

1

Let $[x]$ denote the greatest integer function & $f(x)$ be defined in a neighbourhood of 2 by

$$f(x) = \begin{cases} \frac{\exp\{(x-2)(\ln 4)\}^4 - 16}{4^x - 16}, & x < 2 \\ A \frac{1 - \cos(x-2)}{(x-2)\tan(x-2)}, & x > 2 \end{cases}$$

$A=1, f(2) = \frac{1}{2}$

2

Find the values of A & $f(2)$ in order that $f(x)$ may be continuous at $x=2$.

Let $f(x) = x + 2, \quad -4 \leq x \leq 0$
 $= 2 - x^2, \quad 0 < x \leq 4$

*from $x \in [-4, 16]$
cont. in both domain*

then find $f(f(x))$, domain of $f(f(x))$ and also comment upon the continuity of $f(f(x))$.

3

Let $g(x) = \lim_{n \rightarrow \infty} \frac{x^n f(x) + h(x) + 1}{2x^n + 3x + 3}, x \neq 1$ and $g(1) = \lim_{x \rightarrow 1} \frac{\sin^2(\pi 2^x)}{\ln(\sec(\pi 2^x))}$ be a continuous function at $x=1$, find the value of $4g(1) + 2f(1) - h(1)$. Assume that $f(x)$ and $h(x)$ are continuous at $x=1$. 5

4

$$f(x) = \frac{a^{\tan x} - a^{\sin x}}{\tan x - \sin x} \text{ for } x > 0$$

$$= \frac{\ln(1+x+x^2) + \ln(1-x+x^2)}{\sec x - \cos x} \text{ for } x < 0, \text{ if } f \text{ is continuous at } x=0, \text{ find 'a'}$$

$a = \frac{1}{e}$

now if $g(x) = \ln\left(2 - \frac{x}{a}\right) \cdot \cot(x-a)$ for $x \neq a, a \neq 0, a > 0$. If g is continuous at $x=a$ then show that $g(e^{-1}) = -e$.

5

Let $f(x+y) = f(x) + f(y)$ for all x, y & if the function $f(x)$ is continuous at $x=0$, then show that $f(x)$ is continuous at all x .

6

If $f(x+y) = f(x)f(y)$ for all x, y and $f(x)$ is continuous at $x=1$. Prove that $f(x)$ is continuous for all x except at $x=0$. Given $f(1) \neq 0$.

7

Given $f(x) = \sum_{r=1}^n \tan\left(\frac{x}{2^r}\right) \sec\left(\frac{x}{2^r-1}\right); r, n \in \mathbb{N}$ $f(r) = \tan r - \tan \frac{x}{2^n}$

$$g(x) = \lim_{n \rightarrow \infty} \frac{(n(f(x) + \tan \frac{x}{2^n}) - (f(x) - \tan \frac{x}{2^n})^n) \cdot \left[\sin\left(\tan \frac{x}{2}\right)\right]}{1 - (f(x) - \tan \frac{x}{2^n})^n}$$

$k=0$

$g(r) = \begin{cases} \ln(\tan r) & 0 < x < \frac{\pi}{4} \\ 0 & \pi/4 < x < \pi/2 \end{cases}$

$= k$ for $x = \frac{\pi}{4}$ and the domain of $g(x)$ is $(0, \pi/2)$.

where $[]$ denotes the greatest integer function.

Find the value of k , if possible, so that $g(x)$ is continuous at $x = \pi/4$. Also state the points of discontinuity of $g(x)$ in $(0, \pi/4)$, if any.

8

No.

- 8 Let $f(x) = x^3 - x^2 - 3x - 1$ and $h(x) = \frac{f(x)}{g(x)}$ where h is a function such that
 (a) it is continuous every where except when $x = -1$, (b) $\lim_{x \rightarrow \infty} h(x) = \infty$ and (c) $\lim_{x \rightarrow -1} h(x) = \frac{1}{2}$
 Find $\lim_{x \rightarrow 0} (3h(x) + f(x) - 2g(x)) = -\frac{34}{4}$ $g(x) = 4(x+1)$
- 9 Let f be continuous on the interval $[0, 1]$ to \mathbb{R} such that $f(0) = f(1)$. Prove that there exists a point c in $[0, \frac{1}{2}]$ such that $f(c) = f(c + \frac{1}{2})$ IVT or BC
- 10 The function defined as $f(x) = \lim_{n \rightarrow \infty} \frac{\cos \pi x - x^{2n} \sin(x-1)}{1 + x^{2n-1} - x^{2n}}$
 (A) is discontinuous at $x = 1$ because $f(1^+) \neq f(1^-)$
 (B) is discontinuous at $x = 1$ because $f(1)$ is not defined
 (C) is discontinuous at $x = 1$ because $f(1^+) = f(1^-) \neq f(1)$
 (D) is continuous at $x = 1$
- 11 Let f be a continuous function on \mathbb{R} . If $f(1/4^n) = (\sin e^n) e^{-n^2} + \frac{n^2}{n^2+1}$ then $f(0)$ is:
 (A) not unique $\sqrt{(B)}$ 1
 (C) data sufficient to find $f(0)$ (D) data insufficient to find $f(0)$
- 12 If $f(x) = \text{sgn}(\cos 2x - 2 \sin x + 3)$, where $\text{sgn}(\cdot)$ is the signum function, then $f(x)$
 (A) is continuous over its domain (B) has a missing point discontinuity
 (C) has isolated point discontinuity (D) has irremovable discontinuity.
- 13 Let $g(x) = \tan^{-1}|x| - \cot^{-1}|x|$, $f(x) = \frac{[x]}{[x+1]}$ ($[x]$), $h(x) = |g(f(x))|$ where $\{x\}$ denotes fractional part and $[x]$ denotes the integral part then which of the following holds good?
 (A) h is continuous at $x = 0$ (B) h is discontinuous at $x = 0$
 (C) $h(0^-) = \frac{\pi}{2}$ (D) $h(0^+) = -\frac{\pi}{2}$
- 14 Consider $f(x) = \lim_{n \rightarrow \infty} \frac{x^n - \sin x^n}{x^n + \sin x^n}$ for $x > 0, x \neq 1$
 $f(1) = 0$
 then
 (A) f is continuous at $x = 1$
 (B) f has a finite discontinuity at $x = 1$
 (C) f has an infinite or oscillatory discontinuity at $x = 1$.
 (D) f has a removable type of discontinuity at $x = 1$.

15 Given $f(x) = \frac{\{[x]\} e^{x^2} \{x + \{x\}\}}{\left(e^{\frac{1}{x^2}} - 1\right) \operatorname{sgn}(\sin x)}$ for $x \neq 0$
 $= 0$ for $x = 0$

where $\{x\}$ is the fractional part function, $[x]$ is the step up function and $\operatorname{sgn}(x)$ is the signum function of x then $f(x)$

- (A) is continuous at $x = 0$ (B) is discontinuous at $x = 0$
 (C) has a removable discontinuity at $x = 0$ (D) has an irremovable discontinuity at $x = 0$

16 Consider $f(x) = \begin{cases} x[x]^2 \log_{10} 2 & \text{for } -1 < x < 0 \\ \frac{\ln(e^{x^2} + 2\sqrt{\{x\}})}{\tan \sqrt{x}} & \text{for } 0 < x < 1 \end{cases}$

where $[*]$ & $\{*\}$ are the greatest integer function & fractional part function respectively, then
 (A) $f(0) = \ln 2 \Rightarrow f$ is continuous at $x = 0$ (B) $f(0) = 2 \Rightarrow f$ is continuous at $x = 0$
 (C) $f(0) = e^2 \Rightarrow f$ is continuous at $x = 0$ (D) f has an irremovable discontinuity at $x = 0$

17 Consider $f(x) = \frac{\sqrt{1+x} - \sqrt{1-x}}{\{x\}}$ $x \neq 0$

$g(x) = \cos 2x$ $-\frac{\pi}{4} < x < 0$

$h(x) = \begin{cases} \frac{1}{\sqrt{2}} f(g(x)) & \text{for } x < 0 \\ 1 & \text{for } x = 0 \\ f(x) & \text{for } x > 0 \end{cases}$

then, which of the following holds good. where $\{x\}$ denotes fractional part function.

- (A) h is continuous at $x = 0$ (B) h is discontinuous at $x = 0$
 (C) $f(g(x))$ is an even function (D) $f(x)$ is an even function

18 Let $f(0) = 0$ and $f'(0) = 1$. For a positive integer k , show that

$\lim_{x \rightarrow 0} \frac{1}{x} \left(f(x) + f\left(\frac{x}{2}\right) + \dots + f\left(\frac{x}{k}\right) \right) = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{k}$

Ans 19

19 $f(x) = x \cdot \left(\frac{e^{|x|+|x|-2}}{|x|-|x|} \right)$, $x \neq 0$ & $f(0) = -1$ where $[x]$ denotes greatest integer less than or equal to x .
 Test the differentiability of $f(x)$ at $x = 0$.

20 Discuss the continuity & the derivability in $[0, 2]$ of $f(x) = \begin{cases} [2x-3][x] & \text{for } x \geq 1 \\ \sin \frac{\pi x}{2} & \text{for } x < 1 \end{cases}$
 where $[]$ denote greatest integer function.

21 If $f(x) = -1 + |x-1|$, $-1 \leq x \leq 3$; $g(x) = 2 - |x+1|$, $-2 \leq x \leq 2$, then calculate $(f \circ g)(x)$ & $(g \circ f)(x)$. Draw their graph. Discuss the continuity of $(f \circ g)(x)$ at $x = -1$ & the differentiability of $(g \circ f)(x)$ at $x = \frac{1}{2}$.
 continuous

22 Discuss the continuity on $0 \leq x \leq 1$ & differentiability at $x = 0$ for the function.
 $f(x) = x \sin \frac{1}{x} \sin \frac{1}{x \sin \frac{1}{x}}$ where $x \neq 0$, $x \neq 1/r\pi$ & $f(0) = f(1/r\pi) = 0$,
 $r = 1, 2, 3, \dots$

23 Consider the function, $f(x) = \begin{cases} x^2 \left| \cos \frac{\pi}{2x} \right| & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

- (a) Show that $f'(0)$ exists and find its value $\Rightarrow 0$
- (b) Show that $f'\left(\frac{1}{3}\right)$ does not exist.
- (c) For what values of x , $f'(x)$ fails to exist. $x = \frac{1}{2m+1}$

24 Let $f(x)$ be a function defined on $(-a, a)$ with $a > 0$. Assume that $f(x)$ is continuous at $x = 0$ and $\lim_{x \rightarrow 0} \frac{f(x) - f(kx)}{x} = \alpha$, where $k \in (0, 1)$ then compute $f'(0^+)$ and $f'(0^-)$, and comment upon the differentiability of f at $x = 0$.
 $\frac{\alpha}{1-k}$

25 A function $f: \mathbb{R} \rightarrow \mathbb{R}$ satisfies the equation $f(x+y) = f(x) \cdot f(y)$ for all x, y in \mathbb{R} & $f(x) \neq 0$ for any x in \mathbb{R} . Let the function be differentiable at $x = 0$ & $f'(0) = 2$. Show that $f'(x) = 2f(x)$ for all x in \mathbb{R} . Hence determine $f(x)$.

26 Let $f(x)$ be a real valued function not identically zero satisfies the equation, $f(x+y)^n = f(x) + (f(y))^n$ for all real x & y and $f'(0) \geq 0$ where $n (> 1)$ is an odd natural number. Find $f(10)$.

27 A function $f: \mathbb{R} \rightarrow \mathbb{R}$ where \mathbb{R} is a set of real numbers satisfies the equation $f\left(\frac{x+y}{3}\right) = \frac{f(x)+f(y)+f(0)}{3}$ for all x, y in \mathbb{R} . If the function is differentiable at $x = 0$ then show that it is differentiable for all x in \mathbb{R} .
 $f(1) = x \Rightarrow f(10) = 10$

28 Given $f(x) = \begin{cases} \log_a (a^{\lfloor x \rfloor} - [-x])^{\lfloor \frac{2}{|x|+1} - 5 \rfloor} & \text{for } |x| \neq 0; a > 1 \\ 0 & \text{for } x = 0 \end{cases}$ where $\lfloor \cdot \rfloor$ represents the integral part function, then :

- (A) f is continuous but not differentiable at $x = 0$
 (B) f is cont. & diff. at $x = 0$
 (C) the differentiability of f at $x = 0$ depends on the value of a
 (D) f is cont. & diff. at $x = 0$ and for $a = e$ only.

29 Consider $f(x) = \begin{cases} \frac{2(\sin x - \sin^3 x) - \lfloor \sin x - \sin^3 x \rfloor}{2(\sin x - \sin^3 x) - \lfloor \sin x - \sin^3 x \rfloor} & , x \neq \frac{\pi}{2} \\ f(\pi/2) = 3 & \text{where } \lfloor \cdot \rfloor \text{ denotes the greatest integer function then,} \end{cases}$ for $x \in (0, \pi)$

- (A) f is continuous & differentiable at $x = \pi/2$
 (B) f is continuous but not differentiable at $x = \pi/2$
 (C) f is neither continuous nor differentiable at $x = \pi/2$
 (D) none of these

30 Given the function $f(x) = 2x\sqrt{x^2-1} + 5\sqrt{x}\sqrt{1-x^4} + 7x^2\sqrt{x-1} + 3x + 2$ then :

- (A) the function is continuous but not differentiable at $x = 1$
 (B) the function is discontinuous at $x = 1$
 (C) the function is both cont. & differentiable at $x = 1$
 (D) the range of $f(x)$ is \mathbb{R}^+ .

isolated point
cont.
but diff not defined.

31 If $f(x+y) = f(x) + f(y) + |x|y + xy^2, \forall x, y \in \mathbb{R}$ and $f'(0) = 0$, then

- (A) f need not be differentiable at every non zero x
 (B) f is differentiable for all $x \in \mathbb{R}$
 (C) f is twice differentiable at $x = 0$
 (D) none

32 Let f be a differentiable function on the open interval (a, b) . Which of the following statements must be true?

- I. f is continuous on the closed interval $[a, b]$
 II. f is bounded on the open interval (a, b)
 III. If $a < a_1 < b_1 < b$, and $f(a_1) < 0 < f(b_1)$, then there is a number c such that $a_1 < c < b_1$ and $f(c) = 0$
 (A) I and II only (B) I and III only (C) II and III only (D) only III

33 If $f(x) = \begin{cases} a + \frac{\sin[x]}{2^x} & , x > 0 \\ \frac{x}{2} & , x = 0 \\ b + \frac{\sin x - x}{x^3} & , x < 0 \end{cases}$ (where $[\cdot]$ denotes the greatest integer function). If $f(x)$ is continuous at $x = 0$, then b is equal to :
 (A) $a - 1$ (B) $a + 1$ (C) $a + 2$ (D) $a - 2$

34 Let $f(x)$ be a function defined on $[0, 1]$ such that $f(x) = \begin{cases} x & ; x \in \mathbb{Q} \\ 1-x & ; x \in \mathbb{Q}^c \end{cases}$, then $f \circ f(x)$ is
 (A) continuous for all $x \in [0, 1]$ (B) continuous for only one value of x
 (C) not possible to define (D) none of these

35 The jump of discontinuity for the function, (where $[x]$ denotes greatest integer function and $\{x\}$ denotes fractional part of x).

$$f(x) = \begin{cases} \frac{e^{[x]} - e^{\{x\}}}{e^x} & , x < 0 \\ \frac{\sin [x]}{\tan x} & , x > 0 \\ 2 & , x = 0 \end{cases}$$

(A) $e^{-1} - e - 1$ (B) $e^{-1} - e - 2$ (C) 1 (D) $1 + e - e^{-1}$

36 Number of points of non-differentiability of the function $g(x) = [x^2] (\cos^4 x) + \{x^2\} [\cos^4 x] + x^2 \sin^2 4x + [x^2] |\cos^4 x| + \{x^2\} (\cos^4 x)$ in $(-50, 50)$. (where $[\cdot]$ denotes the greatest integer function and $\{ \cdot \}$ denotes the fractional part of function), is equal to
 (A) 98 (B) 99
 (C) 100 (D) 0

37 Let $f(x) = (x + |x|) |x|$, then for all x
 (A) f is continuous (B) f is differentiable $\forall x \in \mathbb{R}$
 (C) f' is continuous (D) f'' is continuous

37) Let $f(x) = \begin{cases} [x] & ; x \in [-2, 0) \\ |x| & ; x \in [0, 2] \end{cases}$; where $[\]$ represent G.I.F. and $g(x) = \sec x, x \in \mathbb{R} - (2n + 1)\frac{\pi}{2}$.

Match the following statements in column-I with their values in column-II in interval $(-\frac{3\pi}{2}, \frac{3\pi}{2})$.

Column - I	Column - II
(A) Limit of fog exist at	(p) -1
(B) Limit of gof does not exist at	(q) π
(C) Points of discontinuity of fog is/are	(r) $\frac{5\pi}{6}$
(D) Points of differentiability of fog is/are	(s) $-\pi$
	(t) $\frac{\pi}{3}$

A → p, q, r, s, t; B → r; C → q, s; D → p, r

38) Let $f(x) = \begin{cases} x+1 & , 0 \leq x \leq 1 \\ 2x^2 - 6x + 6 & , 1 \leq x \leq 2 \end{cases}$

And $g(t) = \int_{t-1}^1 f(x)dx, t \in [1, 2]$. Which of the following is true statement

- (A) $f(x)$ is continuous and differentiable $\forall x \in [0, 2]$
 (B) $g(t)$ is maximum at $t = \frac{3}{2}$
 (C) $g(t)$ is minimum at $t = 2$
 (D) all the above

- 39) $f(x)$ is defined for $x > -1$ and has a continuous derivate. If f satisfies $f(0) = 1, f'(0) = 0;$
 $(1 + f(x)) f'(x) = 1 + x$. If x is positive then $f'(x)$ is
 (A) always positive (B) always negative
 (C) always non-negative (D) none of these

40) Let $f(x) = x^2 - 2|x|$ and $g(x) = \begin{cases} \text{Min}\{f(t) : -2 \leq t \leq x, -2 \leq x < 0\} \\ \text{Max}\{f(t) : 0 \leq t \leq x, 0 \leq x \leq 3\} \end{cases}$.

- (a) Draw the graph of $g(x)$.
 (b) At how many points is $g(x)$ discontinuous in its domain?
 (A) 0 (B) 1 (C) 2 (D) 3 (E) None of these
 (c) At how many points is $g(x)$ non-differentiable in its domain?
 (A) 0 (B) 1 (C) 2 (D) 3 (E) None of these

- 41) Consider the function $f(x) = [x] + \sqrt{|x|}$. This function is
 (A) continuous at all integer points. (B) discontinuous at all integer points.
 (C) differentiable at all integer points. (D) non-differentiable at all integer points.

42) Let $f(\frac{x+y}{2}) = \frac{f(x)+f(y)+2}{3} \forall x, y \in \mathbb{R}$. Also, it is given that $f'(0) = 2$. Which of the following statements is / are true?

- (A) $f'(x) = f'(0) \forall x \in \mathbb{R}$ (C) $f(x)$ is a linear function
 (B) $f'(x) = f(x) \forall x \in \mathbb{R}$ (D) none of these

43) Let $\alpha \in \mathbb{R}$. Prove that a function $f: \mathbb{R} \rightarrow \mathbb{R}$ is differentiable at α if and only if there exists a function $g: \mathbb{R} \rightarrow \mathbb{R}$ which is continuous at α and satisfies $f(x) - f(\alpha) = g(x)(x - \alpha)$ for all $x \in \mathbb{R}$.

44) Let f be an injective function with domain $[a, b]$ and range $[c, d]$. If α is a point in (a, b) such that f has left hand derivative l and right hand derivative r at $x = \alpha$ with both l and r non-zero different and negative, then left hand derivative and right hand derivative of f^{-1} at $x = f(\alpha)$ respectively, is :

- (a) $\frac{1}{r}, \frac{1}{l}$ (b) r, l (c) $\frac{1}{l}, \frac{1}{r}$ (d) l, r

- 46 If $f(x) = (p^2 - 1) [\tan^{-1} x] + 4(q^2 + 2q - 3) \left\{ \frac{1}{2+x^2} \right\} + (p+q) \operatorname{sgn}(x^2 - x + 2)$ is continuous in R and $f(x_1) = f(x_2) \forall x_1, x_2 \in R$ then largest value of $|p+q|$ is:
 (a) 0 (b) 2 (c) 4 (d) 5

[Note: $\operatorname{sgn}(y)$, $[y]$ and $\{y\}$ denote signum function, greatest integer function and fractional part function respectively.]

- 47 If $f(x) = \begin{cases} (p^2 - 1)([x] + 2\{x\}) - 2, & -2 < x \leq -1 \\ q \left(\frac{e^x + e^{-x}}{2} \right) + p|(x-1)|, & -1 < x < 2 \end{cases}$, $p, q \in R$ is continuous in $(-2, 2)$

then $f\left(f\left(f\left(\frac{-1}{2}\right)\right)\right)$ is:

- (a) -2 (b) -1 (c) 0 (d) not defined

[Note: $[k]$ denotes greatest integer function less than or equal to k and $\{k\}$ denotes fractional part function of k .]

10

- 48 Which of the following statements is(are) correct?

[Note: $[x]$ and $\{x\}$ denote greatest integer less than or equal to x and fractional part of x respectively.]

- (a) $f(x) = [\ln x] + \sqrt{[\ln x]}$, $x > 1$ is continuous at $x = e$.
 (b) If $\lim_{x \rightarrow 2} \left(\frac{3x^2 + ax + a + 1}{x^2 + x - 2} \right)$ exists and equals l , then $a + \frac{1}{l} = 10$.
 (c) $f: [-1, 1] \rightarrow [-1, 1]$, $f(x) = x^2 \operatorname{sgn}(x)$ is a bijective function, where $\operatorname{sgn} x$ denotes signum function of x .
 (d) If f is continuous on $[-1, 1]$, $f(-1) = 4$ and $f(1) = 3$, then there exists a number r such that $|r| < 1$ and $f(r) = \pi$.

49 Column I

(a) $f_1(x) = \left[\frac{4x}{\pi} \right] \operatorname{sgn}(x^2 - x + 1)$

(b) $f_2(x) = \cos^{-1} \left(\operatorname{sgn} \left(\cos \frac{2x-1}{2} \pi \right) \right)$

(c) $f_3(x) = \max. \{(x+1), (5-x)\}$

(d) $f_4(x) = \sqrt{x^2} + [x]^2$
 $a \rightarrow p, q, r, s, t \quad b \rightarrow p, q, r, s, t$
 $c \rightarrow p, r \quad d \rightarrow p, q, r, s, t$

Column II

(p) discontinuous at more than 3 points but less than 6 points in $[-2, 2]$

(q) non derivable at more than 2 points but atmost 5 points in $[-2, 2]$

(r) range contains atleast one integer but not more than seven and no irrational value in $[-2, 2]$

(s) many one but not even function in $[-2, 2]$

(t) neither odd nor periodic in $[-2, 2]$

- 50 Let f be a differentiable function satisfying the functional rule $f(xy) = f(x) + f(y) + \frac{x+y-1}{xy} \forall x, y > 0$ and $f'(1) = 2$. Find the value of $[f(e^{100})]$

Note: $[k]$ denotes the greatest integer less than or equal to k .

- 51 Let $f(x) = \begin{cases} \sqrt{(2n+1)x - x^2 - (n^2 + n)} & n \leq x < n + \frac{1}{2} \\ n + 1 - x & n + \frac{1}{2} \leq x < n + 1 \end{cases}$ ($n \in I$). Find the number of values of x where $f(x)$ is non-derivable in $(-5, 5)$.

52 If for a differentiable function $y = f(x)$, ($f'(x) > 0$) $\lim_{x \rightarrow 1} \frac{e^{2f(x)} - 2e^{f(x)+1} + e^2 \cos(x-1)}{\sec^2(x-1) - 1} = \frac{7}{2}e^2$ and area of the triangle formed by the tangent drawn to the curve $y = f(x)$ at $[1, f(1)]$ and co-ordinate axes is Δ , then find the value of $\frac{1}{\Delta} = 4$.

53 If $f(x) = 23 - 2^{-x^2+2x+3} \forall x \in R$ and $g(x) = \left[\frac{f(x)}{\mu} \right]$, where $[k]$ denotes greatest integer function less than or equal to k and $\mu \in N$, then find the sum of all values of μ for which $g(x)$ is discontinuous for at least one real value of x .

54 Let $f(x) = (x^2 - 3x + 2) |x^3 - 6x^2 + 11x - 6| + \left| \sin\left(x + \frac{\pi}{4}\right) \right|$.

Number of points at which the function $f(x)$ is non-differentiable in $[0, 2\pi]$, is :
 (a) 5 (b) 4 (c) 3 (d) 2

55 Let f be a differentiable function satisfying $f'(x) = f'(-x) \forall x \in R$. Then

(a) If $f(1) = f(2)$, then $f(-1) = f(-2)$

(b) $\frac{1}{2}f(x) + \frac{1}{2}f(y) = f\left(\frac{1}{2}(x+y)\right)$ for all real values of x, y

(c) Let $f(x)$ be an even function, then $f(x) = 0 \forall x \in R$

(d) $f(x) + f(-x) = 2f(0) \forall x \in R$

56 If $g(x)$ is a polynomial satisfying $g(x)g(y) = g(x) + g(y) + g(xy) - 2$ for all real x and y and $g(2) = 5$

then $\lim_{x \rightarrow 3} g(x)$ is

(a) 9

(c) 10

(b) 25

(d) none of these

Consider the function

$$\textcircled{57} \quad f(x) = x \left[\frac{1}{x(1+x)} + \frac{1}{(1+x)(1+2x)} + \frac{1}{(1+2x)(1+3x)} + \dots \infty \right] \text{ for } x \neq 0.$$

Find $f(0)$ if $f(x)$ is continuous at $x=0$. $\textcircled{2}$

$$\textcircled{58} \quad \text{Given } f(x) = \sum_{r=1}^n (x^r + x^{-r})^2, \quad x \neq \pm 1 \text{ and}$$

$$g(x) = \begin{cases} \lim_{n \rightarrow \infty} (f(x) - 2n)x^{-2n-2}(1-x^2) & \text{for } x \neq \pm 1 \\ -1, & \text{for } x = \pm 1 \end{cases}$$

show that $g(x)$ is continuous for all x in the domain.

$\textcircled{59}$ Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function which satisfies $f(x+y) = f(x) \cdot (f(y))^3 \quad \forall x, y \in \mathbb{R}$. If f is continuous at $x=0$, prove that f is continuous everywhere.

$\textcircled{60}$ If $f(x+y) = f(x) \cdot f(y)$ for all $x, y \in \mathbb{R}$ and $f(x) = 1 + g(x) \cdot G(x)$ where $\lim_{x \rightarrow 0} g(x) = 0$ and $\lim_{x \rightarrow 0} G(x)$ exists, prove that $f(x)$ is continuous at all $x \in \mathbb{R}$.

60 Let $f(x) = \operatorname{cosec} 2x + \operatorname{cosec} 2^2 x + \operatorname{cosec} 2^3 x + \dots + \operatorname{cosec} 2^n x$, $x \in (0, \pi/2)$

and $g(x) = f(x) + \cot 2^n x$

If

$$H(x) = \begin{cases} (\cos x)^{g(x)} + (\sec x)^{\operatorname{cosec} x} & \text{if } x > 0 \\ p & \text{if } x = 0 \\ \frac{e^x + e^{-x} - 2 \cos x}{x \sin x} & \text{if } x < 0 \end{cases}$$

find the value of p , if possible to make the function $H(x)$ continuous at $x=0$.

61 Prove that

$f(x) = [\tan x] + \sqrt{\tan x - [\tan x]}$ (where $[.]$ denotes greatest integer function) is continuous in $\left[0, \frac{\pi}{2}\right)$.

62 Check differentiability of $f(x)$ in $[-2, 2]$, if

$$f(x) = \begin{cases} \cos \frac{\pi}{2} (|x| - \{x\}) & x < 1 \\ \sqrt{4x^2 - 12x + 9} \{x\} & x \geq 1 \end{cases}$$

where $\{.\}$ denotes the fractional part function.

no. of cases $x = -1, 0, 1, 3/2, 2$

63 A function $f: (0, \infty) \rightarrow \mathbb{R}$ satisfies the equation $f(x/y) = f(x) - f(y)$. If $f(x)$ is differentiable on $(0, \infty)$ and $\lim_{x \rightarrow 0} \frac{f(1+x)}{x} = 3$, then determine $f(x)$. = $3 \ln x$

64 A differentiable function satisfies the relation $f(x+y) = f(x) + f(y) + 2xy - 1 \quad \forall x, y \in \mathbb{R}$. If $f'(0) = \sqrt{3+a-a^2}$, find $f(x)$ and prove that $f(x) > 0 \quad \forall x \in \mathbb{R}$.

65 If $f\left(\frac{x+y}{3}\right) = \frac{2+f(x)+f(y)}{3}$ for all real x and y and $f'(2) = 2$ then determine $y = f(x)$. $f(x) = 2(x+1)$

66 Let $f(xy) = xf(y) + yf(x)$ for all $x, y \in \mathbb{R}^+$ and $f(x)$ be differentiable in $(0, \infty)$ then determine $f(x)$. = $\frac{1}{2} \ln(x^2)$

67 If $e^{-xy}f(xy) = e^{-x}f(x) + e^{-y}f(y) \quad \forall x, y \in \mathbb{R}^+$, and $f'(1) = e$, determine $f(x)$. = $e^x \ln x$

68 A function $f: (-1, 1) \rightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

satisfies the equation

$$f(x) + f(y) = f(x\sqrt{1-y^2} + y\sqrt{1-x^2}).$$

- (i) Show that $f(x)$ is odd.
 (ii) If $f(x)$ is differentiable on $(-1, 1)$ and $f'(0) = 1$, then

show that $f'(x) = \frac{1}{\sqrt{1-x^2}}$

(iii) Hence, determine $f(x)$. = $\arcsin x$

69 A twice differentiable function f

satisfies the relation $f(x^2 + y^2) = f(x^2 - y^2) + f(2xy) \forall x,$

$y \in \mathbb{R}$. If $f(0) = 0$ and $f''(0) = 2$, find $f(x)$. = x^2

70 $f(x)$ is a differentiable function: satisfy the relationship $f^2(x) + f^2(y) + 2(xy-1) = f^2(x+y) \sqrt{x^2+2}$



71

function $f: (0, \infty) \rightarrow \mathbb{R}$ satisfies the equation $f(xy) = 2f(x) - f\left(\frac{x}{y}\right)$. If f is differentiable on \mathbb{R}^+ and $f(1) = 0, f'(1) = 1$, then show that

(i) $f(y) = -f\left(\frac{1}{y}\right)$ (ii) $f(x \cdot y) = f\left(\frac{x}{y}\right)$

$\forall x, y \in \mathbb{R}$. Also $f(x) > 0 \forall x \in \mathbb{R}$, and $f(\sqrt{2}) = 2$ and hence determine $f(x)$. Determine $f(x)$.

72

A differentiable function f satisfies the relation $f(x+y) + f(xy-1) = f(x) + f(y) + f(xy) \forall x, y \in \mathbb{R}$. If $f(1) = 2, f'(0) = 1$ and $f'(-1) = -1$, find $f(x)$. = $x^2 + x$

73 Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be such that for all x and y in \mathbb{R} , $|f(x) - f(y)| \leq |x - y|^3$. Prove that $f(x)$ is a constant function.

74 Suppose
 If $p(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$
 $|p(x)| \leq |e^{x-1} - 1|$ for all $x \geq 0$, prove that
 $|a_1 + 2a_2 + \dots + na_n| \leq 1$.

75 Find a function continuous and derivable for all x and satisfying the functional relation, $f(x+y) \cdot f(x-y) = f^2(x)$, where x and y are independent variables and $f(0) \neq 0$. $f(x) = Ae^{kx}$

76 Let f be a function such that
 $f(x+f(y)) = f(f(x)) + f(y) \quad \forall x, y \in \mathbb{R}$
 and $f(h) = h$ for $0 < h < \epsilon$ where ϵ is a small positive quantity. Determine $f'(x)$ and $f(x)$.

77 Let $f(x)$ be a thrice differentiable function satisfying $f(x+y) = f(x-y) + y[f'(x+y) + f'(x-y)]$ where $f(0) = 0$, $f'(0) = 1$, and $f(1) = 2$. Find $f(x)$. $f(x) = x^2 + x$

78 Let $f(x)$ be a positive differentiable function satisfying
 $f\left(\frac{x+y}{2}\right) = \sqrt{f(x)f(y)} \quad \forall x, y \in \mathbb{R}$, where $f'(0) = 2$. $f(x) = e^{2x}$
 Find $f(x)$.

79 If $f: \mathbb{R} \rightarrow \mathbb{R}$ is a function defined by $f(x) = |x| \cos\left(\frac{2x-1}{2}\right)\pi$, where $|x|$ denotes the greatest integer function, then f is

- 1) continuous for every real x .
- 2) discontinuous only at $x = 0$.
- 3) discontinuous only at non-zero integral values of x .
- 4) continuous only at $x = 0$.

80) If $[.]$ denotes the greatest integer function then the number of points where $f(x) = [x] + \left[x + \frac{1}{3}\right] + \left[x + \frac{2}{3}\right]$ is discontinuous for $x \in (0, 3)$ are

- 1) 2 2) 9 3) 8 4) 10

81) Let f and g be real valued functions defined on interval $(-1, 1)$ such that $g'(x)$ is continuous, $g(0) \neq 0, g'(0) = 0, g''(0) \neq 0$, and $f(x) = g(x)\sin x$.

Statement 1 : $\lim_{x \rightarrow 0} [g(x)\cot x - g(0)\operatorname{cosec} x] = f''(0)$ b

Statement 2 : $f'(0) = g(0)$

- a) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for Statement 1.
 b) Statement 1 is true, Statement 2 is true; Statement 2 is NOT a correct explanation for Statement 1
 c) Statement 1 is true, statement 2 is false.
 d) Statement 1 is false, Statement 2 is true.

82) $f(x) = \max\{x/n, \sin \pi x\}, n \in N$, has maximum points of non-differentiability for $x \in (0, 4)$. Then n cannot be

- 1) 4 2) 2 3) 5 4) 6

83) If $f(x) = (x^2 - 4)^2 e^x - 6x^3 + 11x - 6 + \frac{x}{1 + |x|}$, then the set of points at which the function $f(x)$ is not differentiable is

- a) $\{-2, 2, 1, 3\}$ b) $\{-2, 0, 3\}$ c) $\{-2, 2, 0\}$ d) $\{1, 3\}$

84) If the function $f(x) = \left[\frac{(x-2)^3}{a}\right] \sin(x-2) + a \cos(x-2)$ (where $[x]$ denotes the greatest integer function) is continuous and differentiable in $(4, 6)$, then 'a' can be

- a) 61 b) 63 c) 65 d) 66

85) If a is the number of points of continuity of $f(x) = \begin{cases} x-1, & x \text{ is rational} \\ x^2-x-2, & x \text{ is irrational} \end{cases}$, b is the number of points of discontinuity of $f(x) = \operatorname{sgn}(x^3 - 3x + 1)$, c is the number of points of non-differentiability of $f(x) = (\log x) |x^2 - 4x + 3| + 2(x-2)^{1/3}$, and d is the number of points where the graph of $f(x) = (\log x) |x^2 - 4x + 3| + 2(x-2)^{1/3}$ has a sharp turn then the value of $a + b + c + d$ is 8

86) The number of points of discontinuity of the function $f(x) = \left[\frac{6x}{\pi}\right] \cos\left[\frac{3x}{\pi}\right]$, in the interval $\left(\frac{\pi}{10}, \frac{11\pi}{10}\right)$ is $([.]$ represents the greatest integer function) 6

87 Let $f: [a, b] \rightarrow [1, \infty)$ be a continuous function and let $g: \mathbb{R} \rightarrow \mathbb{R}$ be defined as

$$g(x) = \begin{cases} 0 & \text{if } x < a, \\ \int_a^x f(t) dt & \text{if } a \leq x \leq b, \\ \int_a^b f(t) dt & \text{if } x > b \end{cases}$$

Then

- (A) $g(x)$ is continuous but not differentiable at a
 (B) $g(x)$ is differentiable on \mathbb{R}
 (C) $g(x)$ is continuous but not differentiable at b
 (D) $g(x)$ is continuous and differentiable at either a or b but not both

88 Let $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be respectively given by $f(x) = |x| + 1$ and $g(x) = x^2 + 1$. Define $h: \mathbb{R} \rightarrow \mathbb{R}$ by

$$h(x) = \begin{cases} \max \{f(x), g(x)\} & \text{if } x \leq 0 \\ \min \{f(x), g(x)\} & \text{if } x > 0 \end{cases}$$

Then number of points at which $h(x)$ is not differentiable is 3

89 Let $f_1: \mathbb{R} \rightarrow \mathbb{R}, f_2: [0, \infty) \rightarrow \mathbb{R}, f_3: \mathbb{R} \rightarrow \mathbb{R}$ and $f_4: \mathbb{R} \rightarrow [0, \infty)$ be defined by

$$f_1(x) = \begin{cases} |x| & \text{if } x < 0 \\ e^x & \text{if } x \geq 0 \end{cases}; f_2(x) = x^2; f_3(x) = \begin{cases} \sin x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases} \text{ and } f_4(x) = \begin{cases} f_2(f_1(x)) & \text{if } x < 0 \\ f_3(f_1(x)) - 1 & \text{if } x \geq 0 \end{cases}$$

	List - I	List - II
(P)	f_1 is	(1) onto but not one-one
(Q)	f_2 is	(2) neither continuous nor one-one
(R)	$f_3 \circ f_1$ is	(3) differentiable but not one-one
(S)	f_4 is	(4) continuous and one-one

Codes:

	P	Q	R	S
(A)	3	1	4	2
(B)	1	3	4	2
(C)	3	1	2	4
(D)	1	3	2	4

90 Let $g: \mathbb{R} \rightarrow \mathbb{R}$ be a differential function with $g(0) = 0, g'(0) = 0$ and $g'(1) \neq 0$.

$$\text{Let } f(x) = \begin{cases} \frac{x}{|x|} g(x), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

and $h(x) = e^{2x}$ for all $x \in \mathbb{R}$. Let $(f \circ h)(x)$ denote $f(h(x))$ and $(h \circ f)(x)$ denote $h(f(x))$. Then which of the following is (are) true?

- (A) f is differentiable at $x = 0$
 (B) h is differentiable at $x = 0$
 (C) $f \circ h$ is differentiable at $x = 0$
 (D) $h \circ f$ is differentiable at $x = 0$

Let $a, b \in \mathbb{R}$ and $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = a \cos(|x^3 - x|) + b|x| \sin(|x^3 + x|)$. Then f is

97. (A) differentiable at $x = 0$ if $a = 0$ and $b = 1$
 (B) differentiable at $x = 1$ if $a = 1$ and $b = 0$
 (C) NOT differentiable at $x = 0$ if $a = 1$ and $b = 0$
 (D) NOT differentiable at $x = 1$ if $a = 1$ and $b = 1$

(A, B)

98. Let $f: [-\frac{1}{2}, 2] \rightarrow \mathbb{R}$ and $g: [-\frac{1}{2}, 2] \rightarrow \mathbb{R}$ be functions defined by $f(x) = [x^2 - 3]$ and $g(x) = |x| f(x) + 4x - 7 f(x)$, where $[y]$ denotes the greatest integer less than or equal to y for $y \in \mathbb{R}$. Then

- (A) f is discontinuous exactly at three points in $[-\frac{1}{2}, 2]$
 (B) f is discontinuous exactly at four points in $[-\frac{1}{2}, 2]$
 (C) g is NOT differentiable exactly at four points in $[-\frac{1}{2}, 2]$
 (D) g is NOT differentiable exactly at five points in $[-\frac{1}{2}, 2]$

(B, C)

99. Let $[x]$ be the greatest integer less than or equals to x . Then, at which of the following point(s) the function $f(x) = x \cos(\pi(x + [x]))$ is discontinuous?

- (A) $x = -1$
 (B) $x = 0$
 (C) $x = 2$
 (D) $x = 1$

A, C, D

Let $f(x) = x - \log_a x - x \log_a x$, $x \in (0, \infty)$.

- Column 1 contains information about zeros of $f(x)$, $f'(x)$ and $f''(x)$.
- Column 2 contains information about the limiting behavior of $f(x)$, $f'(x)$ and $f''(x)$ at infinity.
- Column 3 contains information about increasing/decreasing nature of $f(x)$ and $f'(x)$.

Column 1	Column 2	Column 3
(I) $f(x) = 0$ for some $x \in (1, e^2)$	(P) $\lim_{x \rightarrow \infty} f(x) = 0$	(P) f is increasing in $(0, 1)$
(II) $f'(x) = 0$ for some $x \in (1, e)$	(Q) $\lim_{x \rightarrow \infty} f'(x) = -\infty$	(Q) f is decreasing in (e, e^2)
(III) $f''(x) = 0$ for some $x \in (0, 1)$	(R) $\lim_{x \rightarrow \infty} f''(x) = -\infty$	(R) f' is increasing in $(0, 1)$
(IV) $f''(x) = 0$ for some $x \in (1, e)$	(S) $\lim_{x \rightarrow \infty} f''(x) = 0$	(S) f' is decreasing in (e, e^2)

Q.52 Which of the following options is the only CORRECT combination?
 [A] (IV) (i) (S)
 [B] (I) (ii) (R)
 [C] (III) (iv) (P)
 [D] (II) (iii) (S)

Sol. D

Q.53 Which of the following options is the only CORRECT combination?
 [A] (III) (iii) (R)
 [B] (I) (i) (P)
 [C] (IV) (iv) (S)
 [D] (II) (ii) (Q)

Sol. D

Q.54 Which of the following options is the only INCORRECT combination?
 [A] (II) (iii) (P)
 [B] (IV) (iv) (Q)
 [C] (I) (iii) (P)
 [D] (III) (i) (R)

Sol. D

Q.7. For every twice differentiable function $f: \mathbb{R} \rightarrow [-2, 2]$ with $(f(0))^2 + (f'(0))^2 = 85$, which of the following statement(s) is (are) TRUE?

- * (A) There exist $r, s \in \mathbb{R}$, where $r < s$, such that f is one-one on the open interval (r, s)
 (B) There exists $x_0 \in (-4, 0)$ such that $|f'(x_0)| \leq 1$
 (C) $\lim_{x \rightarrow \infty} f(x) = 1$
 (D) There exist $\alpha \in (-4, 4)$ such that $f(\alpha) + f''(\alpha) = 0$ and $f'(\alpha) \neq 0$

A, B, D

Q.8. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function with $f(0) = 1$ and satisfying the equation $f(x+y) = f(x)f'(y) + f'(x)f(y)$ for all $x, y \in \mathbb{R}$. Then, the value of $\log_e(f(4))$ is 2.

Let $f_1: \mathbb{R} \rightarrow \mathbb{R}$, $f_2: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \rightarrow \mathbb{R}$, $f_3: (-1, e^{e^2} - 2) \rightarrow \mathbb{R}$ and $f_4: \mathbb{R} \rightarrow \mathbb{R}$ be functions defined by

(i) $f_1(x) = \sin\left(\sqrt{1 - e^{-x^2}}\right)$.

(ii) $f_2(x) = \begin{cases} \frac{|\sin x|}{\tan^{-1} x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$, where the inverse trigonometric function $\tan^{-1} x$ assumes values in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

(iii) $f_3(x) = [\sin(\log_e(x+2))]$, where, for $t \in \mathbb{R}$, $[t]$ denotes the greatest integer less than or equal to t .

(iv) $f_4(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$.

LIST-I		LIST-II	
P.	The function f_1 is	1.	NOT continuous at $x = 0$
Q.	The function f_2 is	2.	continuous at $x = 0$ and NOT differentiable at $x = 0$
R.	The function f_3 is	3.	differentiable at $x = 0$ and its derivative is NOT continuous at $x = 0$
S.	The function f_4 is	4.	differentiable at $x = 0$ and its derivative is continuous at $x = 0$

The correct option is :

- (A) P \rightarrow 2; Q \rightarrow 3; R \rightarrow 1; S \rightarrow 4
 (B) P \rightarrow 4; Q \rightarrow 1; R \rightarrow 2; S \rightarrow 3
 (C) P \rightarrow 4; Q \rightarrow 2; R \rightarrow 1; S \rightarrow 3
 (D) P \rightarrow 2; Q \rightarrow 1; R \rightarrow 4; S \rightarrow 3

D

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be given by $f(x) = \begin{cases} x^5 + 5x^4 + 10x^3 + 10x^2 + 3x + 1, & x < 0; \\ x^2 - x + 1, & 0 \leq x < 1; \\ \frac{2}{3}x^3 - 4x^2 + 7x - \frac{8}{3}, & 1 \leq x < 3; \\ (x-2)\log_e(x-2) - x + \frac{10}{3}, & x \geq 3. \end{cases}$

Then which of the following options is/are correct?

- A. f has a local maximum at $x = 1$
 B. f is NOT differentiable at $x = 1$
 C. f is onto
 D. f is increasing on $(-\infty, 0)$

A, B, C

101 Let the function $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^3 - x^2 + (x-1)\sin x$ and let $g: \mathbb{R} \rightarrow \mathbb{R}$ be an arbitrary function. Let $fg: \mathbb{R} \rightarrow \mathbb{R}$ be the product function defined by $(fg)(x) = f(x)g(x)$. Then which of the following statements is/are TRUE?

- (A) If g is continuous at $x = 1$, then fg is differentiable at $x = 1$
 (B) If g is differentiable at $x = 1$, then g is continuous at $x = 1$
 (C) If g is differentiable at $x = 1$, then fg is differentiable at $x = 1$
 (D) If fg is differentiable at $x = 1$, then g is differentiable at $x = 1$
- A, C

102 Let $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be functions satisfying

$$f(x+y) = f(x) + f(y) + f(x)f(y) \text{ and } f(x) = xg(x)$$

for all $x, y \in \mathbb{R}$. If $\lim_{x \rightarrow 0} g(x) = 1$, then which of the following statements is/are TRUE?

- (A) f is differentiable at every $x \in \mathbb{R}$
 (B) If $g(0) = 1$, then g is differentiable at every $x \in \mathbb{R}$
 (C) The derivative $f'(1)$ is equal to 1
 (D) The derivative $f'(0)$ is equal to 1

A, B, D

103 Let the functions $f: (-1, 1) \rightarrow \mathbb{R}$ and $g: (-1, 1) \rightarrow (-1, 1)$ be defined by

$$f(x) = |2x-1| + |2x+1| \text{ and } g(x) = x - [x]$$

where $[x]$ denotes the greatest integer less than or equal to x . Let $f \circ g: (-1, 1) \rightarrow \mathbb{R}$ be the composite function defined by $(f \circ g)(x) = f(g(x))$. Suppose c is the number of points in the interval $(-1, 1)$ at which $f \circ g$ is NOT continuous, and suppose d is the number of points in the interval $(-1, 1)$ at which $f \circ g$ is NOT differentiable. Then the value of $c + d$ is _____

4

104 Let the function f, g and h be defined as follows:

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & \text{for } -1 \leq x \leq 1 \text{ and } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$$

$$g(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{for } -1 \leq x \leq 1 \text{ and } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$$

$$h(x) = |x|^3 \text{ for } -1 \leq x \leq 1$$

Which of these functions are differentiable at $x = 0$?

- (A) f and g only (B) f and h only (C) g and h only (D) none

105 Let $f(x) = \lim_{n \rightarrow \infty} \ln \left(\sqrt{e^{\cos x}} \sqrt[3]{e^{3 \cos x}} \sqrt[5]{e^{5 \cos x}} \dots \sqrt[2n+1]{e^{(2n+1) \cos x}} \right)$. If $g(x) = \left\lfloor \frac{1}{3} f(x) \right\rfloor$, then find

the number of points in $[0, 2\pi]$ where $g(x)$ is discontinuous. (4)

[Note: $\lfloor y \rfloor$ denotes greatest integer function less than or equal to y .]

106 Let $f(x) = \operatorname{sgn}(x^2 - 4x + 4 + k^2)$, $x \in \mathbb{R}$. If $f(x)$ is discontinuous at exactly one point then the value of $(\tan^{-1} k + \cos^{-1} k + \operatorname{cosec}^{-1}(2k-1))$ is equal to $\frac{m\pi}{2}$ where m is a

whole number. Find the value of m . (0)

107 The function $f(x) = [x]^2 - [x^2]$ (where, $[x]$ is the greatest integer less than or equal to x), is discontinuous at

- (a) all integers
 (b) all integers except 0 and 1
 (c) all integers except 0
 (d) all integers except 1

108 For every integer n , let a_n and b_n be real numbers. Let function $f: R \rightarrow R$ be given by

$$f(x) = \begin{cases} a_n + \sin \pi x, & \text{for } x \in [2n, 2n + 1] \\ b_n + \cos \pi x, & \text{for } x \in (2n - 1, 2n) \end{cases}$$

for all integers n .

If f is continuous, then which of the following hold(s) for all n ?

- (a) $a_{n-1} - b_{n-1} = 0$
- (b) $a_n - b_n = 1$
- (c) $a_n - b_{n+1} = 1$
- (d) $a_{n-1} - b_n = -1$

109 If $f(x) = \frac{1}{2}x - 1$, then on the interval $[0, \pi]$

- (a) $\tan f(x)$ and $1/f(x)$ are both continuous
- (b) $\tan f(x)$ and $1/f(x)$ are both discontinuous
- (c) $\tan f(x)$ and $f^{-1}(x)$ are both continuous
- (d) $\tan f(x)$ is continuous but $1/f(x)$ is not continuous

110 The following functions are continuous on $(0, \pi)$

- (a) $\tan x$
- (b) $\int_0^x t \sin \frac{1}{t} dt$ *not of form*
- (c) $\begin{cases} 1, & 0 \leq x \leq 3\pi/4 \\ 2 \sin \frac{2}{9}x, & \frac{3\pi}{4} < x < \pi \end{cases}$
- (d) $\begin{cases} x \sin x, & 0 < x \leq \pi/2 \\ \frac{\pi}{2} \sin(\pi + x), & \frac{\pi}{2} < x < \pi \end{cases}$

111 Let $[x]$ denotes the greatest integer less than or equal to x . If $f(x) = [x \sin \pi x]$, then $f(x)$ is

- (a) continuous at $x = 0$
- (b) continuous in $(-1, 0)$
- (c) differentiable at $x = 1$
- (d) differentiable in $(-1, 1)$

112 For the function $f(x) = x \cos \frac{1}{x}, x \geq 1$,

- (a) for atleast one x in the interval $[1, \infty), f(x+2) - f(x) < 2$
- (b) $\lim_{x \rightarrow \infty} f'(x) = 1$
- (c) for all x in the interval $[1, \infty), f(x+2) - f(x) > 2$
- (d) $f'(x)$ is strictly decreasing in the interval $[1, \infty)$

113 Let $f(x) = \begin{cases} x + a, & \text{if } x < 0 \\ |x - 1|, & \text{if } x \geq 0 \end{cases}$ and $g(x) = \begin{cases} x + 1, & \text{if } x < 0 \\ (x - 1)^2 + b, & \text{if } x \geq 0 \end{cases}$ $a = 1, b = \infty$

where, a and b are non-negative real numbers. Determine the composite function $g \circ f$. If $(g \circ f)(x)$ is continuous for all real x determine the values of a and b . Further, for these values of a and b , is $g \circ f$ differentiable at $x = 0$? Justify your answer.

114 Let $f: R \rightarrow R$ be differentiable at $c \in R$ and $f(c) = 0$. If

- (a) $g(x) = |f(x)|$ then at $x = c$, g is not differentiable
- (b) differentiable if $f'(c) \neq 0$
- (c) not differentiable if $f'(c) = 0$
- (d) differentiable if $f'(c) = 0$

115 Let $f(x) = 15 - |x - 10|$; $x \in \mathbb{R}$. Then, the set of all values of x , at which the function, $g(x) = f(f(x))$ is not differentiable, is

- (a) $\{5, 10, 15, 20\}$ (b) $\{5, 10, 15\}$
 (c) $\{10\}$ (d) $\{10, 15\}$

116 Let $f(x) = \begin{cases} -1, & -2 \leq x < 0 \\ x^2 - 1, & 0 \leq x \leq 2 \end{cases}$ and

$g(x) = |f(x)| + f(|x|)$. Then, in the interval $(-2, 2)$, g is

- (a) not differentiable at one point
 (b) not differentiable at two points
 (c) differentiable at all points
 (d) not continuous

117 Let $f: (-1, 1) \rightarrow \mathbb{R}$ be a function defined by $f(x) = \max\{-|x|, -\sqrt{1-x^2}\}$. If K be the set of all points at which f is not differentiable, then K has exactly

- (a) three elements (b) five elements
 (c) two elements (d) one element

118 Let $f(x) = \begin{cases} \max\{|x|, x^2\}, & |x| \leq 2 \\ 8 - 2|x|, & 2 < |x| \leq 4 \end{cases}$

Let S be the set of points in the interval $(-4, 4)$ at which f is not differentiable. Then, S

- (a) equals $\{-2, -1, 0, 1, 2\}$ (b) equals $\{-2, 2\}$
 (c) is an empty set (d) equals $\{-2, -1, 1, 2\}$

119 Let f be a differentiable function from \mathbb{R} to \mathbb{R} such that

$$|f(x) - f(y)| \leq 2|x - y|^{\frac{3}{2}}, \text{ for all } x, y \in \mathbb{R} \text{ If } f(0) = 1, \text{ then}$$

- $\int_0^1 f^2(x) dx$ is equal to
 (a) 2 (b) $\frac{1}{2}$ (c) 1 (d) 0

120 Let $S = \{t \in \mathbb{R} : f(x) = |x - \pi|(e^{|t|} - 1)\sin|x| \text{ is not differentiable at } t\}$. Then, the set S is equal to

- (a) \emptyset (an empty set) (b) $\{0\}$
 (c) $\{\pi\}$ (d) $\{0, \pi\}$

121 For $x \in \mathbb{R}$, $f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then

- (a) g is not differentiable at $x = 0$
 (b) $g'(0) = \cos(\log 2)$
 (c) $g'(0) = -\cos(\log 2)$
 (d) g is differentiable at $x = 0$ and $g'(0) = -\sin(\log 2)$

122 Let $f(x) = \begin{cases} x^2 \cos \frac{\pi}{x}, & x \neq 0, x \in \mathbb{R} \\ 0, & x = 0 \end{cases}$, then f is

- (a) differentiable both at $x = 0$ and at $x = 2$
 (b) differentiable at $x = 0$ but not differentiable at $x = 2$
 (c) not differentiable at $x = 0$ but differentiable at $x = 2$
 (d) differentiable neither at $x = 0$ nor at $x = 2$

123 If f is a differentiable function satisfying

$$f\left(\frac{1}{n}\right) = 0, \forall n \geq 1, n \in \mathbb{I}, \text{ then}$$

- (a) $f(x) = 0, x \in (0, 1]$
 (b) $f'(0) = 0 = f(0)$
 (c) $f(0) = 0$ but $f'(0)$ not necessarily zero
 (d) $|f(x)| \leq 1, x \in (0, 1]$

124 For a real number y , let $[y]$ denotes the greatest integer less than or equal to y . Then, the function

$$f(x) = \frac{\tan \pi [(x - \pi)]}{1 + [x]^2} \text{ is}$$

- (a) discontinuous at some x
 (b) continuous at all x , but the derivative $f'(x)$ does not exist for some x
 (c) $f'(x)$ exists for all x , but the derivative $f''(x)$ does not exist for some x
 (d) $f'(x)$ exists for all x

125 Let $f: \mathbb{R} \rightarrow \mathbb{R}$, $g: \mathbb{R} \rightarrow \mathbb{R}$ and $h: \mathbb{R} \rightarrow \mathbb{R}$ be differentiable functions such that $f(x) = x^3 + 3x + 2$, $g(f(x)) = x$ and $h(g(g(x))) = x$ for all $x \in \mathbb{R}$. Then,

- (a) $g'(2) = \frac{1}{15}$
 (b) $h'(1) = 666$
 (c) $h(0) = 16$
 (d) $h(g(3)) = 36$

- 126 The function $f(x) = \begin{cases} |x-3|, & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$ is
 (a) continuous at $x=1$ (b) differentiable at $x=1$
 (c) discontinuous at $x=1$ (d) differentiable at $x=3$

In the following, $[x]$ denotes the
 than or equal to x .

127

Column I	
A. $x x $	p. continuous in $(-1, 1)$
B. $\sqrt{ x }$	q. differentiable in $(-1, 1)$
C. $x + [x]$	r. strictly increasing $(-1, 1)$
D. $ x-1 + x+1 $ in $(-1, 1)$	s. not differentiable atleast at one point in $(-1, 1)$

A → p, q, r
 B → p, q
 C → r, s
 D → p, q

- 128 For the function $f(x) = \begin{cases} \frac{x}{1+e^{3/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$
 the derivative from the right, $f'(0^+) = 0$ and the
 derivative from the left, $f'(0^-) = \dots$

- 129 Let $f(x) = \begin{cases} (x-1)^2 \sin \frac{1}{(x-1)} - |x|, & \text{if } x \neq 1 \\ -1, & \text{if } x = 1 \end{cases}$ be a real
 valued function. Then, the set of points, where $f(x)$ is
 not differentiable, is

130 $f(x) = \begin{cases} b \sin^{-1} \left(\frac{x+c}{2} \right), & -\frac{1}{2} < x < 0 \\ \frac{1}{2}, & x = 0 \\ \frac{e^{ax^2} - 1}{x}, & 0 < x < \frac{1}{2} \end{cases}$

If $f(x)$ is differentiable at $x=0$ and $|c| < \frac{1}{2}$, then find the
 value of a and prove that $64b^2 = (4-c^2)$.

- 131 Let $f(x) = \begin{cases} x e^{-\left(\frac{1}{|x|} + \frac{1}{x}\right)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$
 Test whether
 (i) $f(x)$ is continuous at $x=0$
 (ii) $f(x)$ is differentiable at $x=0$

132 Let $f(x)$ be defined in the interval $[-2, 2]$ such that

$$f(x) = \begin{cases} -1, & -2 \leq x \leq 0 \\ x-1, & 0 < x \leq 2 \end{cases}$$

and $g(x) = f(|x|) + |f(x)|$
 Test the differentiability of $g(x)$ in $(-2, 2)$.

- 133 Let $f(x) = x^3 - x^2 + x + 1$
 and $g(x) = \begin{cases} \max\{f(t); 0 \leq t \leq x\}, & 0 \leq x \leq 1 \\ 3-x, & 1 < x \leq 2 \end{cases}$
 Discuss the continuity and differentiability of the
 function $g(x)$ in the interval $(0, 2)$.

- 134 $f(x) = \begin{cases} (x-e)2^{-\frac{1}{(x-e)}}, & x \neq e \\ 0, & x = e \end{cases}$ Discuss the differentiability of
 $f(x) = \sin^{-1} \frac{2x}{1+x^2}$.

- 136 Let $y_n(x) = x^2 + \frac{x^2}{1+x^2} + \frac{x^2}{(1+x^2)^2} + \dots + \frac{x^2}{(1+x^2)^{n-1}}$
 and $y(x) = \lim_{n \rightarrow \infty} y_n(x)$
 Discuss the continuity of $y_n(x)$ ($n = 1, 2, 3, \dots, n$) and $y(x)$ at $x=0$.