

- (1) A vertical line passing through the point $(h, 0)$ intersects the ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$ at the points P and Q. Let the tangents to the ellipse at P and Q meet at the point R. If $\Delta(h)$ = area of the triangle PQR. $\Delta_1 = \max_{1 \leq h \leq 1} \Delta(h)$ and $\Delta_2 = \min_{1 \leq h \leq 1} \Delta(h)$, then $\frac{\Delta_1}{\sqrt{5}} - 8\Delta_2 =$ (9)

Sol. (9) Leibniz integral rule L2. SDE, LH, LG
 For $a \in \mathbb{R}$ (the set of all real numbers), $a \neq -1$, $\lim_{n \rightarrow \infty} \frac{(n+1)^{a+1} [(na+1) + (na+2) + \dots + (na+n)]}{(n^a + 2n^a + \dots + n^a)} = \frac{1}{60}$
 Then $a =$
 (A) 5
 (C) $-\frac{15}{2}$ pb bu 1
 (B) 7
 (D) $-\frac{17}{2}$

Sol. (B, D)

- (4) The largest value of the non-negative integer a for which $\lim_{x \rightarrow 1} \left[\frac{-ax + \sin(x-1) + a}{x + \sin(x-1) - 1} \right]^{\frac{1-x}{x-1}} = \frac{1}{4}$ is 0

- (5) Let m and n be two positive integers greater than 1. If $\lim_{\alpha \rightarrow 0} \left(\frac{e^{\cos \alpha} \alpha^{n+1} - e}{\alpha^m} \right) = -\left(\frac{e}{2} \right)$
 then the value of $\frac{m}{n}$ is (2)

- (6) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a continuous odd function, which vanishes exactly at one point and $f(1) = \frac{1}{2}$. Suppose that $F(x) = \int_{-1}^x f(t) dt$ for all $x \in [-1, 2]$ and $G(x) = \int_{-1}^x t |f(f(t))| dt$ for all $x \in [-1, 2]$. If $\lim_{x \rightarrow 1} \frac{F(x)}{G(x)} = \frac{1}{14}$, then the value of $f\left(\frac{1}{2}\right)$ is (7)

- (7) Let $f: (0, \infty) \rightarrow \mathbb{R}$ be a differentiable function such that $f'(x) = 2 - \frac{f(x)}{x}$ for all $x \in (0, \infty)$ and $f(1) = 1$.
 Then
 (A) $\lim_{x \rightarrow 0^+} f\left(\frac{1}{x}\right) = 1$
 (C) $\lim_{x \rightarrow 0^+} x^2 f'(x) = 0$
 (B) $\lim_{x \rightarrow 0^+} xf\left(\frac{1}{x}\right) = 2$
 (D) $|f(x)| \leq 2$ for all $x \in (0, 2)$

Sol. (A)

- (8) Let $\alpha, \beta \in \mathbb{R}$ be such that $\lim_{x \rightarrow 0} \frac{x^2 \sin(\beta x)}{\alpha x - \sin x} = 1$. Then $6(\alpha + \beta)$ equals (7)
 (7)

2. Let $f(x) = \lim_{n \rightarrow \infty} \left(\frac{n^n (x+n) \left(x + \frac{n}{2}\right) \dots \left(x + \frac{n}{n}\right)}{n! \left(x^2 + n^2\right) \left(x^2 + \frac{n^2}{4}\right) \dots \left(x^2 + \frac{n^2}{n^2}\right)} \right)^{\frac{x}{n}}$, for all $x > 0$. Then

- (A) $f\left(\frac{1}{2}\right) \geq f(1)$ (B) $f\left(\frac{1}{3}\right) \leq f\left(\frac{2}{3}\right)$
 (C) $f'(2) \leq 0$ (D) $\frac{f'(3)}{f(3)} \geq \frac{f'(2)}{f(2)}$

(B, C)

Let $f: \mathbb{R} \rightarrow (0, \infty)$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be twice differentiable functions such that f'' and g'' are continuous functions on \mathbb{R} . Suppose $f(2) = g(2) = 0$, $f'(2) \neq 0$ and $g'(2) \neq 0$. If $\lim_{x \rightarrow 2} \frac{f(x)g(x)}{f'(x)g'(x)} = 1$, then

- (A) f has a local minimum at $x = 2$ (B) f has a local maximum at $x = 2$
 (C) $f'(2) > f(2)$ (D) $f(x) - f'(x) = 0$ for at least one $x \in \mathbb{R}$

(A, D)

11. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function such that $f(0) = 0$, $f\left(\frac{\pi}{2}\right) = 3$ and $f'(0) = 1$. If

$$g(x) = \int_x^{\pi/2} [f'(t) \operatorname{cosec} t - \cot t \operatorname{cosec} t f(t)] dt$$

for $x \in \left(0, \frac{\pi}{2}\right)$, then $\lim_{x \rightarrow 0} g(x) = 2$

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12. Let $f(x) = \frac{1-x(1+|1-x|)}{|1-x|} \cos\left(\frac{1}{1-x}\right)$ for $x \neq 1$. Then

- (A) $\lim_{x \rightarrow 1^-} f(x) = 0$ (B) $\lim_{x \rightarrow 1^-} f(x)$ does not exist
 (C) $\lim_{x \rightarrow 1^+} f(x) = 0$ (D) $\lim_{x \rightarrow 1^+} f(x)$ does not exist

A, D

For each positive integer n , let

$$y_n = \frac{1}{n} ((n+1)(n+2) \dots (n+n))^{1/n}$$

For $x \in \mathbb{R}$, let $[x]$ be the greatest integer less than or equal to x . If $\lim_{n \rightarrow \infty} y_n = L$, then the value of $[L]$ is

1

For any positive integer n , define $f_n : (0, \infty) \rightarrow \mathbb{R}$ as

(14)
$$f_n(x) = \sum_{j=1}^n \tan^{-1} \left(\frac{1}{1+(x+j)(x+j-1)} \right) \text{ for all } x \in (0, \infty)$$

(Here, the inverse trigonometric function $\tan^{-1}x$ assumes values in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$)

Then, which of the following statement(s) is (are) TRUE ?

(A) $\sum_{j=1}^5 \tan^2(f_j(0)) = 55$

(B) $\sum_{j=1}^{10} (1+f'_j(0)) \sec^2(f_j(0)) = 10$

(C) For any fixed positive integer n , $\lim_{x \rightarrow \infty} \tan(f_n(x)) = \frac{1}{n}$

(D) For any fixed positive integer n , $\lim_{x \rightarrow \infty} \sec^2(f_n(x)) = 1$

D

(15) Let $f : (0, \pi) \rightarrow \mathbb{R}$ be a twice differentiable function such that

$$\lim_{t \rightarrow x} \frac{f(x) \sin t - f(t) \sin x}{t - x} = \sin^2 x \text{ for all } x \in (0, \pi)$$

If $f\left(\frac{\pi}{6}\right) = -\frac{\pi}{12}$, then which of the following statement(s) is (are) TRUE ?

(A) $f\left(\frac{\pi}{4}\right) = \frac{\pi}{4\sqrt{2}}$

(B) $f(x) < \frac{x^4}{6} - x^2$ for all $x \in (0, \pi)$

(C) There exists $\alpha \in (0, \pi)$ such that $f'(\alpha) = 0$

(D) $f\left(\frac{\pi}{2}\right) + f\left(\frac{\pi}{2}\right) = 0$

B, C, D

For non-negative integers n , let

$$f(n) = \frac{\sum_{k=0}^n \sin\left(\frac{k+1}{n+2}\pi\right) \sin\left(\frac{k+2}{n+2}\pi\right)}{\sum_{k=0}^n \sin^2\left(\frac{k+1}{n+2}\pi\right)}$$

Assuming $\cos^{-1}x$ takes values in $[0, \pi]$, which of the following options is/are correct?

A. $f(4) = \frac{\sqrt{3}}{2}$

B. If $\alpha = \tan(\cos^{-1}f(6))$, then $\alpha^2 + 2\alpha - 1 = 0$

C. $\sin(7 \cos^{-1}f(5)) = 0$

D. $\lim_{n \rightarrow \infty} f(n) = \frac{1}{2}$

A, B, C

For $a \in \mathbb{R}$ $|a| > 1$, let

$$\lim_{n \rightarrow \infty} \left(\frac{1 + \sqrt[3]{2} + \dots + \sqrt[3]{n}}{n^{7/3} \left(\frac{1}{(an+1)^2} + \frac{1}{(an+2)^2} + \dots + \frac{1}{(an+n)^2} \right)} \right) = 54$$

Then the possible value(s) of a is/are

A. 8
 C. 7

B. -6
 D. -9

A, D

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function. We say that f has

18 PROPERTY 1 if $\lim_{h \rightarrow 0} \frac{f(h) - f(0)}{\sqrt{|h|}}$ exists and is finite, and

PROPERTY 2 if $\lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h^2}$ exists and is finite.

Then which of the following options is/are correct?

A. $f(x) = x|x|$ has PROPERTY 2
 C. $f(x) = |x|$ has PROPERTY 1

B. $f(x) = \sin x$ has PROPERTY 2
 D. $f(x) = x^{2/3}$ has PROPERTY 1

C, D

Let e denote the base of the natural logarithm. The value of the real number a for which the right hand limit

$$\lim_{x \rightarrow 0^+} \frac{(1-x)^{1/x} - e^{-1}}{x^a}$$

is equal to a nonzero real number, is 19

20 Let the functions $f: (-1, 1) \rightarrow \mathbb{R}$ and $g: (-1, 1) \rightarrow (-1, 1)$ be defined by $f(x) = |2x - 1| + |2x + 1|$ and $g(x) = x - [x]$, where $[x]$ denotes the greatest integer less than or equal to x . Let $\text{fog}: (-1, 1) \rightarrow \mathbb{R}$ be the composite function defined by $(\text{fog})(x) = f(g(x))$. Suppose c is the number of points in the interval $(-1, 1)$ at which fog is NOT continuous, and suppose d is the number of points in the interval $(-1, 1)$ at which fog is NOT differentiable. Then the value of $c + d$ is _____

4 The value of the limit

20
$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{4\sqrt{2}(\sin 3x + \sin x)}{2\sin 2x \sin \frac{3x}{2} + \cos \frac{5x}{2} - (\sqrt{2} + \sqrt{2} \cos 2x + \cos \frac{3x}{2})}$$
 8

is _____

8

For any positive integer n , let $S_n: (0, \infty) \rightarrow \mathbb{R}$ be defined by

$$S_n(x) = \sum_{k=1}^n \cot^{-1} \left(\frac{1+k(k+1)x^2}{x} \right)$$

where for any $x \in \mathbb{R}$, $\cot^{-1}(x) \in (0, \pi)$ and $\tan^{-1}(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then which of the following statements

is(are) TRUE?

- 2021
- (A) $S_{10}(x) = \frac{\pi}{2} - \tan^{-1} \left(\frac{1+11x^2}{10x} \right)$, for all $x > 0$ (B) $\lim_{n \rightarrow \infty} \cot(S_n(x)) = x$, for all $x > 0$
- (C) The equation $S_3(x) = \frac{\pi}{4}$ has a root in $(0, \infty)$ (D) $\tan(S_n(x)) \leq \frac{1}{2}$, for all $n \geq 1$ and $x > 0$

Let $f: \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \rightarrow \mathbb{R}$ be a continuous function such that $f(0) = 1$ and $\int_0^{\frac{\pi}{3}} f(t) dt = 0$. Then which of the following statements is(are) TRUE?

- 2021
- (A) The equation $f(x) - 3 \cos 3x = 0$ has at least one solution in $\left(0, \frac{\pi}{3}\right)$
- (B) The equation $f(x) - 3 \sin 3x = -\frac{6}{\pi}$ has at least one solution in $\left(0, \frac{\pi}{3}\right)$

(C) $\lim_{x \rightarrow 0} \frac{x \int_0^x f(t) dt}{1 - e^{x^2}} = -1$

(D) $\lim_{x \rightarrow 0} \frac{\sin x \int_0^x f(t) dt}{x^2} = -1$

A, B, C

$\lim_{x \rightarrow \frac{3\pi}{4}} \frac{1 + \sqrt[3]{\tan x}}{1 - 2\cos^2 x}$ (a) $\lim_{x \rightarrow 0} \tan^{-1} \frac{a}{x^2}$ where $a \in \mathbb{R}$
 (b) Plot the graph of the function $f(x) = \lim_{t \rightarrow 0} \left(\frac{2x}{\pi} \tan^{-1} \frac{x}{t^2} \right)$

Find the sum of an infinite geometric series whose first term is the limit of the function

$f(x) = \frac{\tan x - \sin x}{\sin^2 x}$ as $x \rightarrow 0$ and whose common ratio is the limit of the function
 $g(x) = \frac{1 - \sqrt{x}}{(\cos^{-1} x)^2}$ as $x \rightarrow 1$. (Use of series expansion or L'Hospital's rule is not allowed.)

$\lim_{x \rightarrow 0} \frac{8}{x^3} \left[1 - \cos \frac{x^2}{2} - \cos \frac{x^2}{4} + \cos \frac{x^2}{2} \cos \frac{x^2}{4} \right]$ $\lim_{x \rightarrow 1} \frac{(1x(1+x) - 1x2)(3.4^{x-1} - 3x)}{[(7+x)^2 - (1+3x)^2] \sin(x-1)}$

Let $f(x) = \frac{x}{\sin x}$, $x > 0$ and $g(x) = x + 3$, $x < 1$
 $= 2 - x$, $x \leq 0$ $= x^2 - 2x - 2$, $1 \leq x < 2$
 $= x - 5$, $x \geq 2$

find LHL and RHL of $g(f(x))$ at $x = 0$ and hence find $\lim_{x \rightarrow 0} g(f(x))$

Let $P_n = a^{2n-1} - 1$, $\forall n = 2, 3, \dots$ and Let $P_1 = a^n - 1$ where $a \in \mathbb{R}^+$ then evaluate $\lim_{x \rightarrow 0} \frac{P_n}{x} = (na)^n$

$\lim_{x \rightarrow \infty} \frac{(3x^4 + 2x^2) \sin \frac{1}{x} + |x|^3 + 5}{|x|^3 + |x|^2 + |x| + 1} = -2$

If $f(x) = \begin{cases} h \operatorname{cosec}(x\pi) & 0 < x < 1 \\ = h \sin(2x\pi) & 1 < x < 3/2 \end{cases}$ and $g(x) = \frac{2^{f(x)} + 1}{3^{f(x)} + 1}$ then
 find $\tan^{-1}(g(l^-))$ and $\sec^{-1}(g(l^+))$.

At the end-points and the midpoint of a circular arc AB tangent lines are drawn, and the points A and B are joined with a chord. Prove that the ratio of the areas of the two triangles thus formed tends to 4 as the arc AB decreases indefinitely.

$\lim_{x \rightarrow 0} \left[\frac{(1+x)^{1/x}}{e} \right]^{1/x} \lim_{x \rightarrow 0} \left[\sin^2 \left(\frac{\pi}{2-2x} \right) \right]^{1/x} \lim_{x \rightarrow \infty} x^2 \sin \left(\sqrt[n]{\cos \frac{\pi}{x}} \right) = -\frac{\pi^2}{4}$

$\lim_{x \rightarrow \infty} \left[\cos \left(2x \left(\frac{x}{1+x} \right)^a \right) \right]^{1/x^2}$ $a \in \mathbb{R}$

35 Let $f(x) = \frac{\sin^{-1}(1-\{x\}) \cdot \cos^{-1}(1-\{x\})}{\sqrt{2\{x\}} \cdot (1-\{x\})}$ then find $\lim_{x \rightarrow 0^+} f(x)$ and $\lim_{x \rightarrow 0^-} f(x)$, where $\{x\}$ denotes the fractional part function.

36 Find the values of a, b & c so that $\lim_{x \rightarrow 0} \frac{ae^x - b \cos x + ce^{-x}}{x \sin x} = 2$ $a=c=1 \quad b=2$

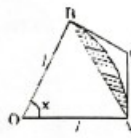
37 $\lim_{x \rightarrow a} \frac{1}{(a^2 - x^2)^2} \left(\frac{a^2 + x^2}{ax} - 2 \sin\left(\frac{a\pi}{2}\right) \sin\left(\frac{\pi x}{2}\right) \right)$ where a is an odd integer $\frac{4 + \pi^2 a^2}{16a^4}$

38 $\lim_{y \rightarrow 0} \left[\lim_{x \rightarrow z} \frac{\exp\left(x \ln\left(1 + \frac{ay}{x}\right)\right) - \exp\left(x \ln\left(1 + \frac{by}{x}\right)\right)}{y} \right] = a - b$

39 If s_n be the sum of n terms of the series, $\sin x + \sin 2x + \sin 3x + \dots + \sin nx$ then show that $\lim_{n \rightarrow \infty} \frac{s_1 - s_2 + \dots + s_n}{n} = \frac{1}{2} \cot \frac{x}{2}$ ($x \neq 2k\pi, k \in \mathbb{I}$)

40 $\lim_{x \rightarrow 0} \left[\frac{(n(1+x))^{1+x}}{x^2} - \frac{1}{x} \right]$ Let $P_n = \frac{2^3-1}{2^3+1} \cdot \frac{3^3-1}{3^3+1} \cdot \frac{4^3-1}{4^3+1} \dots \frac{n^3-1}{n^3+1}$. Evaluate $\lim_{n \rightarrow \infty} P_n = \frac{2}{3}$

42 A circular arc of radius 1 subtends an angle of x radians, $0 < x < \frac{\pi}{2}$ as shown in the figure. The point C is the intersection of the two tangent lines at A & B. Let T(x) be the area of triangle ABC & let S(x) be the area of the shaded region. Compute:



(a) $\lim_{x \rightarrow \infty} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{x} + \sqrt{x}} = \frac{1}{3}$ (b) $\lim_{x \rightarrow \infty} \left[\sqrt{x + \sqrt{x + \sqrt{x}}} - \sqrt{x} \right] = \frac{1}{2}$

43 If $f(n, \theta) = \prod_{r=1}^n \left(1 - \tan^2 \frac{\theta}{2^r} \right)$, then compute $\lim_{n \rightarrow \infty} f(n, \theta) = \frac{2(\tan x - \sin x) - x^3}{x^3} = \frac{1}{4}$

44 Through a point A on a circle, a chord AP is drawn & on the tangent at A a point T is taken such that AT = AP. If TP produced meet the diameter through A at Q, prove that the limiting value of AQ when P moves upto A is double the diameter of the circle.

45 Find a & b if: (i) $\lim_{x \rightarrow \infty} \left[\frac{x^2+1}{x+1} - ax - b \right] = 0$ (ii) $\lim_{x \rightarrow -\infty} \left[\sqrt{x^2-x+1} - ax - b \right] = 0$

46 $\lim_{x \rightarrow 0} \frac{\sin(\pi \cos^2 x)}{x^2}$ equals (A) $-\pi$ (B) π (C) $\frac{\pi}{2}$ (D) 1

47 Evaluate $\lim_{x \rightarrow 0} \frac{a \tan x - a \sin x}{\tan x - \sin x}, a > 0$ $\frac{a}{2}$

48 The integer n for which $\lim_{x \rightarrow 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n}$ is a finite non-zero number is (A) 1 (B) 2 (C) 3 (D) 4

49 If $\lim_{x \rightarrow 0} \frac{\sin(nx)[(a-n)x - \tan x]}{x^2} = 0$ ($n > 0$) then the value of 'a' is equal to (A) $\frac{1}{n}$ (B) $n^2 + 1$ (C) $\frac{n^2 + 1}{n}$ (D) None

50 Find the value of $\lim_{n \rightarrow \infty} \left[\frac{2}{\pi} (n+1) \cos^{-1}\left(\frac{1}{n}\right) - n \right] = -\frac{2}{\pi} + 1$

32. If α, β are the roots of the quadratic equation $ax^2 + bx + c = 0$ then $\lim_{x \rightarrow \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - \alpha)^2}$ equals

- (A) 0 (B) $\frac{1}{2}(\alpha - \beta)^2$ (C) $\frac{a^2}{2}(\alpha - \beta)^2$ (D) $-\frac{a^2}{2}(\alpha - \beta)^2$

33. ABC is an isosceles triangle inscribed in a circle of radius r . If $AB = AC$ & h is the altitude from A to BC and P be the perimeter of ABC then $\lim_{h \rightarrow 0} \frac{\Delta}{P^3}$ equals (where Δ is the area of the triangle)

- (A) $\frac{1}{32r}$ (B) $\frac{1}{64r}$ (C) $\frac{1}{128r}$ (D) none

34. If $[x]$ denotes the greatest integer $\leq x$, then $\lim_{n \rightarrow \infty} \frac{1}{n^4} ([1^2 x] + [2^2 x] + \dots + [n^2 x])$ equals

- (A) $x/2$ (B) $x/3$ (C) $x/6$ (D) $x/4$

Let the function f, g and h be defined as follows:

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & \text{for } -1 \leq x \leq 1 \text{ and } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$$

$$g(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{for } -1 \leq x \leq 1 \text{ and } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$$

$h(x) = |x|^3$ for $-1 \leq x \leq 1$

Which of these functions are differentiable at $x = 0$?

(A) f and g only (B) f and h only (C) g and h only (D) none

35. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin x}{\cos^{-1}\left[\frac{1}{4}(3 \sin x - \sin 3x)\right]}$ where $[\]$ denotes greatest integer function, is

- (A) $\frac{2}{\pi}$ (B) 1 (C) $\frac{4}{\pi}$ (D) does not exist

36. The limiting value of the function $f(x) = \frac{2\sqrt{2} - (\cos x + \sin x)^3}{1 - \sin 2x}$ when $x \rightarrow \frac{\pi}{4}$ is

- (A) $\sqrt{2}$ (B) $\frac{1}{\sqrt{2}}$ (C) $3\sqrt{2}$ (D) $\frac{3}{\sqrt{2}}$

57. $\lim_{n \rightarrow \infty} \frac{1^2 n + 2^2(n-1) + 3^2(n-2) + \dots + n^2 \cdot 1}{1^3 + 2^3 + 3^3 + \dots + n^3}$ is equal to:
 (A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{1}{2}$ (D) $\frac{1}{6}$

58. For the function $f(x) = \lim_{n \rightarrow \infty} \frac{1}{1 + n \sin^2(\pi x)}$, which of the following holds?
 (A) The range of f is a singleton set (B) f is continuous on \mathbb{R}
 (C) f is discontinuous for all $x \in \mathbb{I}$ (D) f is discontinuous for some $x \in \mathbb{R}$

59. $\lim_{x \rightarrow 1/\sqrt{2}} \frac{x - \cos(\sin^{-1} x)}{1 - \tan(\sin^{-1} x)}$ is
 (A) $\frac{1}{\sqrt{2}}$ (B) $-\frac{1}{\sqrt{2}}$ (C) $\sqrt{2}$ (D) $-\sqrt{2}$

60. Limit $\lim_{x \rightarrow \infty} \frac{\cot^{-1}(\sqrt{x+1} - \sqrt{x})}{\sec^{-1}\left(\left[\frac{2x+1}{x-1}\right]^x\right)}$ is equal to
 (A) 1 (B) 0 (C) $\pi/2$ (D) non-existent

61. Let $f(x) = \frac{\ln(x^2 + e^x)}{\ln(x^4 + e^{2x})}$. If $\lim_{x \rightarrow \infty} f(x) = l$ and $\lim_{x \rightarrow -\infty} f(x) = m$ then:
 (A) $l = m = \frac{1}{2}$ (B) $l = 2m$ (C) $2l = m$ (D) $l + m = 0$

62. $\lim_{n \rightarrow \infty} \cos(\pi\sqrt{n^2 + n})$ when n is an integer:
 (A) is equal to 1 (B) is equal to -1 (C) is equal to zero (D) does not exist

63. Limit $\lim_{x \rightarrow 0} \frac{(\sin x - \tan x)^2 - (1 - \cos 2x)^4 + x^5}{7(\tan^{-1} x)^7 + (\sin^{-1} x)^5 + 3 \sin^5 x}$ is equal to
 (A) 0 (B) $\frac{1}{7}$ (C) $\frac{1}{3}$ (D) 1

64. The value of $\lim_{x \rightarrow 0} \frac{\tan(\{x\} - 1) \sin\{x\}}{\{x\}(\{x\} - 1)}$ where $\{x\}$ denotes the fractional part function:
 (A) is 1 (B) is $\tan 1$ (C) is $\sin 1$ (D) is non-existent

65 The value of $\lim_{x \rightarrow 0} (\cos ax)^{\cos ax}$ is

- (A) $\left(-\frac{8a^2}{a^2}\right)$ (B) $\left(-\frac{8a^2}{a^2}\right)$ (C) $\left(-\frac{a^2}{2a^2}\right)$ (D) $\left(-\frac{a^2}{2a^2}\right)$

66 $\lim_{x \rightarrow \infty} \frac{2+2x+\sin 2x}{(2x+\sin 2x)e^{2x}}$ is:

- (A) equal to zero (B) equal to 1 (C) equal to -1 (D) non-existent

67 Let $f(x) = \begin{cases} \frac{\tan^2(x)}{x^2 - [x]^2} & \text{for } x > 0 \\ 1 & \text{for } x = 0 \\ \sqrt{[x] \cot(x)} & \text{for } x < 0 \end{cases}$ where $[x]$ is the step up function and $\{x\}$ is the fractional part function of x , then:

- (A) $\lim_{x \rightarrow 0^+} f(x) = 1$ (B) $\lim_{x \rightarrow 0^-} f(x) = 1$
 (C) $\cot^{-1} \left(\lim_{x \rightarrow 0^+} f(x) \right) = 1$ (D) f is continuous at $x = 1$.

68 $\lim_{x \rightarrow 0} \frac{\sin(6x^2)}{(n \cos(2x^2 - x))} =$

(A) 12 (B) -12 (C) 6 (D) -6

69 $\lim_{x \rightarrow \pi/2} \left[\frac{x - \frac{\pi}{2}}{\cos x} \right]$ is: (where $[]$ represents greatest integer function.)

(A) -1 (B) 0 (C) $(e^e) - 2$ (D) does not exist

70 If $f(x) = \begin{cases} x^2 + 2 & x \geq 2 \\ 1 - x & x < 2 \end{cases}$ and $g(x) = \begin{cases} 2x & x > 1 \\ 3 - x & x \leq 1 \end{cases}$, evaluate $\lim_{x \rightarrow 1} f(g(x))$. (6)

71 Evaluate $\lim_{n \rightarrow \infty} \frac{[1 \cdot 2x] + [2 \cdot 3x] + \dots + [n \cdot (n+1)x]}{n^3}$ where $[\cdot]$ denotes the greatest integer function $\approx 1/3$

72 If $f(x) = \begin{cases} x-1 & x \geq 1 \\ 2x^2-2 & x < 1 \end{cases}$, $g(x) = \begin{cases} x+1 & x > 0 \\ -x^2+1 & x \leq 0 \end{cases}$ and $h(x) = |x|$

then find $\lim_{x \rightarrow 0} f(g(h(x)))$

(A) 1 (B) 0 (C) -1 (D) does not exist

72. $\lim_{x \rightarrow a} \left(\frac{[x]^3}{a} - \left[\frac{x}{a} \right]^3 \right)$ ($a > 0$), where $[x]$ denotes the greatest integer less than or equal to x is

(A) $a^2 + 1$ (B) $a^2 - 1$ (C) a^2 (D) $-a^2$

74. If α and β be the roots of $ax^2 + bx + c = 0$, then $\lim_{x \rightarrow \alpha} (1 + ax^2 + bx + c)^{\frac{1}{x-\alpha}}$ is

(A) $a(\alpha - \beta)$ (B) $\ln |a(\alpha - \beta)|$ (C) $e^{a(\alpha - \beta)}$ (D) $e^{a|\alpha - \beta|}$

75. $\lim_{x \rightarrow 0} \frac{(1 - a^x)^{\sin x}}{[x]}$ is (where $[]$ represents greatest integral part function)

(A) $\frac{1}{1}$ (B) 1 (C) 0 (D) does not exist

76. $\lim_{x \rightarrow 0^+} \log_{\sin(x/2)} \sin x$ is equal to

(A) 1 (B) 0 (C) 4 (D) $\frac{1}{4}$

77. If $\ell = \lim_{x \rightarrow 0} (\sin \sqrt{x+1} - \sin \sqrt{x})$ and $m = \lim_{x \rightarrow 0} [\sin \sqrt{x+1} - \sin \sqrt{x}]$ where $[]$ denotes the greatest integer function then:

(A) $\ell = m = 0$ (B) $\ell = 0$; m is undefined (C) ℓ, m both do not exist (D) $\ell = 0, m = 0$ (although m exist)

78. The limit $\lim_{\theta \rightarrow 0} \left(\left[\frac{n \sin \theta}{\theta} \right] + \left[\frac{n \tan \theta}{\theta} \right] \right)$, where $[x]$ is the greatest integer function and $n \in \mathbb{N}$, is

(A) $2n$ (B) $2n + 1$ (C) $2n - 1$ (D) does not exist

79. The limit $\lim_{x \rightarrow 0} \frac{1 - \cos x \sqrt{\cos 2x}}{x^2}$ is equal to

(A) $1/2$ (B) $-1/2$ (C) $3/2$ (D) 1

80. $\lim_{n \rightarrow \infty} \left(\frac{1}{\sqrt{n^2}} + \frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \dots + \frac{1}{\sqrt{n^2+2n}} \right)$

(A) 1 (B) $1/2$ (C) 0 (D) 2

81. The value of $\lim_{x \rightarrow 0} \frac{\cos(\sin x) - \cos x}{x^4}$ is equal to

(A) $1/5$ (B) $1/6$ (C) $1/4$ (D) $1/2$

82. Limit $\lim_{x \rightarrow \infty} \frac{e^x \left((2^{x^2})^{\frac{1}{e^x}} - (3^{x^2})^{\frac{1}{e^x}} \right)}{x^n}$, $n \in \mathbb{N}$ is equal to:

(A) 0 (B) $\ln(2/3)$ (C) $\ln(3/2)$ (D) none

83. Show that $\lim_{n \rightarrow \infty} \frac{1}{n^2} \left(\sum_{k=1}^n k + 3 \sum_{k=1}^{n-1} k + 5 \sum_{k=1}^{n-2} k + \dots + (2n-1) \cdot 1 \right) = \frac{1}{12}$

84. Let $f(x) = \lim_{n \rightarrow \infty} \left\{ \lim_{m \rightarrow \infty} (\cos^{2m}(n^2 x)) \right\}$ where $x \in \mathbb{R}$. Prove that

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$$

85. Evaluate $\lim_{x \rightarrow 0^+} \left\{ \lim_{n \rightarrow \infty} \left(\frac{[1^2(\sin x)^x] + [2^2(\sin x)^x] + \dots + [n^2(\sin x)^x]}{n^3} \right) \right\} = \frac{1}{3}$

where $[]$ denotes the greatest integer function.

86 Consider two functions $f(x) = \lim_{n \rightarrow \infty} \left(\cos \frac{x}{\sqrt{n}} \right)^n$ and $g(x) = -x^{4b}$ where $b = \lim_{x \rightarrow \infty} (\sqrt{x^2+x+1} - \sqrt{x^2+1})$

Then

$f(x)$ is

(A) e^{-x^2}

(B) $e^{-\frac{x^2}{2}}$

(C) e^{x^2}

(D) $e^{\frac{x^2}{2}}$

$g(x)$ is

(A) $-x^2$

(B) x^2

(C) x^4

(D) $-x^4$

Number of solutions of $f(x) + g(x) = 0$ is

(A) 2

(B) 4

(C) 0

(D) 1

87 For $x > 0$, $\lim_{x \rightarrow 0} (\sin x)^{1/x} + (1/x)^{\sin x}$ is

(A) 0

(B) -1

(C) 1

(D) 2

88 Limit $\lim_{x \rightarrow \pi/2} \frac{(1 - \tan \frac{x}{2})(1 - \sin x)}{(1 + \tan \frac{x}{2})(\pi - 2x)^3}$ is :

(A) 1/16

(B) -1/16

(C) 1/32

(D) -1/32

89 $\lim_{x \rightarrow \infty} \frac{\log^x - [x]}{[x]}$, $n \in \mathbb{N}$, $[x]$ denotes greatest integer less than or equal to x .

(A) has value -1

(B) has value 0

(C) has value 1

(D) does not exist

If

$$f(x) = \begin{cases} \frac{[x]^2 + \sin [x]}{[x]} & \text{for } [x] \neq 0 \\ 0 & \text{for } [x] = 0 \end{cases}$$

Q1

where $[x]$ denotes the greatest integer less than or equal to x , then $\lim_{x \rightarrow 0} f(x)$ equals

- (a) 1
(b) 0
(c) -1
(d) $\lim_{x \rightarrow 0^+} f(x) = 0$

Ans. (d)

Q2

$$\lim_{x \rightarrow a} \left[\left(\frac{a^{1/2} + x^{1/2}}{a^{1/4} - x^{1/4}} \right)^{-1} \right]$$

$$= \left[\frac{2(ax)^{1/4}}{x^{3/4} - a^{1/4} x^{1/2} + a^{1/2} x^{1/4} - a^{3/4}} \right]^{-1} - \sqrt{2}^{\log_4 a} \Bigg\}^8$$

is

- (a) a
(b) $a^{3/4}$
(c) a^2
(d) none of these

Ans. (c)

Let

$$f(x) = \lim_{n \rightarrow \infty} \frac{\log(2+x) - x^{2n} \sin x}{1+x^{2n}}$$

Then

- (a) f is continuous at $x = 1$
(b) $\lim_{x \rightarrow 1^+} f(x) \neq \lim_{x \rightarrow 1^-} f(x)$
(c) $\lim_{x \rightarrow 1^+} f(x) = \sin 1$
(d) $\lim_{x \rightarrow 1^-} f(x)$ doesn't exist

Ans. (b)

Q3

The value of $\lim_{n \rightarrow \infty} \cos \frac{x}{2} \cos \frac{x}{4} \dots \cos \frac{x}{2^n}$ is

- (a) 1
(b) $\frac{\sin x}{x}$
(c) $\frac{x}{\sin x}$
(d) none of these

Ans. (b)

686

If $\lim_{x \rightarrow a} (f(x) g(x))$ exists for any functions f

and g then

- (a) $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ exist
- (b) $\lim_{x \rightarrow a} f(x)$ exist but $\lim_{x \rightarrow a} g(x)$ may not exist
- (c) $\lim_{x \rightarrow a} f(x)$ mayn't exist but $\lim_{x \rightarrow a} g(x)$ exist
- (d) $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ may not exist

Ans. (d)

67

The $\lim_{x \rightarrow 0} x^8 \left[\frac{1}{x^3} \right]$ (where $[x]$ is greatest

integer function) is

- (a) a nonzero real number
- (b) a rational number
- (c) an integer
- (d) zero

Ans. (b), (c) and (d)

68

$$\Delta(x) = \begin{vmatrix} \tan x & \tan(x+h) & \tan(x+2h) \\ \tan(x+2h) & \tan x & \tan(x+h) \\ \tan(x+h) & \tan(x+2h) & \tan x \end{vmatrix}$$

Find the value of $\lim_{h \rightarrow 0} \frac{\sqrt{3} \Delta(\pi/3)}{h^2}$

$$\frac{2\sqrt{3} \times 3}{432}$$

94

(a) $f(x) = x \operatorname{sgn}(x - 1)$

(b) $f(x) = \frac{\sin(\sin(\tan(x^2/2)))}{\log \cos 3x}$

(c) $f(x) = \frac{\sqrt[3]{1 + \tan^{-1} 3x} - \sqrt[3]{1 - \sin^{-1} 3x}}{\sqrt{1 + \sin^{-1} 2x} - \sqrt{1 + \tan^{-1} 2x}}$

(d) $f(x) = \frac{e^{1/x} - 1}{e^{1/x} + 1}$

100 **Statement-1** If a and b are positive and $[x]$ denotes the greatest integer $\leq x$, then $\lim_{x \rightarrow 0^+} \frac{x [b/x]}{a [x]} = \frac{b}{a}$.

Statement-2 $\lim_{x \rightarrow \infty} \frac{\{x\}}{x} = 0$, where $\{x\}$ denotes fractional part of x .

Ans. (a)

101) Let $L = \lim_{x \rightarrow 0} \frac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}, a > 0$

If L is finite, then

(a) $a = 2$

(b) $a = 1$

(c) $L = \frac{1}{64}$

(d) $L = \frac{1}{32}$

102) Evaluate $\lim_{x \rightarrow 1^-} \frac{1 - \sqrt{x}}{(\cos^{-1} x)^2} \cdot \frac{1}{4}$

103) Evaluate $\lim_{x \rightarrow -1^+} \frac{\sin^{-1}(\sqrt{x - \sqrt{\cos^{-1} x}})}{\sqrt{1 - x^2}} \cdot \frac{1}{\sqrt{17}}$

104) Solve $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan x - \sin(\tan^{-1}(\tan x))}{\tan x + \cos^2(\tan x)} \quad (1)$

105) Evaluate $\lim_{x \rightarrow 0} \frac{\sin x - x^2 - \{x\} \cdot \{-x\}}{x \cos x - x^2 - \{x\} \cdot \{-x\}}$

where $\{.\}$ denotes the fractional part

RHL = 1 LHL = $\frac{1}{3}$ DNE

106) Evaluate $\lim_{x \rightarrow 0} \frac{x + \ln(\sqrt{1+x^2} - x)}{x^3} = \frac{1}{6}$

Find a polynomial of the least

107) degree, such that $\lim_{x \rightarrow 0} \left(1 + \frac{x^2 + f(x)}{x^2}\right)^{1/x} = e^2$. $f(x) = -x^2 + 2x$

108) Evaluate $\lim_{x \rightarrow 0} \lim_{n \rightarrow \infty} \frac{\tan x^2 + (x+1)^n \cdot \sin x}{x^2 + (x+1)^n}$ DNE

LHL 1 RHL 0 DNE

107 $\lim_{n \rightarrow \infty} \prod_{r=3}^n \frac{r^3 - 8}{r^3 + 8} = \frac{2}{7}$

108 Evaluate $\lim_{x \rightarrow 0} \left(\frac{(1 + \{x\})^{1/\{x\}}}{e} \right)^{1/\{x\}}$ if it exist, where $\{x\}$ denotes the fractional part of x .

111 $\lim_{x \rightarrow 0} \left(\frac{(1+x)^x}{e^2} \right)^{\frac{4}{\sin x}}$ is:

(a) e^4 (b) e^{-4} (c) e^8 (d) e^{-8}

112 For $n \in \mathbb{N}$, let $f_n(x) = \tan \frac{x}{2} (1 + \sec x) (1 + \sec 2x) (1 + \sec 4x) \dots (1 + \sec 2^{n-1}x)$, the $\lim_{x \rightarrow 0} \frac{f_n(x)}{2x}$ is equal to:

(a) 0 (b) 2^n (c) 2^{n-1} (d) 2^{n+1}

113 The value of $\lim_{x \rightarrow \frac{\pi}{4}} (1 + [x])^{\frac{1}{\ln(\tan x)}}$ is:

(where $[.]$ denotes greatest integer function).

(a) 0 (b) 1 (c) e (d) $\frac{1}{e}$

114 The value of $\lim_{n \rightarrow \infty} \left(\frac{n!}{n^n} \right)^{\frac{3n^3+4}{4n^4-1}}$, $n \in \mathbb{N}$ is equal to:

(a) $\left(\frac{1}{e} \right)^{3/4}$ (b) $e^{3/4}$ (c) e^{-1} (d) 0

115 If $f(x) = \lim_{n \rightarrow \infty} x \left(\frac{3}{2} + [\cos x] (\sqrt{n^2+1} - \sqrt{n^2-3n+1}) \right)$ where $[y]$ denotes largest integer $\leq y$, then identify the correct statement(s).

(a) $\lim_{x \rightarrow 0} f(x) = 0$ (b) $\lim_{x \rightarrow \frac{\pi}{2}} f(x) = \frac{3\pi}{4}$

(c) $f(x) = \frac{3x}{2} \forall x \in \left[0, \frac{\pi}{2} \right]$ (d) $f(x) = 0 \forall x \in \left(\frac{\pi}{2}, \frac{3\pi}{2} \right)$

116 The value of $\lim_{n \rightarrow \infty} \cos^2 \left(\pi \left(\sqrt[3]{n^3 + n^2 + 2n} \right) \right)$ (where $n \in \mathbb{N}$):

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{9}$

117 If $\alpha, \beta \in \left(-\frac{\pi}{2}, 0 \right)$ such that $(\sin \alpha + \sin \beta) + \frac{\sin \alpha}{\sin \beta} = 0$ and $(\sin \alpha + \sin \beta) \frac{\sin \alpha}{\sin \beta} = -1$ and $\lambda = \lim_{n \rightarrow \infty} \frac{1 + (2 \sin \alpha)^{2n}}{(2 \sin \beta)^{2n}}$ then:

(a) $\alpha = -\frac{\pi}{6}$ (b) $\lambda = 2$ (c) $\alpha = -\frac{\pi}{3}$ (d) $\lambda = 1$

118 Let $f(x) = \begin{cases} |x-2| + a^2 - 6a + 9, & x < 2 \\ 5 - 2x, & x \geq 2 \end{cases}$

If $\lim_{x \rightarrow 2} [f(x)]$ exists, the possible values a can take is/are (where $[\]$ represents the greatest integer function)

- (a) 2 ✓ (b) $\frac{5}{2}$ ✓ (c) 3 ✓ (d) $\frac{7}{2}$

119 Let $f(x) = \begin{cases} x+3 & ; -2 < x < 0 \\ 4 & ; x = 0 \\ 2x+5 & ; 0 < x < 1 \end{cases}$, then

120 $\lim_{x \rightarrow 0} f([x - \tan x])$ is: ($[\]$ denotes greatest integer function) (d) None of these

- (a) 2 ✓ (b) 4 ✓ (c) 5

121 $\lim_{x \rightarrow 0} f\left(\left\{\frac{x}{\tan x}\right\}\right)$ is: ($\{ \}$ denotes fractional part of function) (d) None of these

- (a) 4 (b) 5 ✓ (c) 7

122 If $\lim_{x \rightarrow 0} \frac{\ln \cot\left(\frac{\pi}{4} - \beta x\right)}{\tan \alpha x} = 1$, then $\frac{\alpha}{\beta} = 2$

123 If $\lim_{x \rightarrow 0} \frac{f(x)}{\sin^2 x} = 8$, $\lim_{x \rightarrow 0} \frac{g(x)}{2 \cos x - xe^x + x^3 + x - 2} = \lambda$ and $\lim_{x \rightarrow 0} (1 + 2f(x))^{g(x)} = \frac{1}{e}$, then $\lambda = 8$

124 If α, β are two distinct real roots of the equation $ax^3 + x - 1 - a = 0$, ($a \neq -1, 0$), none of which is equal to unity, then the value of $\lim_{x \rightarrow 0} \frac{(1+a)x^3 - x^2 - a}{(e^{1-ax} - 1)(x-1)} = \frac{a(k\alpha - \beta)}{\alpha}$. Find the value of kl . 1

125 Find $\lim_{x \rightarrow \alpha} \left[\frac{\min(\sin x, \{x\})}{x-1} \right]$ where α is root of equation $\sin x + 1 = x$ (here $[\]$ represent greatest integer and $\{ \}$ represent fractional part function) 0

126 Let $f(x) = \cos 2x \cdot \cos 4x \cdot \cos 6x \cdot \cos 8x \cdot \cos 10x$, then $\lim_{x \rightarrow 0} \frac{1 - (f(x))^3}{5 \sin^2 x}$ equals: (a) 660 (b) 135 (c) 132 ✓ (d) 66

127 If $\lim_{x \rightarrow 0} \left(\left[\frac{\sin^{-1} x}{x} \right] + \left[\frac{2^2 \sin^{-1} 2x}{x} \right] + \left[\frac{3^2 \sin^{-1} 3x}{x} \right] + \dots + \left[\frac{n^2 \sin^{-1} nx}{x} \right] \right) = 100$, then the value of n , is: 4

126 Let $f: R \rightarrow (0, \infty)$ be such that $f(x) + \frac{e^{x+x^2}}{f(x)} \leq e^x + e^{x^2} \forall x > 0$, then $\lim_{x \rightarrow 1} f(x)$ is
 (a) 1 (b) $\frac{1}{e}$ (c) e (d) $2e$

127 Let $f(x) = \begin{cases} \lim_{n \rightarrow \infty} \left(\frac{px}{n} \sum_{r=1}^n \frac{[r^2 - e^{-x} + r - 1]}{r(r+1)} \right) + \lambda, & x > 0 \\ q, & x = 0 \\ \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{[r^2 + r + e^x - 1]}{r(r+1)}, & x < 0 \end{cases}$

is differentiable in R (the set of all real numbers)

[Note: $\{y\}$ and $\{y\}$ denote greatest integer and fractional part function of y .]

128 The value of $p + q + \lambda$ is equal to:
 (a) -2 (b) 2 (c) 1 (d) 3

129 The value of $f'(\ln 2) + f'\left(\ln \frac{1}{2}\right) + f'\left(\ln \frac{3}{2}\right) + f'\left(\ln \frac{5}{2}\right) + \dots \infty$ is equal to:
 (a) 3 (b) 4 (c) 6 (d) 2

130 If g is the inverse of f then $g'\left(\frac{1}{2}\right)$ is equal to:
 (a) $\ln 2$ (b) $-\ln 2$ (c) 2 (d) $\frac{1}{2}$

131 The possible value(s) of k for which $\lim_{x \rightarrow \frac{\pi}{8}} \frac{2x^3 - (\tan^{-1} x)^3}{\pi x^3 \cot^{-1} |kx| + k^2 x^6 \sin \frac{1}{x^3} - 3kx^3} = \frac{1}{2}$ is
 (a) 0 (b) -1 (c) 2 (d) 4

Let $f(x) = \frac{\left(\frac{\pi}{2} - \cos^{-1}(1 - \{x\}^2)\right) \cdot (\cos^{-1}(1 - (x)))^2}{2(\{x\} - \{x\}^3)}$. If $f(0^+) = p$ and $f(0^-) = q$, then

find the value of $\left(\frac{p\pi}{q}\right)$ 4

[Note: $\{k\}$ denotes the fractional part of k .]

If α and β ($\alpha < \beta$) are the roots of the equation

132 $\lim_{t \rightarrow \infty} \cos^{-1} \left[\sin \left(\tan^{-1} \left(\frac{\sqrt{tx}}{\sqrt{tx^2 - 3tx + t - 1 - x}} \right) \right) \right] = \frac{\pi}{6}$

then find the value of $(8^\alpha + 2^\beta - \alpha\beta)$. 9

133 If $S_n = \sum_{r=1}^n \tan^{-1} \left[\frac{2(2r-1)}{4+r^2(r^2-2r+1)} \right]$ then find the value of

$\lim_{n \rightarrow \infty} \sum_{n=2}^n (\cot(S_{n-1}) - \cot(S_n))$. 2

134 Let $\lim_{n \rightarrow \infty} \frac{1 + cn^2}{(2n+3+2\sin n)^2} = \frac{1}{2}$. If $c \leq \alpha \leq \beta$ where α and β are the roots of the quadratic equation $x^2 - 2px + p^2 - 1 = 0$, then find the minimum integral value of p . 3

135 If $f(x) = \lim_{n \rightarrow \infty} \frac{\left(1 - \cos \left\{1 - \tan \left(\frac{\pi}{4} - x\right)\right\}\right) (x+1)^n + \lambda \sin((n - \sqrt{n^2 - 8n})x)}{x^2 (x+1)^n + x}$, $x \neq 0$ is continuous at $x = 0$, then find the value of $(f(0) + 2\lambda)$. 3

136 If $f: (0, \infty) \rightarrow \mathbb{N}$ and
 $f(x) = \left\lfloor \frac{x^2 + x + 1}{x^2 + 1} \right\rfloor + \left\lfloor \frac{4x^2 + x + 2}{2x^2 + 1} \right\rfloor + \left\lfloor \frac{9x^2 + x + 3}{3x^2 + 1} \right\rfloor + \dots + \left\lfloor \frac{n^2 x^2 + x + n}{nx^2 + 1} \right\rfloor, n \in \mathbb{N}$ then

find the value of $\lim_{n \rightarrow \infty} \left[\frac{f(x) - n}{(f(x))^2 - \frac{n^3(n+2)}{4}} \right] = \textcircled{1}$

[Note: $\lfloor y \rfloor$ denotes the greatest integer less than or equal to y .]

137 $\lim_{x \rightarrow 0} \left[\min(y^2 - 4y + 11), \frac{\sin x}{x} \right]$ (where $\lfloor \cdot \rfloor$ denotes the greatest integer function) is
 1) 5 2) 6 3) 7 4) Does not exist

138 Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a positive increasing function with $\lim_{x \rightarrow \infty} \frac{f(3x)}{f(x)} = 1$ then $\lim_{x \rightarrow \infty} \frac{f(2x)}{f(x)} =$

- 1) $\frac{2}{3}$ 2) $\frac{3}{2}$ 3) 3 4) 1