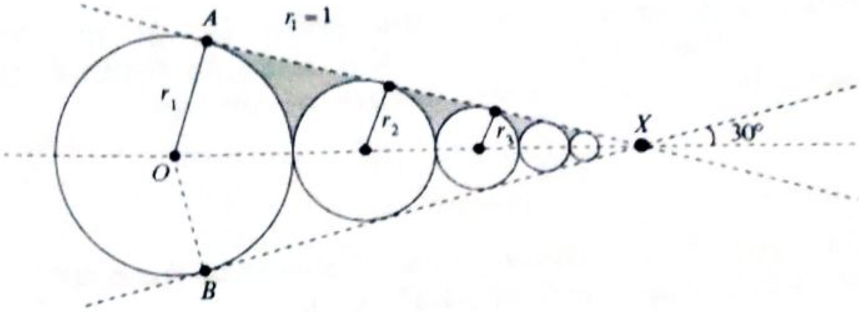
(A) 1 (B) 2 (C) 4 (D) 8

It is given that two circles, both of which pass through the point $(0, a)$ and $(0, -a)$ and touch the line $y = mx + c$, will intersect orthogonally if $c^2 = a^2(\lambda + m^2)$. The value of λ is

(A) 1 (B) 2 (C) 3 (D) 4 (E) None of these

Prove that the locus of the centres of the circles cutting two given circles orthogonally is their radical axis.

Let S be the sum of this (decreasing) infinite sequence of shaded areas:



The value of $[S]$ (the greatest integer part of S) is

(A) 0 (B) 1 (C) 2 (D) 3

Three circles with radii r_1, r_2, r_3 (where $r_1 < r_2 < r_3$) touch each other externally. If they have a common tangent, the value of $\sqrt{\frac{r_1}{r_2}}+\sqrt{\frac{r_1}{r_3}}$ is

(A) 1 (B) $\sqrt{2}$ (C) 2 (D) $2\sqrt{2}$

Let $L_1 : 2x+3y+p-3=0$ and $L_2 : 2x+3y+p+3=0$ be two lines and $p \in \mathbb{Z}$. Let $C : x^2+y^2+6x+10y+30=0$. If it is given that at least one of the lines L_1, L_2 is a chord of C, the probability that both are chords of C is

(A) $\frac{2}{7}$ (B) $\frac{3}{7}$ (C) $\frac{4}{11}$ (D) $\frac{5}{11}$ (E) None of these

Consider a family of circles passing through the intersection point of the lines $\sqrt{3}(y-1) = x-1$ and $y-1=\sqrt{3}(x-1)$ and having its centre on the acute angle bisector of the given lines.

(a) Show that the common chords of each member of the family and the circle $x^2+y^2+4x-6y+5=0$ are concurrent.

(b) If the point of concurrency is (a, b), the value of $a+b$ is

(A) 0 (B) 1 (C) 2 (D) 3

(131) Consider two fixed circles $x^2 + y^2 + 4|x| + 3 = 0$. A triangle ABC is initially located so that its vertices have the following positions:

$$\{A \equiv (0, 2), B \equiv (2, 2\sqrt{3}+2), C \equiv (-2, 2\sqrt{3}+2)\}$$

It starts translating downwards perpendicular to the x-axis, and stops when its edges hit the circles (AB at the point P_1, and AC at P_2). The ratio in which P_1 divides AB is

(A) $\frac{3-\sqrt{3}}{\sqrt{3}}$ (B) $\frac{4-\sqrt{3}}{\sqrt{3}}$ (C) $\frac{1+\sqrt{3}}{\sqrt{3}}$ (D) $\frac{2+\sqrt{3}}{\sqrt{3}}$ (E) None of these

(133) Consider the straight line $x + y = 0$ and the circle $x^2 + y^2 - 2x + \frac{1}{2} = 0$. At $t = 0$, the line starts rotating anti-clockwise about the origin with angular speed ω. The circle too starts moving (anti-clockwise) such that the locus of its center is a circle of radius 1 centered at the origin. The angular speed of the circle's center is 2ω. Find the equation of the line and the circle at the instant when they first meet after $t = 0$.

(134) **Tangents are drawn from the point (h, k) to the circle $x^2+y^2=a^2$; prove that the area of the triangle formed by them and the straight line joining their points of contact is**

$$\frac{a(h^2+k^2-a^2)^{\frac{3}{2}}}{h^2+k^2}.$$

(135) **Find the locus of the middle points of chords of the circle $x^2+y^2=a^2$ which pass through the fixed point (h, k).**

(136) **Find the locus of the middle points of chords of the circle $x^2+y^2=a^2$ which subtend a right angle at the point $(c, 0)$.**

(137) **Whatever be the value of a, prove that the locus of the intersection of the straight lines**

$$x \cos a + y \sin a = a \text{ and } x \sin a - y \cos a = b$$

is a circle.

(138) **From a point P on a circle perpendiculars PM and PN are drawn to two radii of the circle which are not at right angles; find the locus of the middle point of MN.**

(139) **Shew that the locus of a point, which is such that the tangents from it to two given concentric circles are inversely as the radii, is a concentric circle, the square of whose radius is equal to the sum of the squares of the radii of the given circles.**

(140) **Prove that a common tangent to two circles of a coaxal system subtends a right angle at either limiting point of the system.**

141 **Prove that the circle of similitude of the two circles**

$$x^2+y^2-2kx+\delta=0 \text{ and } x^2+y^2-2k'x+\delta=0$$

(*i.e.* the locus of the points at which the two circles subtend the same angle) is the coaxal circle

$$x^2+y^2-2\frac{kk'+\delta}{k+k'}x+\delta=0.$$

142 **Find the equation to the circle cutting orthogonally the three circles**

$$x^2+y^2=a^2, \quad (x-c)^2+y^2=a^2, \quad \text{and} \quad x^2+(y-b)^2=a^2.$$

143 Three concentric circles of which the biggest is $x^2+y^2=1$, have their radii in A.P. If the line $y=x+1$ cuts all the circles in real and distinct points. The interval in which the common difference of the A.P. will lie is:

(a) $\left(0, \frac{1}{4}\right)$ (b) $\left(0, \frac{1}{2\sqrt{2}}\right)$ (c) $\left(0, \frac{2-\sqrt{2}}{4}\right)$ (d) none

144 A circle of radius unity is centred at origin. Two particles start moving at the same time from the point (1, 0) and move around the circle in opposite direction. One of the particle moves counter clockwise with constant speed v and the other moves clockwise with constant speed $3v$. After leaving (1, 0), the two particles meet first at a point P, and continue until they meet next at point Q. The coordinates of the point Q are:

(a) (1, 0) (b) (0, 1)

(c) (0, −1) (d) (−1, 0)

145 If $a=\max\{(x+2)^2+(y-3)^2\}$ and $b=\min\{(x+2)^2+(y-3)^2\}$ where x, y satisfying $x^2+y^2+8x-10y-40=0$, then :

(a) $a+b=18$ (b) $a+b=178$ (c) $a-b=4\sqrt{2}$ (d) $a-b=72\sqrt{2}$

146 The locus of points of intersection of the tangents to $x^2+y^2=a^2$ at the extremeties of a chord of circle $x^2+y^2=a^2$ which touches the circle $x^2+y^2-2ax=0$ is/are :

(a) $y^2=a(a-2x)$ (b) $x^2=a(a-2y)$

(c) $x^2+y^2=(x-a)^2$ (d) $x^2+y^2=(y-a)^2$

Question Nos. 1 to 3

Let each of the circles,

$S_1 \equiv x^2 + y^2 + 4y - 1 = 0,$ $\quad S_2 \equiv x^2 + y^2 + 6x + y + 8 = 0,$

$S_3 \equiv x^2 + y^2 - 4x - 4y - 37 = 0$

touches the other two. Let P_1, P_2, P_3 be the points of contact of S_1 and S_2, S_2 and S_3, S_3 and S_1 respectively and C_1, C_2, C_3 be the centres of S_1, S_2, S_3 respectively.

The co-ordinates of P_1 are :

(a) $(2, -1)$ (b) $(2, 1)$ (c) $(-2, 1)$ (d) $(-2, -1)$

The ratio $\dfrac{\text{area}(\Delta P_1P_2P_3)}{\text{area}(\Delta C_1C_2C_3)}$ is equal to :

(a) $3:2$ (b) $2:5$ (c) $5:3$ (d) $2:3$

P_2 and P_3 are image of each other with respect to line :

(a) $y = x + 1$ (b) $y = -x$ (c) $y = x$ (d) $y = -x + 2$

Let $A(3, 7)$ and $B(6, 5)$ are two points. $C : x^2 + y^2 - 4x - 6y - 3 = 0$ is a circle.

- The chords in which the circle C cuts the members of the family S of circle passing through A and B are concurrent at :

(a) $(2, 3)$ (b) $\left(2, \frac{23}{3}\right)$ (c) $\left(3, \frac{23}{2}\right)$ (d) $(3, 2)$

Equation of the member of the family of circles S that bisects the circumference of C is :

(a) $x^2 + y^2 - 5x - 1 = 0$ (b) $x^2 + y^2 - 5x + 6y - 1 = 0$

(c) $x^2 + y^2 - 5x - 6y - 1 = 0$ (d) $x^2 + y^2 + 5x - 6y - 1 = 0$

If O is the origin and P is the center of C, then absolute value of difference of the squares of the lengths of the tangents from A and B to the circle C is equal to :

(a) $(AB)^2$ (b) $(OP)^2$ (c) $|(AP)^2 - (BP)^2|$ (d) $(AP)^2 + (BP)^2$

Let L_1, L_2 and L_3 be the lengths of tangents drawn from a point P to the circles $x^2 + y^2 = 4$, $x^2 + y^2 - 4x = 0$ and $x^2 + y^2 - 4y = 0$ respectively. If $L_1^4 = L_2^2 L_3^2 + 16$ then the locus of P are the curves, C_1 (a straight line) and C_2 (a circle).

Circum centre of the triangle formed by C_1 and two other lines which are at angle of 45° with C_1 and tangent to C_2 is :

(a) (1, 1) (b) (0, 0) (c) (−1, −1) (d) (2, 2)

If S_1, S_2 and S_3 are three circles congruent to C_2 and touch both C_1 and C_2; then the area of triangle formed by joining centres of the circles S_1, S_2 and S_3 is (in square units)

(a) 2 (b) 4 (c) 8 (d) 16

	Column-I		Column-II
(A)	A ray of light coming from the point (1, 2) is reflected at a point A on the x-axis then passes through the point (5, 3). The coordinates of the point A are :	(P)	$\left(\frac{13}{5}, 0\right)$
(B)	The equation of three sides of triangle ABC are $x + y = 3$, $x - y = 5$ and $3x + y = 4$. Considering the sides as diameter, three circles S_1, S_2, S_3 are drawn whose radical centre is at :	(Q)	(4, −1)
(C)	If the straight line $x - 2y + 1 = 0$ intersects the circle $x^2 + y^2 = 25$ at the points P and Q, then the coordinate of the point of intersection of tangents drawn at P and Q to the circle is	(R)	(−25, 50)
(D)	The equation of three sides of a triangle are $4x + 3y + 9 = 0$, $2x + 3 = 0$ and $3y - 4 = 0$. The circum centre of the triangle is :	(S)	$\left(\frac{-19}{8}, \frac{1}{6}\right)$
		(T)	(−1, 2)

Tangents are drawn to circle $x^2 + y^2 = 1$ at its intersection points (distinct) with the circle $x^2 + y^2 + (\lambda - 3)x + (2\lambda + 2)y + 2 = 0$. The locus of intersection of tangents is a straight line, then the slope of that straight line is.

Let $S = \{(x, y) \mid x, y \in R, x^2 + y^2 - 10x + 16 = 0\}$. The largest value of $\frac{y}{x}$ can be put in the form $\frac{m}{n}$ where m, n are relatively prime natural numbers, then $m^2 + n^2 =$

(6) Three circles with radii 3 cm, 4 cm and 5 cm touch each other externally. If A is the point of intersection of tangents to these circles at their points of contact, then the distance of A from the points of contact is

(a) $\sqrt{3}$ (b) 2

(c) $\sqrt{5}$ (d) $\sqrt{6}$

Ans. (c)

(7) A line meets the coordinate axes in A and B. A circle is circumscribed about the triangle OAB. If m and n are the distances of the tangent to the circle at the origin from the points A and B respectively, the diameter of the circle is

(a) $m(m + n)$ (b) $m + n$

(c) $n(m + n)$ (d) $(1/2)(m + n)$.

Ans. (b)

(8) If O is the origin and OP, OQ are distinct tangents to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, the circumcentre of the triangle OPQ is

(a) $(-g, -f)$ (b) (g, f)

(c) $(-f, -g)$ (d) none of these.

Ans. (d)

Tangents drawn from the point $P(1, 8)$ to the circle $x^2 + y^2 - 6x - 4y - 11 = 0$ touch the circle at the points A and B. The equation of the circumcircle of the triangle in PAB is

(a) $x^2 + y^2 + 4x - 6y + 19 = 0$

(b) $x^2 + y^2 - 4x - 10y + 19 = 0$

(c) $x^2 + y^2 - 2x + 6y - 29 = 0$

(d) $x^2 + y^2 - 6x - 4y + 19 = 0$

Ans. (b)

C_1 and C_2 are circles of unit radius with centres at $(0, 0)$ and $(1, 0)$ respectively. C_3 is a circle of unit radius, passes through the centres of the circles C_1 and C_2 and have its centre above x-axis. Equation of the common tangent to C_1 and C_3 which does not pass through C_2 is

(a) $x - \sqrt{3}\,y + 2 = 0$

(b) $\sqrt{3}\,x - y + 2 = 0$

(c) $\sqrt{3}\,x - y - 2 = 0$

(d) $x + \sqrt{3}\,y + 2 = 0$

Ans. (b)

A circle C_1 of radius b touches the circle $x^2 + y^2 = a^2$ externally and has its centre on the positive x-axis; another circle C_2 of radius c touches the circle C_1 externally and has its centre on the positive x-axis. Given $a < b < c$, then the three circles have a common tangent if a, b, c are in

(a) A.P. (b) G.P.
(c) H.P. (d) none of these

Ans. (b)

A and B are two points on the x-axis and y-axis respectively. Two circles are drawn passing through the origin and having centre at A and B.

(a) Equation of the common chord is $ax - by = 0$

(b) mid-point of the common chord is

$$\left(\frac{ab^2}{a^2+b^2}, \frac{a^2 b}{a^2+b^2}\right)$$

(c) AB bisects the common chord.

(d) AB is perpendicular to the common chord.

Ans. (a), (b), (c), (d)

Equation of the straight line which meets the circle $x^2 + y^2 = a^2$ at points which are at a distance d from a point $A(\alpha, \beta)$ on the circle is

(a) $2\alpha x + 2\beta y = 2a^2 - d^2$ (b) $2\alpha x - 2\beta y = 2a^2 + d^2$

(c) $2\alpha x + 2\beta y = 2a^2 + d^2$ (d) $2\alpha x + 2\beta y + 2a^2 = d^2$

Ans. (a), (d)

$P(a, 5a)$ and $Q(4a, a)$ are two points. Two circles are drawn through these points touching the axis of y.

1 Centre of these circles are at

(a) (a, a)

(b) $\left(\frac{205a}{18}, \frac{29a}{3}\right)$

(c) $\left(\frac{5a}{2}, 3a\right)$

(d) $\left(3a, \frac{29a}{3}\right)$

2 Angle of intersection of these circles is

(a) $\tan^{-1}(4/3)$

(b) $\tan^{-1}(40/9)$

(c) $\tan^{-1}(84/187)$

(d) none of these

3 If C_1, C_2 are the centres of these circles then area of $\Delta\, OC_1C_2$, where O is the origin, is

(a) a^2

(b) $5a^2$

(c) $10a^2$

(d) $20a^2$

A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ, QR, RP are D, E, F respectively. The line PQ is given by the equation $\sqrt{3}\ x + y - 6 = 0$ and the point D is $(3\sqrt{3}/2,\ 3/2)$. Further it is given that the origin and the centre of C are on the same side of PQ.

The equation of circle C is

(a) $(x - 2\sqrt{3})^2 + (y - 1)^2 = 1$

(b) $(x - 2\sqrt{3})^2 + (y + 1/2)^2 = 1$

(c) $(x - \sqrt{3})^2 + (y + 1)^2 = 1$

(d) $(x - \sqrt{3})^2 + (y - 1)^2 = 1$

Points E and F are given by

(a) $(\sqrt{3}/2,\ 3/2)$, $(\sqrt{3},\ 0)$

(b) $(\sqrt{3}/2,\ 1/2)$, $(\sqrt{3},\ 0)$

(c) $(\sqrt{3}/2,\ 3/2)$, $(\sqrt{3}/2,\ 1/2)$

(d) $(3/2,\ \sqrt{3}/2)$, $(\sqrt{3}/2,\ 1/2)$

Equations of the sides QR, RP are

(a) $y = (2/\sqrt{3})x + 1$, $y = (-2/\sqrt{3})x - 1$

(b) $y = (1/\sqrt{3})x$, $y = 0$

(c) $y = (\sqrt{3}/2)x + 1$, $y = (-\sqrt{3}/2)x - 1$

(d) $y = \sqrt{3}x$, $y = 0$

A triangle has two of its sides along the axes, its third side touches the circle $x^2 + y^2 - 2ax - 2ay + a^2 = 0$. If the locus of the circumcentre of the triangle passes through the point $(38, -37)$ then $a^2 - 2a$ is equal to

The centres of two circles C_1 and C_2 each of unit radius are at a distance of 6 units from each other. Let P be the mid point of the line segment joining the centres of C_1 and C_2 and C be a circle touching C_1 and C_2 externally. If a common tangent to C_1 and C passing through P is also a common tangent to C_2 and C_1, then the radius of the circle C is

(a) The radical axis of two circles	(p) Subtends a right angle at a point of intersection.
(b) The common tangent to two intersecting circles of equal radii	(q) is perpendicular to the line joining the centres.
(c) The common chord of two intersecting circles	(r) is parallel to the line joining the centres.
(d) The line joining the centres of two circles intersecting orthogonally.	(s) is bisected by the line joining the centres.

Statement-1: From a point P on the circle with centre O, the chord $PA = 8$ cm is drawn. The radius of the circle is 24 cm. Let PB be drawn parallel to OA. Suppose BO extended meet PA extended at M. The length of MA is 9 cm.

Statement-2: OA is a radius of a circle with centre at O. R is a point on OA through which a chord CD perpendicular to OA is drawn. Let a chord through A meet the chord CD at M and the circle at B. Also OS is perpendicular from O on chord AB. The radius of the circle is 18 cm. R is the mid point of AO and AM/MB = 1/2. The length of OS is 9 cm.

Lines $5x + 12y - 10 = 0$ and $5x - 12y - 40 = 0$ touch a circle C_1 of diameter 6. If the centre of C_1 lies in the first quadrant, find the equation of the circle C_2 which is concentric with C_1 and cuts intercepts of length 8 on these lines.

If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = k^2$ orthogonally, then the equation of the locus of its centre is

(a) $2ax + 2by - (a^2 + b^2 + k^2) = 0$
(b) $2ax + 2by - (a^2 - b^2 + k^2) = 0$
(c) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - k^2) = 0$
(d) $x^2 + y^2 - 2ax - 3by + (a^2 - b^2 + k^2) = 0$

If two circles $(x - 1)^2 + (y - 3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then

(a) $2 < r < 8$
(b) $r < 2$
(c) $r = 2$
(d) $r > 2$

Find the intervals of values of a for which the line $y + x = 0$ bisects two chords drawn from a point $\left(\frac{1+\sqrt{2}a}{2}, \frac{1-\sqrt{2}a}{2}\right)$ to the circle.

$$2x^2 + 2y^2 - (1 + \sqrt{2}a)x - (1 - \sqrt{2}a)y = 0$$

Let $ABCD$ be a quadrilateral with area 18, with side AB parallel to the side CD and $AB = 2CD$. Let AD be perpendicular to AB and CD. If a circle is drawn inside the quadrilateral $ABCD$ touching all the sides, then its radius is

(a) 3 (b) 2

(c) 3/2 (d) 1

Tangents are drawn from the point (17, 7) to the circle $x^2 + y^2 = 169$.

Statement-1: The tangents are mutually perpendicular.

Because

Statement-2: The locus of the point from which mutually perpendicular tangents can be drawn to the given circle is $x^2 + y^2 = 338$.

A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ, QR, RP are D, E, F respectively. The line PQ is given by the equation $\sqrt{3}x + y - 6 = 0$ and the point D is $(3\sqrt{3}/2, 3/2)$. Further it is given that the origin and the centre C are on the same side PQ.

The equation of circle C is

(a) $(x - 2\sqrt{3})^2 + (y - 1)^2 = 1$

(b) $(x - 2\sqrt{3})^2 + (y + 1/2)^2 = 1$

(c) $(x-\sqrt{3})^2+(y+1)^2=1$

(d) $(x-\sqrt{3})^2+(y-1)^2=1$

. Points E and F are given by

(a) $(\sqrt{3}/2,\ 3/2),\ (\sqrt{3},\ 0)$

(b) $(\sqrt{3}/2,\ 1/2),\ (\sqrt{3},\ 0)$

(c) $(\sqrt{3}/2,\ 3/2),\ (\sqrt{3}/2,\ 1/2)$

(d) $(3/2,\ \sqrt{3}/2),\ (\sqrt{3}/2,\ 1/2)$

. Equations of the sides QR, RP are

(a) $y=(2/\sqrt{3})x+1,\ y=(-2/\sqrt{3})x-1$

(b) $y=(1/\sqrt{3})x,\ y=0$

(c) $y=(\sqrt{3}/2)x+1,\ y=(-\sqrt{3}/2)x-1$

(d) $y=\sqrt{3}x,\ y=0$

26) The length of the chord of contact of the point $P(x_1,y_1)$ w.r.t to the circle $S=0$ is $2r\sqrt{\frac{S_{11}}{S_{11}+r^2}}$. $S=0$ is a circle in standard form, with centre C and radius r. If $P(x_1,y_1)$ is a point then the area of the triangle formed by pair of tangents from P and chord of contact of P is $\frac{r(S_{11})^{3/2}}{S_{11}+r^2}$.

27) If d is the distance between centres of two circles whose radii are r_1 and r_2 then length of direct common tangent of two circles is $\sqrt{d^2-(r_1-r_2)^2}$ and length of transverse common tangent of two circles is $\sqrt{d^2-(r_1+r_2)^2}$.

28) **The number of lattice points that are interior to the circle $x^2+y^2=25$ is**

a) 81 b) 69 c) 12 d) 70

29) The set of values of 'c' so that $y=|x|+c$ and $x^2+y^2-8|x|-9=0$ have no solution is

a) $(-\infty,-3)\cup(3,\infty)$

b) $(-3,\ 3)$

c) $(-\infty,-5\sqrt{2})\cup(5\sqrt{2},\infty)$

d) $(5\sqrt{2}-4,\infty)$

$c<-4-5\sqrt{2}$

A circle with centre at the origin and radius equal to 'a' meets the axis of X at A and B. $P(\alpha)$ and $Q(\beta)$ are two points on this circle so that $\alpha - \beta = 2\gamma$ where γ is a constant. The locus of the point of intersection of AP and BQ is

a) $x^2 - y^2 - 2ay\tan\gamma = a^2$ b) $x^2 + y^2 - 2ay\tan\gamma = a^2$

c) $x^2 + y^2 + 2ay\tan\gamma = a^2$ d) $x^2 - y^2 + 2ay\tan\gamma = a^2$

A variable straight line through $A(-1, 1)$ is drawn to cut the circle $x^2 + y^2 = 1$ at the points B and C. A point 'P' is chosen on the line ABC satisfying the condition given the Column - I. Let d be the minimum distance of the origin from the locus of P given in the Column - II

COLUMN - I	COLUMN - II
A) AB, AP, AC are in A.P	p) 0
B) AB, AP, AC are in G.P	q) $1/\sqrt{2}$
C) AB, AP, AC are in H.P	r) $\sqrt{2}$
D) $AB, \frac{AP}{2}, AC$ are in A.P	s) $\sqrt{2}-1$

The Base of a triangle $AB = 6$ the third vertex C moves such that $\frac{\sin A}{\sin B} = 2$. Then Locus of C is circle then its radius is

A ray of light incident at the point $(-2,-1)$ gets reflected from the tangents at $(0, -1)$ to the circle $x^2 + y^2 = 1$ The reflected ray touches the circle, the equation of the line along which the incident ray moved is

a) $4x - 3y + 11 = 0$ b) $4x + 3y + 11 = 0$ c) $3x + 4y + 11 = 0$ d) None of these

If C_1 and C_2 are the two concentric circles with radii r_1 and r_2 $(r_1 < r_2)$. If the tangents drawn from any point of C_2 to C_1 meets again C_2 at the ends of its diameter then

a) $r_2 = 2r_1$ b) $r_2 = \sqrt{2}r_1$ c) $r_2^2 < 2r_1^2$ d) $r_2 = 3r_1$

A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ, QR, RP are D, E, F, respectively. The line PQ is given by the equation $\sqrt{3}x+y-6=0$ and the point D is $\left(\frac{3\sqrt{3}}{2},\frac{3}{2}\right)$. Further it is given that the origin and [redacted]ntre C are on the same side of the line PQ

The equation of circle C is

a) $(x-2\sqrt{3})^2+(y-1)^2=1$

b) $(x-2\sqrt{3})^2+\left(y+\frac{1}{2}\right)^2=1$

c) $(x-\sqrt{3})^2+(y+1)^2=1$

d) $(x-\sqrt{3})^2+(y-1)^2=1$

Equation of the sides QR, RP are

a) $y=\frac{2}{\sqrt{3}}x+1, y=-\frac{2}{\sqrt{3}}x-1$

b) $y=\frac{1}{\sqrt{3}}x, y=0$

c) $y=\frac{\sqrt{3}}{2}x+1, y=-\frac{\sqrt{3}}{2}x-1$

d) $y=\sqrt{3}x, y=0$

Points E and F are given by

a) $\left(\frac{\sqrt{3}}{2},\frac{3}{2}\right),(\sqrt{3},0)$

b) $\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right),(\sqrt{3},0)$

c) $\left(\frac{\sqrt{3}}{2},\frac{3}{2}\right),\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$

d) $\left(\frac{3}{2},\frac{\sqrt{3}}{2}\right),\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$

Two circles of radii r_1 & r_2 $(r_1>r_2)$ touch each other externally then the radius of the circle which touches both of them externally & also their direct common tangent is

a) $\frac{r_1r_2}{(\sqrt{r_1}+\sqrt{r_2})^2}$ b) $\sqrt{r_1r_2}$ c) $\frac{r_1r_2}{2}$ d) r_1-r_2

If C, C_1, C_2 be the circles of radii 5, 3, 2 respectively. If C_1 & C_2 touch externally & they touch internally with C. The radius of circle C_3 which touches externally with C_1 & C_2 and internally with C is

a) $\frac{30}{19}$

b) 1

c) 3

d) Can not be determined

A square $OABC$ is formed by line pairs $xy=0$ and $xy+1=x+y$ where 'O' is the origin . A circle with centre C_1 inside the square is drawn to touch the line pair $xy=0$ and another circle with centre C_2 and radius twice that of C_1 is drawn to touch the circle C_1 and other line pair. The radius of the circle with centre C_1 is

a) $\frac{\sqrt{2}}{\sqrt{3}(\sqrt{2}+1)}$ b) $\frac{2\sqrt{2}}{3(\sqrt{2}+1)}$ c) $\frac{\sqrt{2}}{3(\sqrt{2}+1)}$ d) $\frac{\sqrt{2}+1}{3\sqrt{2}}$

C_1, C_2, C_3 are circles of radii 5, 3, 2 respectively. C_1 and C_2 touch each other externally and C internally. The radius of circles C_3 which touches C internally and C_2, C_1 externally is ....

1) $\frac{3}{2}$ b) $\frac{20}{9}$ c) $\frac{35}{19}$ d) $\frac{30}{19}$

If PQR is the triangle formed by the common tangents to the circles $x^2+y^2+6x=0$ and $x^2+y^2-2x=0$ then

a) Centroid of ΔPQR is (1, 0)
b) In-centre of is (1, [illegible]
c) Circum-radius of is 2 units
d) In radius of is 1 unit

Two circles of radii a and b touching each other externally are inscribed in the area bounded by $y=\sqrt{1-x^2}$ and x - axis. If $b=\frac{1}{2}$, then $4a$ is equal to ......

From a point P outside of a circle with centre at O, tangent segments PA and PB are drawn. If $\frac{1}{(AB)^2}+\frac{1}{(PA)^2}=\frac{1}{16}$ then the length of the chord AB is ....

Inside a semi-circle of radius 1 unit, two circles of radii r_1 and r_2 are drawn, each touching the circumference and the diameter of the semi-circle also touches each other externally. Then $[\max.(r_1+r_2)]$ where $[x]$ denotes integral value nearest to x ....

If the line $x+y=n$, $n \in N$ is chord of the circle $x^2+y^2=4$ then

a) Sum of the squares of the length of the chord intercepts by the line on the circle is 22
b) No. of such chords of 4
c) No. of such chords of 2
d) None of these

$f(x, y)=0$ be the equation of a circle, such that $f(0, y)$ has equal roots and $f(x, 0)=0$ has two distinct Real roots the angle between the tangents drawn from P to $f(x,y)=0$ is $\pi/3$. Then locus of P is the circle is whose equation is $g(x,y)=x^2+y^2-5x-4y+c=0$ Let Q be a point from which the tangents to $g(x,y)=0$ are perpendicular. AB is the chord of contact of Q w.r.t. $g(x,y)=0$ then

Equation of $f(x,y)=0$ is

a) $x^2+y^2-5x-4y+5=0$ b) $x^2+y^2-5x-4y-4=0$ c) $x^2+y^2-5x-4y-5=0$ d) $x^2+y^2-5x-4y+4=0$

Area of ΔQAB is

a) $\frac{25}{2}$ b) $\frac{25}{4}$ c) $\frac{25}{8}$ d) $\frac{25}{12}$

The length of X-intercept made by $g(x,y)=0$ is

a) $2\sqrt{80}$ b) $\sqrt{84}$ c) $\sqrt{74}$ d) $\sqrt{78}$

Consider the relation $4l^2 - 5m^2 + 6l + 1 = 0$ where $l, m \in R$ then the line $lx + my + 1 = 0$ touches a fixed circle

Centre and radius of the circle

a) (2, 0), 3 b) (−3, 0), $\sqrt{3}$ c) (3, 0) $\sqrt{5}$ d) (3,0), $\sqrt{3}$

No. of tangents which can be drawn from the point (2, −3) are

a) 0 b) 1 c) 2 d) 1 or 2

From a point $P(2, -3)$ two tangents are drawn to the circle and A, B are points of contact then $PA \cdot PB$

a) 26 b) 5 c) 15 d) 25

Five circle C_1, C_2, C_3, C_4, C_5 with radii r_1, r_2, r_3, r_4, r_5 respectively $(r_1 < r_2 < r_3 < r_4 < r_5)$ be such that C_i and C_{i+1} touch each other externally for all $i = 1,2,3,4$ if all the five circles touch each of the two staright lines L_1 and L_2 and $r_1 = 2$ and $r_5 = 32$ then r_3 is equal to

a) 8 b) 17 c) $\frac{64}{17}$ d) Depends upon r_3 and r_4

Two circles with centres at A and B, touch at T. BD is the tangent at D and TC is a common tangent. AT has length 3 and BT has length 2. The length of CD is

a) 4/3 b) 3/2 c) 5/3 d) 7/4

C_1, C_2 are two circles of radii $a, b (a < b)$ respectively touching both the coordinate axes and their centres in the first quadrant. Then the true statements among the following are

a) If C_1 and C_2 touch each other then $\frac{b}{a} = 3 + 2\sqrt{2}$

b) If C_1 and C_2 are orthogonal then $\frac{b}{a} = 2 + \sqrt{3}$

c) If C_1 and C_2 intersect the such a way that their common chord has maximum length then $\frac{b}{a}$

d) If C_2 passes through centre of C_1 then $\frac{b}{a} = 2 + \sqrt{2}$

Let x, y be real variable satisfying the $x^2 + y^2 + 8x - 10y - 40 = 0$.
Let $a = \max\left(\sqrt{(x+2)^2 + (y-3)^2}\right)$ and $b = \min\left(\sqrt{(x+2)^2 + (y-3)^2}\right)$, then

a) $a + b = 18$ b) $a + b = 4\sqrt{2}$ c) $a - b = 4\sqrt{2}$ d) $ab = 73$

In the parallelogram $ABCD$ with angle $A = 60^0$, the bisector of angle B is drawn which cuts the side CD at a point E. A circle S_1 of radius 'r' is inscribed in the triangle ECB. Another circle 'S_2' is inscribed in the trapezoid $ABED$.

COLUMN - I	COLUMN - II
A) The value of radius of S_2 is	p) $2\sqrt{3}r$
B) The value of distance between the centres of S_1 and S_2 is	q) $\frac{\sqrt{3}}{2}r$
C) The value of the length of common tangent of S_1 and S_2 is	r) $\sqrt{7}r$
D) The value of the length CE is	s) $\frac{3}{2}r$

$A(-2, 0)$ and $B(2, 0)$ are two fixed points and P is a point such that $PA - PB = 2$. Let S be the circle $x^2 + y^2 = r^2$ then match the following

Column-I	Column-II
a) If $r = 2$ then the number of points P satisfying $PA - PB = 2$ and lying on $x^2 + y^2 = r^2$ is	p) 2
b) If $r = 1$ then the number of points P satisfying $PA - PB = 2$ and lying on $x^2 + y^2 = r^2$ is	q) 4
c) For $r = 2$ the number of common tangents is	r) 0
d) For $r^2 = \frac{1}{2}$ the number of common tangents	s) 1

a) A - q; B - r; c - s; d - p
b) A - r; B - s; c - p; d - q
c) A - s; B - p; c - q; d - p
d) A - p; B - s; c - r; d - p

If the circle C_1 touches x-axis and the line $y = x\tan\theta, \theta \in \left(0, \frac{\pi}{2}\right)$ in first quadrant and cirlce C_2 touches the ine $y = x\tan\theta$, y-axis and circle C_1 in such a way that ratio of radius of C_1 to radius of C_2 is 2 : 1, then value of $\tan\frac{\theta}{2} = \frac{\sqrt{a} - b}{c}$ where a, b, c are relatively prime natural numbers thus $\frac{(a+b+c)}{11}$ is

If the circles $x^2+y^2+2ax+cy+a=0$ and $x^2+y^2-3ax+dy-1=0$ intersect in two distinct points P and Q then the line $5x+by-a = 0$ passes through P and Q for

1) exactly one value of 'a'
2) no value of 'a'
3) infinitely many values of 'a'
4) exactly two values of 'a'

If r and r^1 are the radii of the circles $S = 0$ and $S^1 = 0$ respectively then the circles $\frac{S}{r} \pm \frac{S^1}{r^1} = 0$ intersect at an angle of

1) $\frac{\pi}{3}$
2) $\frac{\pi}{4}$
3) $\frac{\pi}{2}$
4) $\frac{\pi}{6}$

61 The points $A(2, 3)$ and $B(-7, -12)$ are conjugate points w.r.t to the circle $x^2+y^2-6x-8y-1=0$. Th centre of the circle passing through A and B and orthogonal to given circle is

1) $(-5, -9)$ 2) $(-9, -15)$ 3) $\left(-\frac{5}{2}, -\frac{9}{2}\right)$ 4) $\left(\frac{1}{2}, \frac{3}{2}\right)$

62 If the circle $x^2+y^2+2gx+2fy+c=0$ bisects the circumference of the circle $x^2+y^2+2g'x+2f'y+c'=0$ the length of the common chord of the circles is

1) $2\sqrt{g^2+f^2-c}$ 2) $2\sqrt{g'^2+f'^2-c'}$ 3) $2\sqrt{g^2+f^2+c}$ 4) $2\sqrt{g'^2+f'^2+c'}$

63 $x^2+y^2 = a^2$ and $(x-c)^2+y^2 = b^2$ are two intersecting circles. If a, b, c are the sides BC, CA, AB of ΔABC. If p_1, p_2, p_3 are the altitudes through A, B, C respectively then the length of the common chord is

1) $2p_1$ 2) $2p_2$ 3) $2p_3$ 4) p_1

64 The circles having radii 1, 2, 3 touch each other externally. Then the radius of the circle which cuts the three circles orthogonally is

1) 1 2) $\frac{3}{2}$ 3) 2 4) 3

65 **Let $C \equiv x^2+y^2+4x=0$ is a given circle and a circle C_1 of radius 2 units rolls on the outer side of the circle 'C' touching it externally. If the line joining the centres of C and C_1 makes an angle 60^0 with the x-axis, then that circle be C_2**

The locus of centre of the circle C_1 is

a) $x^2+y^2+4x-12=0$
b) $x^2+y^2-4\sqrt{3}y+8=0$
c) $x^2+y^2+4x+12=0$
d) $x^2+y^2-4\sqrt{3}y-12=0$

67 The equation of circle joining the centres C and C_1 as a diameter is

a) $x^2+y^2-2x+2\sqrt{3}y=0$
b) $x^2+y^2-2x-2\sqrt{3}y=0$
c) $x^2+y^2+2x-2\sqrt{3}y=0$
d) None of these

68 The equation of least circle containing both the circles C_1 and C_2 is

a) $x^2+y^2-4x+12=0$
b) $x^2+y^2-4x-12=0$
c) $x^2+y^2-2x+2\sqrt{3}y+12=0$
d) $x^2+y^2+2x-2\sqrt{3}y-12=0$

Tangents PA and PB are drawn to the circle $(x-4)^2+(y-5)^2=4$ from the point P on the curve $y=\sin x$ where A, B lie on the circle. consider the function $y=f(x)$ representing by the locus of the centre of the circumcentre of the triangle PAB, then answer the following questions.

Range of $y=f(x)$ is

a) $[-2, 1]$ b) $[-1, 4]$ c) $[0, 2]$ d) $[2, 3]$

Period of $y=f(x)$ is

a) 2π b) 3π c) π d) None

Which of the following is true

a) $f(x)=4$ has real roots

b) $f(x)=1$ has real roots

c) range of $y=f^{-1}(x)$ is $\left[-\frac{\pi}{4}+2, \frac{\pi}{4}+2\right]$

d) none

Let the orthocentre and centroid of a triangle be $A(-3, 5)$ and $B(3, 3)$, respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is

(a) $\sqrt{10}$ (b) $2\sqrt{10}$ (c) $3\sqrt{\frac{5}{2}}$ (d) $\frac{3\sqrt{5}}{2}$

The centre of the circle passing through the point $(0, 1)$ and touching the curve $y=x^2$ at $(2, 4)$ is

(a) $\left(-\frac{16}{5}, \frac{27}{10}\right)$ (b) $\left(-\frac{16}{7}, \frac{53}{10}\right)$

(c) $\left(-\frac{16}{5}, \frac{53}{10}\right)$ (d) None of the above

The abscissae of the two points A and B are the roots of the equation $x^2+2ax-b^2=0$ and their ordinates are the roots of the equation $y^2+2py-q^2=0$. Find the equation and the radius of the circle with AB as diameter.

The straight line $2x-3y=1$ divide the circular region $x^2+y^2\le 6$ into two parts. If $S=\left\{\left(2, \frac{3}{4}\right), \left(\frac{5}{2}, \frac{3}{4}\right), \left(\frac{1}{4}, -\frac{1}{4}\right), \left(\frac{1}{8}, \frac{1}{4}\right)\right\}$, then the number of point (s) in S lying inside the smaller part is ... .

Let S be the circle in the XY-plane defined by the equation

$$x^2 + y^2 = 4.$$

(There are two questions based on above Paragraph, the question given below is one of them)

Let E_1E_2 and F_1F_2 be the chords of S passing through the point P_0 (1, 1) and parallel to the X-axis and the Y-axis, respectively. Let G_1G_2 be the chord of S passing through P_0 and having slope -1. Let the tangents to S at E_1 and E_2 meet at E_3, then tangents to S at F_1 and F_2 meet at F_3, and the tangents to S at G_1 and G_2 meet at G_3. Then, the points E_3, F_3 and G_3 lie on the curve

(a) $x + y = 4$
(b) $(x - 4)^2 + (y - 4)^2 = 16$
(c) $(x - 4)(y - 4) = 4$
(d) $xy = 4$

Let P be a point on the circle S with both coordinates being positive. Let the tangent to S at P intersect the coordinate axes at the points M and N. Then, the mid-point of the line segment MN must lie on the curve

(a) $(x + y)^2 = 3xy$
(b) $x^{2/3} + y^{2/3} = 2^{4/3}$
(c) $x^2 + y^2 = 2xy$
(d) $x^2 + y^2 = x^2y^2$

46. If a variable line, $3x + 4y - \lambda = 0$ is such that the two circles $x^2 + y^2 - 2x - 2y + 1 = 0$ and $x^2 + y^2 - 18x - 2y + 78 = 0$ are on its opposite sides, then the set of all values of λ is the interval

(a) [13, 23]
(b) (2, 17)
(c) [12, 21]
(d) (23, 31)

Let T be the line passing through the points $P(-2,7)$ and $Q(2,-5)$. Let F_1 be the set of all pairs of circles (S_1, S_2) such that T is tangent to S_1 at P and tangent to S_2 at Q, and also such that S_1 and S_2 touch each other at a point, say M. Let E_1 be the set representing the locus of M as the pair (S_1, S_2) varies in F_1. Let the set of all straight line segments joining a pair of distinct points of E_1 and passing through the point $R(1,1)$ be F_2. Let E_2 be the set of the mid-points of the line segments in the set F_2. Then, which of the following statement(s) is (are) TRUE?

(a) The point $(-2, 7)$ lies in E_1

(b) The point $\left(\frac{4}{5}, \frac{7}{5}\right)$ does NOT lie in E_2

(c) The point $\left(\frac{1}{2}, 1\right)$ lies in E_2

(d) The point $\left(0, \frac{3}{2}\right)$ does NOT lie in E_1

77 Let $ABCD$ be a square of side length 2 unit. C_2 is the circle through vertices A, B, C, D and C_1 is the circle touching all the sides of square $ABCD$. L is the line through A.

If P is a point of C_1 and Q is a point on C_2, then $\dfrac{PA^2 + PB^2 + PC^2 + PD^2}{QA^2 + QB^2 + QC^2 + QD^2}$ is equal to

(a) 0.75 (b) 1.25

(c) 1 (d) 0.5

78 A circle touches the line L and the circle C_1 externally such that both the circles are on the same side of the line, then the locus of centre of the circle is

(a) ellipse (b) hyperbola

(c) parabola (d) parts of straight line

79 A line M through A is drawn parallel to BD. Point S moves such that its distances from the line BD and the vertex A are equal. If locus of S cuts M at T_2 and T_3 and AC at T_1, then area of $\Delta T_1T_2T_3$ is

(a) $\frac{1}{2}$ sq unit (b) $\frac{2}{3}$ sq unit

(c) 1 sq unit (d) 2 sq units

Match the Columns

Match the conditional/expressions in Column I with statement in Column II.

Column I		Column II	
A.	Two intersecting circles	p.	have a common tangent
B.	Two mutually external circles	q.	have a common normal
C.	Two circles, one strictly inside the other	r.	do not have a common tangent
D.	Two branches of a hyperbola	s.	do not have a common normal

The tangent and the normal lines at the point $(\sqrt{3}, 1)$ to the circle $x^2 + y^2 = 4$ and the X-axis form a triangle. The area of this triangle (in square units) is

(a) $\frac{1}{3}$ (b) $\frac{4}{\sqrt{3}}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{1}{\sqrt{3}}$

Let RS be the diameter of the circle $x^2 + y^2 = 1$, where S is the point $(1, 0)$. Let P be a variable point (other than R and S) on the circle and tangents to the circle at S and P meet at the point Q. The normal to the circle at P intersects a line drawn through Q parallel to RS at point E. Then, the locus of E passes through the point(s)

(a) $\left(\frac{1}{3}, \frac{1}{\sqrt{3}}\right)$ (b) $\left(\frac{1}{4}, \frac{1}{2}\right)$

(c) $\left(\frac{1}{3}, -\frac{1}{\sqrt{3}}\right)$ (d) $\left(\frac{1}{4}, -\frac{1}{2}\right)$

The circle $C_1 : x^2 + y^2 = 3$ with centre at O intersects the parabola $x^2 = 2y$ at the point P in the first quadrant. Let the tangent to the circle C_1 at P touches other two circles C_2 and C_3 at R_2 and R_3, respectively. Suppose C_2 and C_3 have equal radii $2\sqrt{3}$ and centres Q_2 and Q_3, respectively. If Q_2 and Q_3 lie on the Y-axis, then

(a) $Q_2Q_3 = 12$
(b) $R_2R_3 = 4\sqrt{6}$
(c) area of the ΔOR_2R_3 is $6\sqrt{2}$
(d) area of the ΔPQ_2Q_3 is $4\sqrt{2}$

Tangents are drawn from the point (17, 7) to the circle $x^2 + y^2 = 169$.

Statement I The tangents are mutually perpendicular.

because

Statement II The locus of the points from which a mutually perpendicular tangents can be drawn to the given circle is $x^2 + y^2 = 338$.

(a) Statement I is true, Statement II is true; Statement II is correct explanation of Statement I
(b) Statement I is true, Statement II is true, Statement II is not correct explanation of Statement I.
(c) Statement I is true, Statement II is false.
(d) Statement I is false, Statement II is true.

Passage 1

A tangent PT is drawn to the circle $x^2 + y^2 = 4$ at the point $P(\sqrt{3}, 1)$. A straight line L, perpendicular to PT is a tangent to the circle $(x-3)^2 + y^2 = 1$.

A possible equation of L is

(a) $x - \sqrt{3}y = 1$
(b) $x + \sqrt{3}y = 1$
(c) $x - \sqrt{3}y = -1$
(d) $x + \sqrt{3}y = 5$

A common tangent of the two circles is

(a) $x = 4$
(b) $y = 2$
(c) $x + \sqrt{3}y = 4$
(d) $x + 2\sqrt{2}\,y = 6$

A circle C of radius 1 is inscribed in an equilateral ΔPQR. The points of contact of C with the sides PQ, QR, RP are D, E, F respectively. The line PQ is given by the equation $\sqrt{3}\,x + y - 6 = 0$ and the point D is $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$.

Further, it is given that the origin and the centre of C are on the same side of the line PQ.

The equation of circle C is

(a) $(x - 2\sqrt{3})^2 + (y - 1)^2 = 1$

(b) $(x - 2\sqrt{3})^2 + \left(y + \frac{1}{2}\right)^2 = 1$

(c) $(x - \sqrt{3})^2 + (y + 1)^2 = 1$

(d) $(x - \sqrt{3})^2 + (y - 1)^2 = 1$

Points E and F are given by

(a) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), (\sqrt{3}, 0)$ (b) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), (\sqrt{3}, 0)$

(c) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ (d) $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

Equations of the sides QR, RP are

(a) $y = \frac{2}{\sqrt{3}}x + 1,\ y = -\frac{2}{\sqrt{3}}x - 1$ (b) $y = \frac{1}{\sqrt{3}}x,\ y = 0$

(c) $y = \frac{\sqrt{3}}{2}x + 1,\ y = -\frac{\sqrt{3}}{2}x - 1$ (d) $y = \sqrt{3}x,\ y = 0$

87 Two circles with equal radii are intersecting at the points (0, 1) and (0, −1). The tangent at the point (0,1) to one of the circles passes through the centre of the other circle. Then, the distance between the centres of these circles is

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) 1 (d) 2

88 Three circles of radii $a, b, c\,(a < b < c)$ touch each other externally. If they have X-axis as a common tangent, then

(a) a, b, c are in AP (b) $\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}$

(c) $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in AP (d) $\frac{1}{\sqrt{b}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{c}}$

Find the equation of the circle which passes through the point (2, 0) and whose centre is the limit of the point of intersection of the lines $3x + 5y = 1$, $(2 + c)x + 5c^2y = 1$ as c tends to 1.

The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line $4x - 5y = 20$ to the circle $x^2 + y^2 = 9$ is

(a) $20(x^2 + y^2) - 36x + 45y = 0$
(b) $20(x^2 + y^2) + 36x - 45y = 0$
(c) $36(x^2 + y^2) - 20y + 45y = 0$
(d) $36(x^2 + y^2) + 20x - 45y = 0$

The equations of the tangents drawn from the origin to the circle $x^2 + y^2 + 2rx + 2hy + h^2 = 0$, are

(a) $x = 0$
(b) $y = 0$
(c) $(h^2 - r^2)x - 2rhy = 0$
(d) $(h^2 - r^2)x + 2rhy = 0$

Consider $L_1 : 2x + 3y + p - 3 = 0$

$L_2 : 2x + 3y + p + 3 = 0$

where, p is a real number and

$C : x^2 + y^2 - 6x + 10y + 30 = 0$

Statement I If line L_1 is a chord of circle C, then line L_2 is not always a diameter of circle C.

Statement II If line L_1 is a diameter of circle C, then line L_2 is not a chord of circle C.

Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3} + 1$ apart. If the chords subtend at the centre, angles of π/k and $\frac{2\pi}{k}$, where $k > 0$, then the value of $[k]$ is......

(Q4) If in triangle ABC, $A \equiv (1, 10)$, circumcenter $\equiv (-1/3, 2/3)$, and orthocenter $\equiv (11/3, 4/3)$, then the coordinates of the midpoint of the side opposite to A are

1) $(1, -11/3)$ 2) $(1, 5)$ 3) $(1, -3)$ 4) $(1, 6)$

(Q5) In triangle ABC, the equation of side BC is $x - y = 0$. Circumcentre and orthocentre of the triangle are $(2, 3)$ and $(5, 8)$ respectively. Equation of circumcircle of the triangle is

A $x^2 + y^2 - 4x + 6y - 27 = 0$

B $x^2 + y^2 - 4x - 6y - 27 = 0$

C $x^2 + y^2 + 4x + 6y - 27 = 0$

D $x^2 + y^2 + 4x - 6y - 27 = 0$