Let [x] denote the greatest integer function & f(x) be defined in a neighbourhood of 2 by

$$f(x) = \begin{cases} \dfrac{\left(\exp\{(x+2)\ell n4\}\right)^{\frac{[x+1]}{4}} - 16}{4^x - 16} & , x<2 \\ A\dfrac{1-\cos(x-2)}{(x-2)\tan(x-2)} & , x>2 \end{cases}$$

Find the values of A & f(2) in order that f(x) may be continuous at x = 2.

Let $f(x) = x + 2, \quad -4 \le x \le 0$
$\quad = 2 - x^2, \quad 0 < x \le 4$

then find $f(f(x))$, domain of $f(f(x))$ and also comment upon the continuity of $f(f(x))$.

Let $g(x) = \lim\limits_{n\to\infty} \dfrac{x^n f(x) + h(x) + 1}{2x^n + 3x + 3}$, $x \ne 1$ and $g(1) = \lim\limits_{x\to 1} \dfrac{\sin^2(\pi \cdot 2^x)}{\ln(\sec(\pi \cdot 2^x))}$ be a continuous function at x = 1, find the value of $4g(1) + 2f(1) - h(1)$. Assume that f (x) and h (x) are continuous at x = 1.

$$f(x) = \frac{a^{\sin x} - a^{\tan x}}{\tan x - \sin x} \text{ for } x > 0$$

$$= \frac{\ln(1+x+x^2) + \ln(1-x+x^2)}{\sec x - \cos x} \text{ for } x < 0,$$ if f is continuous at x = 0, find 'a'

now if $g(x) = \ln\left(2 - \dfrac{x}{a}\right) \cdot \cot(x-a)$ for $x \ne a$, $a \ne 0$, $a > 0$. If g is continuous at x = a then show that $g(e^{-1}) = -e$.

Let f(x + y) = f(x) + f(y) for all x , y & if the function f(x) is continuous at x = 0 , then show that f(x) is continuous at all x.

If $f(x \cdot y) = f(x) \cdot f(y)$ for all x , y and f(x) is continuous at x = 1. Prove that f(x) is continuous for all x except at x = 0. Given $f(1) \ne 0$.

Given $f(x) = \sum\limits_{r=1}^{n} \tan\left(\dfrac{x}{2^r}\right) \sec\left(\dfrac{x}{2^{r-1}}\right)$; $r, n \in N$

$$g(x) = \underset{n\to\infty}{\text{Limit}} \frac{\ell n\left(f(x) + \tan\frac{x}{2^n}\right) - \left(f(x) + \tan\frac{x}{2^n}\right)^n \cdot \left[\sin\left(\tan\frac{x}{2}\right)\right]}{1 + \left(f(x) + \tan\frac{x}{2^n}\right)^n}$$

$= k$ for $x = \dfrac{\pi}{4}$ and the domain of g (x) is $(0, \pi/2)$.

where [ ] denotes the greatest integer function.

Find the value of k, if possible, so that g (x) is continuous at $x = \pi/4$. Also state the points of discontinuity of g (x) in $(0, \pi/4)$, if any.

Let $f(x) = x^3 - x^2 - 3x - 1$ and $h(x) = \frac{f(x)}{g(x)}$ where h is a function such that

(a) it is continuous every where except when $x = -1$, (b) $\lim_{x\to\infty} h(x) = \infty$ and (c) $\lim_{x\to-1} h(x) = \frac{1}{2}$.

Find $\lim_{x\to 0} (3h(x) + f(x) - 2g(x))$

Let f be continuous on the interval [0, 1] to R such that f (0) = f (1). Prove that there exists a point c in $\left[0, \frac{1}{2}\right]$ such that $f(c) = f\left(c + \frac{1}{2}\right)$

The function defined as $f(x) = \lim_{n\to\infty} \frac{\cos \pi x - x^{2n} \sin(x-1)}{1 + x^{2n+1} - x^{2n}}$

(A) is discontinuous at $x = 1$ because $f(1^+) \neq f(1^-)$
(B) is discontinuous at $x = 1$ because $f(1)$ is not defined
(C) is discontinuous at $x = 1$ because $f(1^+) = f(1^-) \neq f(1)$
(D) is continuous at $x = 1$

Let 'f' be a continuous function on R. If $f(1/4^n) = \left(\sin e^n\right) e^{-n^2} + \frac{n^2}{n^2+1}$ then f(0) is :

(A) not unique
(B) 1
(C) data sufficient to find f(0)
(D) data insufficient to find f(0)

If $f(x) = \text{sgn}(\cos 2x - 2\sin x + 3)$, where sgn ( ) is the signum function, then f(x)
(A) is continuous over its domain
(B) has a missing point discontinuity
(C) has isolated point discontinuity
(D) has irremovable discontinuity.

Let $g(x) = \tan^{-1}|x| - \cot^{-1}|x|$, $f(x) = \frac{[x]}{[x+1]}\{x\}$, $h(x) = |g(f(x))|$ where {x} denotes fractional part and [x] denotes the integral part then which of the following holds good?
(A) h is continuous at x = 0
(B) h is discontinuous at x = 0
(C) $h(0^-) = \frac{\pi}{2}$
(D) $h(0^+) = -\frac{\pi}{2}$

Consider $f(x) = \lim_{n\to\infty} \frac{x^n - \sin x^n}{x^n + \sin x^n}$ for $x > 0, x \neq 1$

$f(1) = 0$

then
(A) f is continuous at x = 1
(B) f has a finite discontinuity at x = 1
(C) f has an infinite or oscillatory discontinuity at x = 1.
(D) f has a removable type of discontinuity at x = 1.

Given $f(x)=\dfrac{\left[\{|x|\}\right] e^{x^2}\left\{\left[x+\{x\}\right]\right\}}{\left(e^{\frac{1}{x^2}}-1\right)\text{sgn}\,(\sin x)}$ for $x \neq 0$

$= 0$ for $x = 0$

where $\{x\}$ is the fractional part function; $[x]$ is the step up function and sgn(x) is the signum function of x then, f(x)

(A) is continuous at $x = 0$
(B) is discontinuous at $x = 0$
(C) has a removable discontinuity at $x = 0$
(D) has an irremovable discontinuity at $x = 0$

$$\text{Consider } f(x)=\begin{cases} x[x]^2 \log_{(1+x)} 2 & \text{for } -1<x<0 \\ \dfrac{\ln\left(e^{x^2}+2\sqrt{\{x\}}\right)}{\tan\sqrt{x}} & \text{for } 0<x<1 \end{cases}$$

where [*] & {*} are the greatest integer function & fractional part function respectively, then

(A) $f(0) = \ln 2 \Rightarrow f$ is continuous at $x = 0$
(B) $f(0) = 2 \Rightarrow f$ is continuous at $x = 0$
(C) $f(0) = e^2 \Rightarrow f$ is continuous at $x = 0$
(D) f has an irremovable discontinuity at $x = 0$

Consider $f(x)=\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\{x\}}$ $x \neq 0$

$g(x) = \cos 2x$ $\quad -\dfrac{\pi}{4}<x<0$

$$h(x)=\begin{cases} \dfrac{1}{\sqrt{2}} f(g(x)) & \text{for } x<0 \\ 1 & \text{for } x=0 \\ f(x) & \text{for } x>0 \end{cases}$$

then, which of the following holds good.
where $\{x\}$ denotes fractional part function.

(A) 'h' is continuous at $x = 0$
(B) 'h' is discontinuous at $x = 0$
(C) f(g(x)) is an even function
(D) f(x) is an even function

Let $f(0) = 0$ and $f'(0) = 1$. For a positive integer k, show that

$$\lim_{x\to 0}\frac{1}{x}\left(f(x)+f\left(\frac{x}{2}\right)+\ldots\ldots f\left(\frac{x}{k}\right)\right)=1+\frac{1}{2}+\frac{1}{3}+\ldots\ldots+\frac{1}{k}$$

$f(x) = x\left(\frac{e^{[x]+|x|}-2}{[x]+|x|}\right)$, $x \neq 0$ & $f(0) = -1$ where [x] denotes greatest integer less than or equal to x. Test the differentiability of f(x) at x = 0.

Discuss the continuity & the derivability in [0 , 2] of $f(x) = \begin{cases} |2x-3|\,[x] & \text{for } x \geq 1 \\ \sin\frac{\pi x}{2} & \text{for } x < 1 \end{cases}$

where [ ] denote greatest integer function .

If $f(x) = -1 + |x - 1|$, $-1 \leq x \leq 3$; $g(x) = 2 - |x + 1|$, $-2 \leq x \leq 2$, then calculate (fog) (x) & (gof) (x). Draw their graph. Discuss the continuity of (fog) (x) at x = −1 & the differentiability of (gof) (x) at x = 1.

Discuss the continuity on $0 \leq x \leq 1$ & differentiability at x = 0 for the function.

$f(x) = x \sin\frac{1}{x} \sin\frac{1}{x \sin\frac{1}{x}}$ where $x \neq 0$, $x \neq 1/r\pi$ & $f(0) = f(1/r\pi) = 0$,

r = 1, 2, 3,.........

Consider the function, $f(x) = \begin{cases} x^2\left|\cos\frac{\pi}{2x}\right| & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

(a) Show that $f'(0)$ exists and find its value

(b) Show that $f'\left(\frac{1}{3}\right)$ does not exist.

(c) For what values of x, $f'(x)$ fails to exist.

Let $f(x)$ be a function defined on (−a, a) with a > 0. Assume that $f(x)$ is continuous at x = 0 and $\lim_{x\to 0} \frac{f(x) - f(kx)}{x} = \alpha$, where $k \in (0, 1)$ then compute $f'(0^+)$ and $f'(0^-)$, and comment upon the differentiability of f at x = 0.

A function f : R → R satisfies the equation f(x + y) = f(x). f(y) for all x, y in R & $f(x) \neq 0$ for any x in R. Let the function be differentiable at x = 0 & f′(0) = 2. Show that f′(x) = 2f(x) for all x in R. Hence determine f(x).

Let f(x) be a real valued function not identically zero satisfies the equation, $f(x + y^n) = f(x) + (f(y))^n$ for all real x & y and $f'(0) \geq 0$ where n (> 1) is an odd natural number. Find f(10).

A function f : R → R where R is a set of real numbers satisfies the equation

$f\left(\frac{x+y}{3}\right) = \frac{f(x)+f(y)+f(0)}{3}$ for all x ,y in R. If the function is differentiable at x = 0 then show that it is differentiable for all x in R.

Given $f(x) = \begin{cases} \log_a\left(a\left|[x]+[-x]\right|\right)^x \left(\dfrac{a^{\left(\frac{2}{\left(\frac{[x]+[-x]}{|x|}\right)} - 5\right)}}{3 + a^{\frac{1}{|x|}}} \right) & \text{for } |x| \neq 0 \; ; \; a > 1 \\ 0 & \text{for } x = 0 \end{cases}$ where [ ] represents the integral part function, then :

(A) f is continuous but not differentiable at $x = 0$

(B) f is cont. & diff. at $x = 0$

(C) the differentiability of 'f' at $x = 0$ depends on the value of a

(D) f is cont. & diff. at $x = 0$ and for $a = e$ only.

Consider $f(x) = \left[\dfrac{2(\sin x - \sin^3 x) + |\sin x - \sin^3 x|}{2(\sin x - \sin^3 x) - |\sin x - \sin^3 x|} \right]$, $x \neq \dfrac{\pi}{2}$ for $x \in (0, \pi)$

$f(\pi/2) = 3$ where [ ] denotes the greatest integer function then,

(A) f is continuous & differentiable at $x = \pi/2$

(B) f is continuous but not differentiable at $x = \pi/2$

(C) f is neither continuous nor differentiable at $x = \pi/2$

(D) none of these

Given the function $f(x) = 2x\sqrt{x^3 - 1} + 5\sqrt{x}\ \sqrt{1 - x^4} + 7x^2\sqrt{x - 1} + 3x + 2$ then :

(A) the function is continuous but not differentiable at $x = 1$

(B) the function is discontinuous at $x = 1$

(C) the function is both cont. & differentiable at $x = 1$

(D) the range of $f(x)$ is R^+.

If $f(x + y) = f(x) + f(y) + |x|y + xy^2$, $\forall$ x, y $\in$ R and $f'(0) = 0$, then

(A) f need not be differentiable at every non zero x

(B) f is differentiable for all $x \in R$

(C) f is twice differentiable at $x = 0$

(D) none

Let f be a differentiable function on the open interval (a, b). Which of the following statements must be true?

I. f is continuous on the closed interval [a, b]

II. f is bounded on the open interval (a, b)

III. If $a < a_1 < b_1 < b$, and $f(a_1) < 0 < f(b_1)$, then there is a number c such that $a_1 < c < b_1$ and $f(c) = 0$

(A) I and II only (B) I and III only (C) II and III only (D) only III

If $f(x) = \begin{cases} a + \dfrac{\sin[x]}{x} & , \quad x > 0 \\ 2 \quad , & \quad x = 0 \\ b + \left[\dfrac{\sin x - x}{x^3}\right] & , \quad x < 0 \end{cases}$ (where [.] denotes the greatest integer function). If f(x) is continuous at x = 0 , then b is equal to :

(A) a – 1 (B*) a + 1 (C) a + 2 (D) a – 2

Let f(x) be a function defined on [0, 1] such that

$$f(x) = \begin{cases} x & ; x \in Q \\ 1 - x & ; x \notin Q \end{cases}$$, then fof(x) is

(A*) continuous for all $x \in [0, 1]$
(B) continuous for only one value of x
(C) not possible to define
(D) none of these

The jump of discontinuity for the function, (where [x] denotes greastest integer function and {x} denotes fractional part of x.

$$f(x) = \begin{cases} \dfrac{e^{[x]} - e^{\{x\}}}{e^x} & , \quad x < 0 \\ \dfrac{\sin\{x\}}{\{\tan x\}} & , \quad x > 0 \\ 2 & , \quad x = 0 \end{cases}$$ is:

(A) $e^{-1} - e - 1$ (B) $e^{-1} - e - 2$ (C) 1 (D*) $1 + e - e^{-1}$

Number of points of non-differentiability of the function
$g(x) = [x^2]\{\cos^2 4x\} + \{x^2\}[\cos^2 4x] + x^2 \sin^2 4x + [x^2][\cos^2 4x] + \{x^2\}\{\cos^2 4x\}$ in (– 50, 50), (where [.] denotes the greatest integer function and {.} denotes the fractional part of function), is equal to
(A) 98
(B) 99
(C) 100
(D) 0

Let f(x) = (x + |x|) |x|, then for all x
(A) f is continuous
(B) f′ is differentiable $\forall x \in R$
(C) f′ is continuous
(D) f″ is continuous

Let $f(x)=\begin{cases}[x] & ; \ x\in[-2,0) \\ |x| & ; \ x\in[0,2]\end{cases}$; where [.] represent G.I.F. and g(x) = sec x, $x \in R-(2n+1)\frac{\pi}{2}$.

Match the following statements in column–I with their values in column–II in interval $\left(-\frac{3\pi}{2}, \frac{3\pi}{2}\right)$.

	Column – I		Column – II
(A)	Limit of fog exist at	(p)	-1
(B)	Limit of gof does not exist at	(q)	π
(C)	Points of discontinuity of fog is/are	(r)	$\frac{5\pi}{6}$
(D)	Points of differentiability of fog is/are	(s)	$-\pi$
		(t)	$\frac{\pi}{3}$

Let $f(x)=\begin{cases}x+1 & , \ 0\le x\le 1 \\ 2x^2-6x+6 & , \ 1\le x\le 2\end{cases}$

And $g(t)=\int_{t-1}^{t} f(x)dx$, $t \in [1, 2]$. Which of the following is true statement

(A) f(x) is continuous and differentiable $\forall\ x \in [0, 2]$

(B) g(t) is maximum at t = $\frac{3}{2}$

(C) g(t) is minimum at t = 2

(D) all the above

f(x) is defined for x > –1 and has a continuous derivate. If f satisfies f(0) = 1, f′(0) = 0; (1 + f(x)) f″(x) = 1 + x. If x is positive then f′(x) is

(A) always positive
(B) always negative
(C) always non-negative
(D) none of these

Let $f(x)=x^2-2|x|$ and $g(x)=\begin{cases}\text{Min}\{f(t) : -2\le t\le x, & -2\le x<0\} \\ \text{Max}\{f(t) : 0\le t\le x, & 0\le x\le 3\}\end{cases}$.

(a) Draw the graph of $g(x)$.

(b) At how many points is $g(x)$ discontinuous in its domain?

(A) 0 (B) 1 (C) 2 (D) 3 (E) None of these

(c) At how many points is $g(x)$ non-differentiable in its domain?

(A) 0 (B) 1 (C) 2 (D) 3 (E) None of these

Consider the function $f(x)=[x]+\sqrt{\{x\}}$. This function is

(A) continuous at all integer points.
(B) discontinuous at all integer points.
(C) differentiable at all integer points.
(D) non-differentiable at all integer points.

Let $f(\frac{x+y}{3}) = \frac{f(x)+f(y)+2}{3}$ $\forall x, y \in \mathbb{R}$. Also, it is given that $f'(0) = 2$. Which of the following statements is / are true?

(A) $f'(x) = f'(0)$ $\forall x \in \mathbb{R}$ (C) $f(x)$ is a linear function

(B) $f'(x) = f(x)$ $\forall x \in \mathbb{R}$ (D) none of these

Let $\alpha \in \mathbb{R}$. Prove that a function $f : \mathbb{R} \to \mathbb{R}$ is differentiable at α if and only if there exists a function $g : \mathbb{R} \to \mathbb{R}$ which is continuous at α and satisfies $f(x) - f(\alpha) = g(x)(x - \alpha)$ for all $x \in \mathbb{R}$.

Let [x] denote the greatest integer function & f(x) be defined in a neighbourhood of 2 by

$$f(x) = \begin{cases} \dfrac{\left(\exp\{(x+2)\ell n 4\}\right)^{\frac{[x+1]}{4}} - 16}{4^x - 16} & , x < 2 \\ A\dfrac{1-\cos(x-2)}{(x-2)\tan(x-2)} & , x > 2 \end{cases}$$

Find the values of A & f(2) in order that f(x) may be continuous at x = 2.

$$\text{Given } f(x) = \frac{[\{|x|\}]\, e^{x^2}\, \{[x + \{x\}]\}}{\left(e^{\frac{1}{x^2}} - 1\right) \operatorname{sgn}(\sin x)} \quad \text{for } x \neq 0$$

$$= 0 \quad \text{for } x = 0$$

where {x} is the fractional part function; [x] is the step up function and sgn(x) is the signum function of x then, f(x)

(A) is continuous at x = 0 (B) is discontinuous at x = 0

(C) has a removable discontinuity at x = 0 (D) has an irremovable discontinuity at x = 0

$$\text{Consider } f(x) = \begin{cases} x[x]^2 \log_{(1+x)} 2 & \text{for } -1 < x < 0 \\ \dfrac{\ell n\left(e^{x^2} + 2\sqrt{\{x\}}\right)}{\tan\sqrt{x}} & \text{for } 0 < x < 1 \end{cases}$$

where [*] & {*} are the greatest integer function & fractional part function respectively, then

(A) $f(0) = \ell n 2 \Rightarrow$ f is continuous at x = 0 (B) $f(0) = 2 \Rightarrow$ f is continuous at x = 0

(C) $f(0) = e^2 \Rightarrow$ f is continuous at x = 0 (D) f has an irremovable discontinuity at x = 0

Let f be an injective function with domain $[a, b]$ and range $[c, d]$. If α is a point in (a, b) such that f has left hand derivative l and right hand derivative r at $x = \alpha$ with both l and r non-zero different and negative, then left hand derivative and right hand derivative of f^{-1} at $x = f(\alpha)$ respectively, is :

(a) $\frac{1}{r}, \frac{1}{l}$ **(b)** r, l **(c)** $\frac{1}{l}, \frac{1}{r}$ **(d)** l, r

Given $f(x) = \begin{cases} \log_a (a|[x]+[-x]|)^x \left(\dfrac{a^{\left(\frac{2}{\left(\frac{[x]+[-x]}{|x|}\right)} - 5\right)}}{3 + a^{\frac{1}{|x|}}} \right) & \text{for } |x| \neq 0;\ a > 1 \\ 0 & \text{for } x = 0 \end{cases}$

where [ ] represents the integral part function, then :

(a) f is continuous but not differentiable at $x = 0$

(b) f is continuous and differentiable at $x = 0$

(c) the differentiability of 'f' at $x = 0$ depends on the value of a

(d) f is continuous and differentiable at $x = 0$ and for $a = e$ only

If $f(x) = (p^2 - 1)[\tan^{-1} x] + 4(q^2 + 2q - 3)\left\{\dfrac{1}{2+x^2}\right\} + (p+q)\operatorname{sgn}(x^2 - x + 2)$ is continuous in R and $f(x_1) = f(x_2)\ \forall\ x_1, x_2 \in R$ then largest value of $|p+q|$ is :

(a) 0 (b) 2 (c) 4 (d) 5

[**Note :** sgn (y), [y] and {y} denote signum function, greatest integer function and fractional part function respectively.]

If $f(x) = \begin{cases} (p^2 - 1)(\{x\} + 2[x]) - 2, & -2 < x \le -1 \\ q\left(\dfrac{e^x + e^{-x}}{2}\right) + |p|(x-1), & -1 < x < 2 \end{cases}$, $p, q \in R$ is continuous in $(-2, 2)$

then $f\left(f\left(f\left(\dfrac{-1}{2}\right)\right)\right)$ is :

(a) -2 (b) -1 (c) 0 (d) not defined

[**Note :** [k] denotes greatest integer function less than or equal to k and {k} denotes fractional part function of k.]

Which of the following statements is(are) correct?

[Note : $[x]$ and $\{x\}$ denote greatest integer less than or equal to x and fractional part of x respectively.]

(a) $f(x) = [\ln x] + \sqrt{\{\ln x\}}, x > 1$ is continuous at $x = e$.

(b) If $\lim_{x \to -2}\left(\frac{3x^2 + ax + a + 1}{x^2 + x - 2}\right)$ exists and equals l, then $a + \frac{1}{l} = 10$.

(c) $f : [-1, 1] \to [-1, 1], f(x) = x^2 \operatorname{sgn}(x)$ is a bijective function, where sgn x denotes signum function of x.

(d) If f is continuous on $[-1, 1], f(-1) = 4$ and $f(1) = 3$, then there exists a number r such that $|r| < 1$ and $f(r) = \pi$.

Column I	Column II
(a) $f_1(x) = \left[\frac{4x}{\pi}\right] \operatorname{sgn}(x^2 - x + 1)$	(p) discontinuous at more than 3 points but less than 6 points in $[-2, 2]$
(b) $f_2(x) = \cos^{-1}\left(\operatorname{sgn}\left(\cos \frac{2x - 1}{2} \pi\right)\right)$	(q) non derivable at more than 2 points but atmost 5 points in $[-2, 2]$
(c) $f_3(x) = \max. (\{x + 1\}, \{5 - x\})$	(r) range contains atleast one integer but not more than seven and no irrational value in $[-2, 2]$
(d) $f_4(x) = \sqrt{x^2} + [x]^2$	(s) many one but not even function in $[-2, 2]$
	(t) neither odd nor periodic in $[-2, 2]$

Let f be a differentiable function satisfying the functional rule $f(xy) = f(x) + f(y) + \frac{x + y - 1}{xy}$ $\forall x, y > 0$ and $f'(1) = 2$. Find the value of $[f(e^{100})]$.

Note : $[k]$ denotes the greatest integer less than or equal to k.

Let $f(x) = \begin{cases} \sqrt{(2n+1)x - x^2 - (n^2 + n)} & n \le x < n + \frac{1}{2} \\ n + 1 - x & n + \frac{1}{2} \le x < n + 1 \end{cases}$ $(n \in I)$. Find the number of values of x where $f(x)$ is non-derivable in $(-5, 5)$.

If for a differentiable function $y = f(x), (f'(x) > 0)$ $\lim_{x\to 1} \frac{e^{2f(x)} - 2e^{f(x)+1} + e^2 \cos(x-1)}{\sec^2(x-1) - 1} = \frac{7}{2}e^2$ and area of the triangle formed by the tangent drawn to the curve $y = f(x)$ at $[1, f(1)]$ and co-ordinate axes is Δ, then find the value of $\frac{1}{\Delta}$.

If $f(x) = 23 - 2^{-x^2+2x+3} \forall x \in R$ and $g(x) = \left[\frac{f(x)}{\mu}\right]$, where $[k]$ denotes greatest integer function less than or equal to k and $\mu \in N$, then find the sum of all values of μ for which $g(x)$ is discontinuous for at least one real value of x.

Let $f(x) = (x^2 - 3x + 2)|(x^3 - 6x^2 + 11x - 6)| + \left|\sin\left(x + \frac{\pi}{4}\right)\right|$.

Number of points at which the function $f(x)$ is non-differentiable in $[0, 2\pi]$, is :

(a) 5 (b) 4 (c) 3 (d) 2

Let f be a differentiable function satisfying $f'(x) = f'(-x) \ \forall \ x \in R$. Then

(a) If $f(1) = f(2)$, then $f(-1) = f(-2)$

(b) $\frac{1}{2}f(x) + \frac{1}{2}f(y) = f\left(\frac{1}{2}(x+y)\right)$ for all real values of x, y

(c) Let $f(x)$ be an even function, then $f(x) = 0 \ \forall \ x \in R$

(d) $f(x) + f(-x) = 2f(0) \ \forall \ x \in R$

If $g(x)$ is a polynomial satisfying $g(x)\,g(y) = g(x) + g(y) + g(xy) - 2$ for all real x and y and $g(2) = 5$ then $\lim_{x\to 3} g(x)$ is

(a) 9 (b) 25

(c) 10 (d) none of these

Consider the function

$$f(x) = x\left[\frac{1}{x(1+x)} + \frac{1}{(1+x)(1+2x)} + \frac{1}{(1+2x)(1+3x)} + \ldots\infty\right] \text{ for } x \neq 0.$$

Find $f(0)$ if $f(x)$ is continuous at $x = 0$.

Given $f(x) = \sum_{r=1}^{n}(x^r + x^{-r})^2$, $x \neq \pm 1$ and

$$g(x) = \begin{cases} \lim_{n\to\infty}\left(f(x) - 2n\right)x^{-2n-2}(1-x^2)) & \text{for } x \neq \pm 1 \\ -1, & \text{for } x = \pm 1 \end{cases}$$

show that $g(x)$ is continuous for all x in the domain.

Let $f : R \to R$ be a function which satisfies $f(x + y^3) = f(x) \cdot (f(y))^3$ $\forall$ $x, y \in R$. If f is continuous at $x = 0$, prove that f is continuous everywhere.

If $f(x + y) = f(x) \cdot f(y)$ for all $x, y \in R$ and $f(x) = 1 + g(x) \cdot G(x)$ where $\lim_{x\to 0} g(x) = 0$ and $\lim_{x\to 0} G(x)$ exists, prove that $f(x)$ is continuous at all $x \in R$.

Let $f(x) = \text{cosec } 2x + \text{cosec } 2^2 x + \text{cosec } 2^3 x + \ldots\ldots\ldots \text{cosec } 2^n x,\ x \in (0, \pi/2)$

and $g(x) = f(x) + \cot 2^n x$

If

$$H(x) = \begin{cases} (\cos x)^{g(x)} + (\sec x)^{\text{cosec}\, x} & \text{if } x > 0 \\ p & \text{if } x = 0 \\ \dfrac{e^x + e^{-x} - 2\cos x}{x \sin x} & \text{if } x < 0 \end{cases}$$

find the value of p, if possible to make the function H (x) continuous at x = 0.

Prove that

$f(x) = [\tan x] + \sqrt{\tan x - [\tan x]}$ (where [.] denotes greatest integer function) is continuous in $\left[0, \frac{\pi}{2}\right)$.

Check differentiability of f(x) in [–2, 2], if

$$f(x) = \begin{cases} \cos\dfrac{\pi}{2}(|x| - \{x\}) & x < 1 \\ \sqrt{4x^2 - 12x + 9}\{x\} & x \geq 1 \end{cases}$$

where {.} denotes the fractional part function.

A function $f : (0, \infty) \to R$ satisfies the equation $f(x/y) = f(x) - f(y)$. If $f(x)$ is differentiable on $(0, \infty)$ and $\lim_{x \to 0} \frac{f(1+x)}{x} = 3$, then determine $f(x)$.

A differentiable function satisfies the relation $f(x+y) = f(x) + f(y) + 2xy - 1 \ \forall\, x, y \in R$. If $f'(0) = \sqrt{3 + a - a^2}$, find $f(x)$ and prove that $f(x) > 0$ $\forall\, x \in R$.

If $f\left(\frac{x+y}{3}\right) = \frac{2 + f(x) + f(y)}{3}$ for all real x and y and $f'(2) = 2$ then determine $y = f(x)$.

Let $f(xy) = xf(y) + yf(x)$ for all $x, y \in R^{+}$ and $f(x)$ be differentiable in $(0, \infty)$ then determine $f(x)$.

If $e^{-xy} f(xy) = e^{-x} f(x) + e^{-y} f(y)\ \forall\, x, y \in R^{+}$, and $f'(1) = e$, determine $f(x)$.

A function $f:(-1,1) \rightarrow \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

satisfies the equation

$$f(x)+f(y)=f\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right).$$

(i) Show that f(x) is odd.

(ii) If f(x) is differentiable on (–1, 1) and $f'(0)=1$, then show that $f'(x)=\frac{1}{\sqrt{1-x^2}}$

(iii) Hence, determine f(x).

A twice differentiable function f satisfies the relation $f(x^2+y^2)=f(x^2-y^2)+f(2xy)\ \forall\ x, y \in R$. If $f(0)=0$ and $f''(0)=2$, find f(x).

f(x) is a differentiable function satisfy the relationship $f^2(x)+f^2(y)+2(xy-1)=f^2(x+y)$ $\forall\ x, y \in R$. Also $f(x)>0\ \forall\ x \in R$, and $f(\sqrt{2})=2$. Determine f(x).

function $f:(0,\infty) \rightarrow R$ satisfies the equation $f(xy)=2f(x)-f\left(\frac{x}{y}\right)$. If f is differentiable on R^+ and $f(1)=0$, $f'(1)=1$, then show that

(i) $f(y)=-f\left(\frac{1}{y}\right)$ (ii) $f(x)+f(y)=f\left(\frac{x}{y}\right)$ and hence determine f(x).

A differentiable function f satisfies the relation $f(x+y)+f(xy-1)=f(x)+f(y)+f(xy)\ \forall\ x, y \in R$.

If $f(1)=2$, $f'(0)=1$ and $f'(-1)=-1$, find f(x).

Let $f : R \to R$ be such that for all x and y in R, $|f(x) - f(y)| \le |x - y|^3$. Prove that f(x) is a constant function.

Suppose

$$p(x) = a_0 + a_1x + a_2x^2 + ... + a_nx^n.$$

If $|p(x)| \le |e^{x-1} - 1|$ for all $x \ge 0$, prove that $|a_1 + 2a_2 + ... + na_n| \le 1$.

Find a function continuous and derivable for all x and satisfying the functional relation, $f(x+y) . f(x-y) = f^2(x)$, where x and y are independent variables and $f(0) \neq 0$.

Let f be a function such that

$$f(x + f(y)) = f(f(x)) + f(y) \qquad \forall\ x, y \in R$$

and $f(h) = h$ for $0 < h < \varepsilon$ where ε is a small positive quantity. Determine f'(x) and f(x).

Let f(x) be a thrice differentiable function satisfying $f(x+y) = f(x-y) + y[f'(x+y) + f'(x-y)]$ where $f(0) = 0, f'(0) = 1$, and $f(1) = 2$. Find f(x).

Let f(x) be a positive differentiable function satisfying

$$f\left(\frac{x+y}{2}\right) = \sqrt{f(x).f(y)}\ \forall\ x, y \in R, \text{ where } f'(0) = 2$$

Find f(x).

If $f : R \to R$ is a function defined by $f(x) = [x]\cos\left(\frac{2x-1}{2}\right)\pi$, where $[x]$ denotes the greatest integer function, then f is

1) continuous for every real x.
2) discontinuous only at $x = 0$.
3) discontinuous only at non-zero integral values of x.
4) continuous only at $x = 0$.

If [.] denotes the greatest integer function then the number of points where $f(x)=[x]+\left[x+\frac{1}{3}\right]+\left[x+\frac{2}{3}\right]$ is discontinuous for $x\in(0,3)$ are

1) 2 2) 9 3) 8 4) 10

Let f and g be real valued functions defined on interval $(-1,1)$ such that $g''(x)$ is continuous, $g(0)\neq 0, g'(0)=0, g''(0)\neq 0$, and $f(x)=g(x)\sin x$.

Statement 1 : $\lim_{x\to 0}[g(x)\cot x - g(0)\operatorname{cosec} x] = f''(0)$

Statement 2 : $f'(0) = g(0)$

a) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for Statement 1.

b) Statement 1 is true, Statement 2 is true; Statement 2 is NOT a correct explanation for Statement 1

c) Statement 1 is true, statement 2 is false.

d) Statement 1 is false, Statement 2 is true.

$f(x)=\max\{x/n, |\sin \pi x|\}, n\in N$, has maximum points of non- differentiability for $x\in(0,4)$. Then n cannot be

1) 4 2) 2 3) 5 4) 6

If $f(x)=(x^2-4)|x^3-6x^2+11x-6|+\frac{x}{1+|x|}$, then the set of points at which the function $f(x)$ is not differentiable is

a) {–2,2,1,3} b) {–2,0,3} c) {–2,2,0} d) {1,3}

If the function $f(x)=\left[\frac{(x-2)^3}{a}\right]\sin(x-2)+a\cos(x-2)$ (where $[x]$ denotes the greatest integer function) is continuous and differentiable in (4, 6), then 'a' can be

a) 61 b) 63 c) 65 d) 66

If a is the number of points of continuity of $f(x)=\begin{cases} x-1, & x \text{ is rational} \\ x^2-x-2, & x \text{ is irrational} \end{cases}$, b is the number of points of discontinuity of $f(x)=\operatorname{sgn}(x^3-3x+1)$, c is the number of points of non–differentiability of $f(x)=(\log x)|x^2-4x+3|+2(x-2)^{1/3}$, and d is the number of points where the graph of $f(x)=(\log x)|x^2-4x+3|+2(x-2)^{1/3}$ has a sharp turn then the value of $a+b+c+d$ is ______.

The number of points of discontinuity of the function f(x) = $\left[\frac{6x}{\pi}\right]\cos\left[\frac{3x}{\pi}\right]$, in the interval $\left(\frac{\pi}{10},\frac{11\pi}{10}\right)$ is ([.] represents the greatest integer function)

Let $f : [a, b] \to [1, \infty)$ be a continuous function and let $g : R \to R$ be defined as

$$g(x) = \begin{cases} 0 & \text{if} \quad x < a, \\ \int_a^x f(t)\,dt & \text{if} \quad a \le x \le b, \\ \int_a^b f(t)\,dt & \text{if} \quad x > b \end{cases}$$

Then
(A) g(x) is continuous but not differentiable at a
(B) g(x) is differentiable on R
(C) g(x) is continuous but not differentiable at b
(D) g(x) is continuous and differentiable at either a or b but not both

Let $f : R \to R$ and $g : R \to R$ be respectively given by $f(x) = |x| + 1$ and $g(x) = x^2 + 1$. Define $h : R \to R$ by

$$h(x) = \begin{cases} \max \ \{f(x), g(x)\} & \text{if} \quad x \le 0 \\ \min \ \{f(x), g(x)\} & \text{if} \quad x > 0 \end{cases}.$$

Then number of points at which h(x) is not differentiable is __________

Let $f_1 : R \to R$, $f_2 : [0, \infty) \to R$, $f_3 : R \to R$ and $f_4 : R \to [0, \infty)$ be defined by

$$f_1(x) = \begin{cases} |x| & \text{if} \quad x < 0 \\ e^x & \text{if} \quad x \ge 0 \end{cases}; \ f_2(x) = x^2; \ f_3(x) = \begin{cases} \sin x & \text{if} \quad x < 0 \\ x & \text{if} \quad x \ge 0 \end{cases} \text{ and } f_4(x) = \begin{cases} f_2(f_1(x)) & \text{if} \quad x < 0 \\ f_2(f_1(x)) - 1 & \text{if} \quad x \ge 0 \end{cases}$$

List – I		List – II	
(P)	f_4 is	(1)	onto but not one-one
(Q)	f_3 is	(2)	neither continuous nor one-one
(R)	f_2 o f_1 is	(3)	differentiable but not one-one
(S)	f_2 is	(4)	continuous and one-one

Codes:

	P	Q	R	S
(A)	3	1	4	2
(B)	1	3	4	2
(C)	3	1	2	4
(D)	1	3	2	4

Let $g : \mathbb{R} \to \mathbb{R}$ be a differential function with $g(0) = 0$, $g'(0) = 0$ and $g'(1) \ne 0$.

Let $f(x) = \begin{cases} \frac{x}{|x|} g(x), & x \ne 0 \\ 0, & x = 0 \end{cases}$

and $h(x) = e^{|x|}$ for all $x \in \mathbb{R}$. Let (f o h)(x) denote $f(h(x))$ and (h o f)(x) denote $h(f(x))$. Then which of the following is (are) true?

(A) f is differentiable at $x = 0$
(B) h is differentiable at $x = 0$
(C) f o h is differentiable at $x = 0$
(D) h o f is differentiable at $x = 0$

Let $a, b \in \mathbb{R}$ and $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = a\cos(|x^3 - x|) + b|x|\sin(|x^3 + x|)$. Then f is

(A) differentiable at $x = 0$ if $a = 0$ and $b = 1$

(B) differentiable at $x = 1$ if $a = 1$ and $b = 0$

(C) **NOT** differentiable at $x = 0$ if $a = 1$ and $b = 0$

(D) **NOT** differentiable at $x = 1$ if $a = 1$ and $b = 1$

(A, B)

Let $f : \left[-\frac{1}{2}, 2\right] \to \mathbb{R}$ and $g : \left[-\frac{1}{2}, 2\right] \to \mathbb{R}$ be functions defined by $f(x) = [x^2 - 3]$ and $g(x) = |x|\,f(x) + |4x - 7|\,f(x)$, where $[y]$ denotes the greatest integer less than or equal to y for $y \in \mathbb{R}$. Then

(A) f is discontinuous exactly at three points in $\left[-\frac{1}{2}, 2\right]$

(B) f is discontinuous exactly at four points in $\left[-\frac{1}{2}, 2\right]$

(C) g is **NOT** differentiable exactly at four points in $\left(-\frac{1}{2}, 2\right)$

(D) g is **NOT** differentiable exactly at five points in $\left(-\frac{1}{2}, 2\right)$

(B, C)

Let $[x]$ be the greatest integer less than or equals to x. Then, at which of the following point(s) the function $f(x) = x\cos(\pi(x + [x]))$ is discontinuous ?

[A] $x = -1$ [B] $x = 0$

[C] $x = 2$ [D] $x = 1$

A, C, D

Let $f(x) = x + \log_e x - x\log_e x$, $x \in (0, \infty)$.

- Column 1 contains information about zeros of $f(x)$, $f'(x)$ and $f''(x)$.
- Column 2 contains information about the limiting behavior of $f(x)$, $f'(x)$ and $f''(x)$ at infinity.
- Column 3 contains information about increasing/decreasing nature of $f(x)$ and $f'(x)$.

Column 1	Column 2	Column 3
(I) $f(x) = 0$ for some $x \in (1, e^2)$	(i) $\lim_{x\to\infty} f(x) = 0$	(P) f is increasing in (0, 1)
(II) $f'(x) = 0$ for some $x \in (1, e)$	(ii) $\lim_{x\to\infty} f(x) = -\infty$	(Q) f is decreasing in (e, e^2)
(III) $f'(x) = 0$ for some $x \in (0, 1)$	(iii) $\lim_{x\to\infty} f'(x) = -\infty$	(R) f' is increasing in (0, 1)
(IV) $f''(x) = 0$ for some $x \in (1, e)$	(iv) $\lim_{x\to\infty} f''(x) = 0$	(S) f' is decreasing in (e, e^2)

Q.52 Which of the following options is the only CORRECT combination ?

[A] (IV) (i) (S) [B] (I) (ii) (R)

[C] (III) (iv) (P) [D] (II) (iii) (S)

Sol. **D**

Q.53 Which of the following options is the only CORRECT combination ?

[A] (III) (iii) (R) [B] (I) (i) (P)

[C] (IV) (iv) (S) [D] (II) (ii) (Q)

Sol. **D**

Q.54 Which of the following options is the only **INCORRECT** combination ?

[A] (II) (iii) (P) [B] (II) (iv) (Q)

[C] (I) (iii) (P) [D] (III) (i) (R)

Sol. **D**

For every twice differentiable function $f : R \to [-2, 2]$ with $(f(0))^2 + (f'(0))^2 = 85$, which of the following statement(s) is (are) TRUE ?

(A) There exist r, s ∈ R, where r < s, such that f is one-one on the open interval (r, s)

(B) There exists $x_0 \in (-4, 0)$ such that $|f'(x_0)| \le 1$

(C) $\lim_{x\to\infty} f(x) = 1$

(D) There exist $\alpha \in (-4, 4)$ such that $f(\alpha) + f''(\alpha) = 0$ and $f'(\alpha) \ne 0$

A, B, D

Let $f : R \to R$ be a differentiable function with $f(0) = 1$ and satisfying the equation

$$f(x + y) = f(x)f'(y) + f'(x)f(y) \text{ for all } x, y \in R.$$

Then, the value of $\log_e(f(4))$ is ______ .

2

Let $f_1 : R \to R, f_2 : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to R$, $f_3 : (-1, e^{\pi/2} - 2) \to R$ and $f_4 : R \to R$ be functions defined by

(i) $f_1(x) = \sin\left(\sqrt{1-e^{-x^2}}\right)$,

(ii) $f_2(x) = \begin{cases} \dfrac{|\sin x|}{\tan^{-1} x} & if \quad x \neq 0 \\ 1 & if \quad x = 0 \end{cases}$, where the inverse trigonometric function $\tan^{-1}x$ assumes values in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$,

(iii) $f_3(x) = [\sin(\log_e(x+2))]$, where, for $t \in R$, $[t]$ denotes the greatest integer less than or equal to t,

(iv) $f_4(x) = \begin{cases} x^2 \sin\left(\dfrac{1}{x}\right) & if \quad x \neq 0 \\ 0 & if \quad x = 0 \end{cases}$.

LIST–I		LIST–II	
P.	The function f_1 is	1.	**NOT** continuous at $x = 0$
Q.	The function f_2 is	2.	continuous at $x = 0$ and **NOT** differentiable at $x = 0$
R.	The function f_3 is	3.	differentiable at $x = 0$ and its derivative is **NOT** continuous at $x = 0$
S.	The function f_4 is	4.	differentiable at $x = 0$ and its derivative is continuous at $x = 0$

The correct option is :

(A) **P → 2; Q → 3; R → 1; S → 4**
(B) **P → 4; Q → 1; R → 2; S → 3**
(C) **P → 4; Q → 2; R → 1; S → 3**
(D) **P → 2; Q → 1; R → 4; S → 3**

D

Let $f : R \to R$ by given by $f(x) = \begin{cases} x^5 + 5x^4 + 10x^3 + 10x^2 + 3x + 1, & x < 0; \\ x^2 - x + 1, & 0 \le x < 1; \\ \frac{2}{3}x^3 - 4x^2 + 7x - \frac{8}{3}, & 1 \le x < 3; \\ (x-2)\log_e(x-2) - x + \frac{10}{3}, & x \ge 3. \end{cases}$

Then which of the following options is/are correct?

A. f' has a local maximum at x = 1

B. f' is NOT differentiable at x = 1

C. f is onto

D. f is increasing on (–∞ , 0)

A, B, C

Let the function $f : R \to R$ be defined by $f(x) = x^3 - x^2 + (x - 1) \sin x$ and let $g : R \to R$ be an arbitrary function. Let $fg: R \to R$ be the product function defined by $(fg)(x) = f(x)g(x)$. Then which of the following statements is/are TRUE?

(A) If g is continuous at $x = 1$, then fg is differentiable at $x = 1$

(B) If fg is differentiable at $x = 1$, then g is continuous at $x = 1$

(C) If g is differentiable at $x = 1$, then fg is differentiable at $x = 1$

(D) If fg is differentiable at $x = 1$, then g is differentiable at $x = 1$

A, C

Let $f : \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$ be functions satisfying

$$f(x + y) = f(x) + f(y) + f(x)f(y) \text{ and } f(x) = xg(x)$$

for all $x, y \in \mathbb{R}$. If $\lim_{x\to 0} g(x) = 1$, then which of the following statements is/are TRUE?

(A) f is differentiable at every $x \in \mathbb{R}$

(B) If $g(0) = 1$, then g is differentiable at every $x \in \mathbb{R}$

(C) The derivative $f'(1)$ is equal to 1

(D) The derivative $f'(0)$ is equal to 1

A, B, D

Let the functions $f:(-1, 1) \to \mathbb{R}$ and $g: (-1, 1) \to (-1, 1)$ be defined by

$$f(x) = |2x - 1| + |2x + 1| \text{ and } g(x) = x - [x],$$

where [x] denotes the greatest integer less than or equal to x. Let $fog :(-1, 1) \to \mathbb{R}$ be the composite function defined by $(fog)(x) = f(g(x))$. Suppose c is the number of points in the interval $(-1,1)$ at which fog is **NOT** continuous, and suppose d is the number of points in the interval $(-1,1)$ at which fog is **NOT** differentiable. Then the value of $c + d$ is _____

4

Let the function *f*, *g* and *h* be defined as follows :

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & \text{for } -1 \le x \le 1 \text{ and } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$$

$$g(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{for } -1 \le x \le 1 \text{ and } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$$

$$h(x) = |x|^3 \quad \text{for } -1 \le x \le 1$$

Which of these functions are differentiable at $x = 0$?

(A) *f* and *g* only (B) *f* and *h* only (C) *g* and *h* only (D) none

Let $f(x) = \lim_{n\to\infty} \ln\left(\sqrt{e^{\cos x}\sqrt{e^{3\cos x}\sqrt{e^{5\cos x}\,.....\,\sqrt{e^{(2n+1)\cos x}}}}}\right)$. If $g(x) = \left[\frac{1}{3}f(x)\right]$, then find the number of points in $[0, 2\pi]$ where $g(x)$ is discontinuous.

[Note: $[y]$ denotes greatest integer function less than or equal to y.]

Let $f(x) = \text{sgn}\,(x^2 - 4x + 4 + k^2)$, $x \in R$. If $f(x)$ is discontinuous at exactly one point then the value of $(\tan^{-1} k + \cos^{-1} k + \text{cosec}^{-1}(2k-1))$ is equal to $\frac{m\pi}{2}$ where m is a whole number. Find the value of m.

The function $f(x) = [x]^2 - [x^2]$ (where, $[x]$ is the greatest integer less than or equal to x), is discontinuous at

(a) all integers

(b) all integers except 0 and 1

(c) all integers except 0

(d) all integers except 1

The function $f(x) = [x]\cos\left(\frac{2x-1}{2}\right)\pi$, $[\cdot]$ denotes the greatest integer function, is discontinuous at

(a) all x **(1993, 1M)**

(b) all integer points

(c) no x

(d) x which is not an integer

If $f(x) = x(\sqrt{x} + \sqrt{(x+1)})$, then

(a) $f(x)$ is continuous but not differentiable at $x = 0$

(b) $f(x)$ is differentiable at $x = 0$

(c) $f(x)$ is not differentiable at $x = 0$

(d) None of the above

For every integer n, let a_n and b_n be real numbers. Let function $f : R \to R$ be given by

$$f(x) = \begin{cases} a_n + \sin \pi x, & \text{for } x \in [2n, 2n+1] \\ b_n + \cos \pi x, & \text{for } x \in (2n-1, 2n) \end{cases},$$

for all integers n.

If f is continuous, then which of the following hold(s) for all n ?

(a) $a_{n-1} - b_{n-1} = 0$ (b) $a_n - b_n = 1$

(c) $a_n - b_{n+1} = 1$ (d) $a_{n-1} - b_n = -1$

Let $f_1 : R \to R$, $f_2 : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to R$, $f_3 : (-1, e^{\pi/2} - 2) \to R$ and $f_4 : R \to R$ be functions defined by

(i) $f_1(x) = \sin(\sqrt{1 - e^{-x^2}})$,

(ii) $f_2(x) = \begin{cases} \frac{|\sin x|}{\tan^{-1} x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$, where the inverse trigonometric function $\tan^{-1} x$ assumes values in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(iii) $f_3(x) = [\sin(\log_e(x+2))]$, where for $t \in R$, $[t]$ denotes the greatest integer less than or equal to t,

(iv) $f_4(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

	List-I		List-II
P.	The function f_1 is	1.	NOT continuous at $x=0$
Q.	The function f_2 is	2.	continuous at $x=0$ and NOT differentiable at $x=0$
R.	The function f_3 is	3.	differentiable at $x=0$ and its derivative is NOT continuous at $x=0$
S.	The function f_4 is	4.	differentiable at $x=0$ and its derivative is continuous at $x=0$

The correct option is

(a) P → 2; Q → 3; R → 1; S → 4

(b) P → 4; Q → 1; R → 2; S → 3

(c) P → 4; Q → 2; R → 1; S → 3

(d) P → 2; Q → 1; R → 4; S → 3

If $f(x) = \frac{1}{2}x - 1$, then on the interval $[0, \pi]$

(a) $\tan[f(x)]$ and $1/f(x)$ are both continuous

(b) $\tan[f(x)]$ and $1/f(x)$ are both discontinuous

(c) $\tan[f(x)]$ and $f^{-1}(x)$ are both continuous

(d) $\tan[f(x)]$ is continuous but $1/f(x)$ is not continuous

The following functions are continuous on $(0, \pi)$

(a) $\tan x$

(b) $\int_0^x t \sin \frac{1}{t}\, dt$

(c) $\begin{cases} 1, & 0 \le x \le 3\pi/4 \\ 2\sin\frac{2}{9}x, & \frac{3\pi}{4} < x < \pi \end{cases}$

(d) $\begin{cases} x \sin x, & 0 < x \le \pi/2 \\ \frac{\pi}{2}\sin(\pi + x), & \frac{\pi}{2} < x < \pi \end{cases}$

Let $[x]$ denotes the greatest integer less than or equal to x. If $f(x) = [x \sin \pi x]$, then $f(x)$ is

(a) continuous at $x = 0$

(b) continuous in $(-1, 0)$

(c) differentiable at $x = 1$

(d) differentiable in $(-1, 1)$

Let $f(x) = \begin{cases} \frac{x^2}{2}, & 0 \le x < 1 \\ 2x^2 - 3x + \frac{3}{2}, & 1 \le x \le 2 \end{cases}$

Discuss the continuity of f, f' and f'' on $[0, 2]$.

For the function $f(x) = x\cos\frac{1}{x},\ x \ge 1$,

(a) for atleast one x in the interval $[1, \infty)$, $f(x+2) - f(x) < 2$

(b) $\lim_{x \to \infty} f'(x) = 1$

(c) for all x in the interval $[1, \infty)$, $f(x+2) - f(x) > 2$

(d) $f'(x)$ is strictly decreasing in the interval $[1, \infty)$

Let $f(x) = \begin{cases} x + a, & \text{if } x < 0 \\ |x - 1|, & \text{if } x \ge 0 \end{cases}$ and

$g(x) = \begin{cases} x + 1, & \text{if } x < 0 \\ (x-1)^2 + b, & \text{if } x \ge 0 \end{cases}$

where, a and b are non-negative real numbers. Determine the compositie function gof. If $(gof)(x)$ is continuous for all real x determine the values of a and b. Further, for these values of a and b, is gof differentiable at $x = 0$? Justify your answer.

Let $f(x + y) = f(x) + f(y)$ for all x and y. If the function $f(x)$ is continuous at $x = 0$, then show that $f(x)$ is continuous at all x.

Let $f: R \to R$ be differentiable at $c \in R$ and $f(c) = 0$. If $g(x) = |f(x)|$, then at $x = c$, g is

(a) not differentiable

(b) differentiable if $f'(c) \ne 0$

(c) not differentiable if $f'(c) = 0$

(d) differentiable if $f'(c) = 0$

Let $f(x) = 15 - |x - 10|$; $x \in \mathbf{R}$. Then, the set of all values of x, at which the function, $g(x) = f(f(x))$ is not differentiable, is

(a) $\{5, 10, 15, 20\}$ (b) $\{5, 10, 15\}$

(c) $\{10\}$ (d) $\{10, 15\}$

Let $f(x) = \begin{cases} -1, & -2 \le x < 0 \\ x^2 - 1, & 0 \le x \le 2 \end{cases}$ and

$g(x) = |f(x)| + f(|x|)$. Then, in the interval $(-2, 2)$, g is

(a) not differentiable at one point

(b) not differentiable at two points

(c) differentiable at all points

(d) not continuous

Let $f : (-1, 1) \longrightarrow R$ be a function defined by $f(x) = \max\{-|x|, -\sqrt{1 - x^2}\}$. If K be the set of all points at which f is not differentiable, then K has exactly

(a) three elements (b) five elements

(c) two elements (d) one element

Let $f(x) = \begin{cases} \max\{|x|, x^2\}, & |x| \le 2 \\ 8 - 2|x|, & 2 < |x| \le 4 \end{cases}$

Let S be the set of points in the interval $(-4, 4)$ at which f is not differentiable. Then, S

(a) equals $\{-2, -1, 0, 1, 2\}$ (b) equals $\{-2, 2\}$

(c) is an empty set (d) equals $\{-2, -1, 1, 2\}$

Let f be a differentiable function from R to R such that $|f(x) - f(y)| \le 2|x - y|^{\frac{3}{2}}$, for all $x, y \in R$. If $f(0) = 1$, then $\int_0^1 f^2(x)\,dx$ is equal to

(a) 2 (b) $\frac{1}{2}$ (c) 1 (d) 0

Let $S = (t \in \mathbf{R} : f(x) = |x - \pi| \cdot (e^{|x|} - 1)\sin|x|$ is not differentiable at $t\}$. Then, the set S is equal to

(a) ϕ (an empty set) (b) $\{0\}$

(c) $\{\pi\}$ (d) $\{0, \pi\}$

For $x \in R$, $f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then

(a) g is not differentiable at $x = 0$

(b) $g'(0) = \cos(\log 2)$

(c) $g'(0) = -\cos(\log 2)$

(d) g is differentiable at $x = 0$ and $g'(0) = -\sin(\log 2)$

Let $f(x) = \begin{cases} x^2 \left| \cos \frac{\pi}{x} \right|, & x \neq 0, x \in R, \text{ then } f \text{ is} \\ 0, & x = 0 \end{cases}$

(a) differentiable both at $x = 0$ and at $x = 2$

(b) differentiable at $x = 0$ but not differentiable at $x = 2$

(c) not differentiable at $x = 0$ but differentiable at $x = 2$

(d) differentiable neither at $x = 0$ nor at $x = 2$

If f is a differentiable function satisfying $f\left(\frac{1}{n}\right) = 0, \forall\ n \geq 1, n \in I$, then

(a) $f(x) = 0, x \in (0, 1]$

(b) $f'(0) = 0 = f(0)$

(c) $f(0) = 0$ but $f'(0)$ not necessarily zero

(d) $|f(x)| \leq 1, x \in (0, 1]$

There exists a function $f(x)$ satisfying $f(0) = 1$, $f'(0) = -1, f(x) > 0, \forall\ x$ and

(a) $f''(x) < 0, \forall\ x$

(b) $-1 < f''(x) < 0, \forall x$

(c) $-2 \leq f''(x) \leq -1, \forall x$

(d) $f''(x) < -2, \forall x$

For a real number y, let $[y]$ denotes the greatest integer less than or equal to y. Then, the function $f(x) = \frac{\tan \pi [(x - \pi)]}{1 + [x]^2}$ is

(a) discontinuous at some x

(b) continuous at all x, but the derivative $f'(x)$ does not exist for some x

(c) $f'(x)$ exists for all x, but the derivative $f''(x)$ does not exist for some x

(d) $f'(x)$ exists for all x

For every twice differentiable function $f: R \to [-2, 2]$ with $(f(0))^2 + (f'(0))^2 = 85$, which of the following statement(s) is (are) TRUE ?

(a) There exist $r, s \in R$, where $r < s$, such that f is one-one on the open interval (r, s)

(b) There exists $x_0 \in (-4, 0)$ such that $|f'(x_0)| \leq 1$

(c) $\lim_{x \to \infty} f(x) = 1$

(d) There exists $\alpha \in (-4, 4)$ such that $f(\alpha) + f''(\alpha) = 0$ and $f'(\alpha) \neq 0$

Let $f: R \to R$, $g: R \to R$ and $h: R \to R$ be differentiable functions such that $f(x) = x^3 + 3x + 2$, $g(f(x)) = x$ and $h(g(g(x))) = x$ for all $x \in R$. Then,

(a) $g'(2) = \frac{1}{15}$

(b) $h'(1) = 666$

(c) $h(0) = 16$

(d) $h(g(3)) = 36$

The function $f(x)=\begin{cases}|x-3|, & x\ge 1\\ \dfrac{x^2}{4}-\dfrac{3x}{2}+\dfrac{13}{4}, & x<1\end{cases}$ is

(a) continuous at $x=1$ (b) differentiable at $x=1$
(c) discontinuous at $x=1$ (d) differentiable at $x=3$

In the following, $[x]$ denotes the than or equal to x.

	Column I		
A.	$x\lvert x\rvert$	p.	continuous in $(-1, 1)$
B.	$\sqrt{\lvert x\rvert}$	q.	differentiable in $(-1, 1)$
C.	$x+[x]$	r.	strictly increasing $(-1, 1)$
D.	$\lvert x-1\rvert+\lvert x+1\rvert$, in $(-1, 1)$	s.	not differentiable atleast at one point in $(-1, 1)$

For the function $f(x)=\begin{cases}\dfrac{x}{1+e^{1/x}}, & x\neq 0\\ 0, & x=0\end{cases}$;

the derivative from the right, $f'(0^+)=\ldots$ and the derivative from the left, $f'(0^-)=\ldots$.

Let $f(x)=\begin{cases}(x-1)^2\sin\dfrac{1}{(x-1)}-|x|, & \text{if } x\neq 1\\ -1, & \text{if } x=1\end{cases}$ be a real valued function. Then, the set of points, where $f(x)$ is not differentiable, is .... .

$$f(x)=\begin{cases}b\sin^{-1}\left(\dfrac{x+c}{2}\right), & -\dfrac{1}{2}<x<0\\ \dfrac{1}{2}, & x=0\\ \dfrac{e^{ax/2}-1}{x}, & 0<x<\dfrac{1}{2}\end{cases}$$

If $f(x)$ is differentiable at $x=0$ and $|c|<\dfrac{1}{2}$, then find the value of a and prove that $64b^2=(4-c^2)$.

Let $\alpha\in R$. Prove that a function $f:R\to R$ is differentiable at α if and only if there is a function $g:R\to R$ which is continuous at α and satisfies $f(x)-f(\alpha)=g(x)(x-\alpha)$, $\forall x\in R$.

Let $f[(x+y)/2]=\{f(x)+f(y)\}/2$ for all real x and y, if $f'(0)$ exists and equals -1 and $f(0)=1$, find $f(2)$.

Let $f(x)=\begin{cases}x\,e^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, & x\neq 0\\ 0, & x=0\end{cases}$

Test whether
(i) $f(x)$ is continuous at $x=0$.
(ii) $f(x)$ is differentiable at $x=0$.

Let R be the set of real numbers and $f : R \to R$ be such that for all x and y in R, $f(x) - f(y)|^2 \le (x-y)^3$. Prove that $f(x)$ is a constant.

Let $f\ (x)$ be defined in the interval $[-2, 2]$ such that

$$f\ (x) = \begin{cases} -1, & -2 \le x \le 0 \\ x-1, & 0 < x \le 2 \end{cases}$$

and $\quad g\ (x) = f\ (|\,x\,|) + |\,f\ (x)\,|.$

Test the differentiability of $g\ (x)$ in $(-2, 2)$.

Let $f\ (x) = x^3 - x^2 - x + 1$

and $$g\ (x) \begin{cases} = \max\{f(t); 0 \le t \le x\}, & 0 \le x \le 1 \\ = 3 - x, & 1 < x \le 2 \end{cases}$$

Discuss the continuity and differentiability of the function $g\ (x)$ in the interval $(0, 2)$.

Discuss the differentiability of

$$f(x) = \begin{cases} (x-e)2^{-2^{\left(\frac{1}{(e-x)}\right)}}, & x \ne e \\ 0, & x = e \end{cases} \text{ at } x = e.$$

Discuss the differentiability of

$$f(x) = \sin^{-1}\frac{2x}{1+x^2}.$$

Let $y_n(x) = x^2 + \dfrac{x^2}{1+x^2} + \dfrac{x^2}{(1+x^2)^2} + \ldots\ldots\ldots\ldots + \dfrac{x^2}{(1+x^2)^{n-1}}$

and $y(x) = \lim\limits_{n\to\infty} y_n(x)$

Discuss the continuity of $y_n(x)$ ($n = 1, 2, 3 \ldots\ldots\ldots n$) and $y(x)$ at $x = 0$