A vertical line passing through the point (h, 0) intersects the ellipse $\frac{x^2}{4}+\frac{y^2}{3}=1$ at the points P and Q. Let the tangents to the ellipse at P and Q meet at the point R. If $\Delta(h)$ = area of the triangle PQR, $\Delta_1 = \max_{1/2\le h\le 1} \Delta(h)$ and $\Delta_2 = \min_{1/2\le h\le 1} \Delta(h)$, then $\frac{8}{\sqrt{5}}\Delta_1 - 8\Delta_2 =$ ________

Sol. **(9)**

For a ∈ R (the set of all real numbers), $a \ne -1$, $\lim_{n\to\infty}\frac{\left(1^a+2^a+\ldots+n^a\right)}{(n+1)^{a-1}\left[(na+1)+(na+2)+\ldots+(na+n)\right]}=\frac{1}{60}$.

Then a =

(A) 5 (B) 7

(C) $\frac{-15}{2}$ (D) $\frac{-17}{2}$

Sol. **(B, D)**

The largest value of the non-negative integer a for which $\lim_{x\to 1}\left\{\frac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}\right\}^{\frac{1-x}{1-\sqrt{x}}}=\frac{1}{4}$ is ______

Let m and n be two positive integers greater than 1. If

$$\lim_{\alpha\to 0}\left(\frac{e^{\cos(\alpha^n)}-e}{\alpha^m}\right)=-\left(\frac{e}{2}\right)$$

then the value of $\frac{m}{n}$ is

Let f : $\mathbb{R}\to\mathbb{R}$ be a continuous odd function, which vanishes exactly at one point and $f(1)=\frac{1}{2}$. Suppose that $F(x)=\int_{-1}^{x} f(t)\,dt$ for all $x \in [-1, 2]$ and $G(x)=\int_{-1}^{x} t\left|f(f(t))\right|dt$ for all $x\in[-1, 2]$. If $\lim_{x\to 1}\frac{F(x)}{G(x)}=\frac{1}{14}$, then the value of $f\left(\frac{1}{2}\right)$ is

Let f:(0, ∞) → $\mathbb{R}$ be a differentiable function such that $f'(x)=2-\frac{f(x)}{x}$ for all x ∈ (0, ∞) and f(1) ≠ 1. Then

(A) $\lim_{x\to 0+} f'\left(\frac{1}{x}\right)=1$ (B) $\lim_{x\to 0+} xf\left(\frac{1}{x}\right)=2$

(C) $\lim_{x\to 0+} x^2 f'(x)=0$ (D) $|f(x)|\le 2$ for all x ∈(0, 2)

Sol. **(A)**

Let α, β ∈ $\mathbb{R}$ be such that $\lim_{x\to 0}\frac{x^2\sin(\beta x)}{\alpha x-\sin x}=1$. Then 6(α + β) equals

(7)

Let $f(x) = \lim_{n\to\infty}\left(\frac{n^n(x+n)\left(x+\frac{n}{2}\right)\ldots\left(x+\frac{n}{n}\right)}{n!\left(x^2+n^2\right)\left(x^2+\frac{n^2}{4}\right)\ldots\left(x^2+\frac{n^2}{n^2}\right)}\right)^{\frac{x}{n}}$, for all x > 0. Then

(A) $f\left(\frac{1}{2}\right) \ge f(1)$

(B) $f\left(\frac{1}{3}\right) \le f\left(\frac{2}{3}\right)$

(C) $f'(2) \le 0$

(D) $\frac{f'(3)}{f(3)} \ge \frac{f'(2)}{f(2)}$

(B, C)

Let $f:\mathbb{R}\to(0,\infty)$ and $g:\mathbb{R}\to\mathbb{R}$ be twice differentiable functions such that f″ and g″ are continuous functions on $\mathbb{R}$. Suppose $f'(2) = g(2) = 0$, $f''(2) \ne 0$ and $g'(2) \ne 0$. If $\lim_{x\to 2}\frac{f(x)g(x)}{f'(x)g'(x)} = 1$, then

(A) f has a local minimum at x = 2

(B) f has a local maximum at x = 2

(C) $f''(2) > f(2)$

(D) $f(x) - f''(x) = 0$ for at least one $x \in \mathbb{R}$

(A, D)

Let f: R → R be a differentiable function such that $f(0) = 0$, $f\left(\frac{\pi}{2}\right) = 3$ and $f'(0) = 1$. If

$$g(x) = \int_x^{\pi/2}\left[f'(t)\operatorname{cosec} t - \cot t \operatorname{cosec} t\, f(t)\right]dt$$

for $x \in \left(0, \frac{\pi}{2}\right]$, then $\lim_{x\to 0} g(x) =$

2

Let $f(x) = \frac{1 - x(1+|1-x|)}{|1-x|}\cos\left(\frac{1}{1-x}\right)$ for $x \ne 1$. Then

[A] $\lim_{x\to 1^-} f(x) = 0$

[B] $\lim_{x\to 1^-} f(x)$ does not exist

[C] $\lim_{x\to 1^+} f(x) = 0$

[D] $\lim_{x\to 1^+} f(x)$ does not exist

A,D

For each positive integer n, let

$$y_n = \frac{1}{n}\left((n+1)(n+2)\ldots(n+n)\right)^{1/n}.$$

For x ∈ R, let [x] be the greatest integer less than or equal to x. If $\lim_{n\to\infty} y_n = L$, then the value of [L] is _____ .

1

For any positive integer n, define $f_n : (0, \infty) \to R$ as

$$f_n(x) = \sum_{j=1}^{n} \tan^{-1}\left(\frac{1}{1+(x+j)(x+j-1)}\right) \text{ for all } x \in (0, \infty)$$

$\left(\text{Here, the inverse trigonometric function } \tan^{-1}x \text{ assumes values in } \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right)$

Then, which of the following statement(s) is (are) TRUE ?

(A) $\sum_{j=1}^{5} \tan^2(f_j(0)) = 55$

(B) $\sum_{j=1}^{10} (1+f_j'(0))\sec^2(f_j(0)) = 10$

(C) For any fixed positive integer n, $\lim_{x\to\infty} \tan(f_n(x)) = \frac{1}{n}$

(D) For any fixed positive integer n, $\lim_{x\to\infty} \sec^2(f_n(x)) = 1$

D

Let $f : (0, \pi) \to R$ be a twice differentiable function such that

$$\lim_{t\to x} \frac{f(x)\sin t - f(t)\sin x}{t-x} = \sin^2 x \text{ for all } x \in (0, \pi)$$

If $f\left(\frac{\pi}{6}\right) = -\frac{\pi}{12}$, then which of the following statement(s) is (are) TRUE ?

(A) $f\left(\frac{\pi}{4}\right) = \frac{\pi}{4\sqrt{2}}$

(B) $f(x) < \frac{x^4}{6} - x^2$ for all $x \in (0, \pi)$

(C) There exists $\alpha \in (0, \pi)$ such that $f'(\alpha) = 0$

(D) $f''\left(\frac{\pi}{2}\right) + f\left(\frac{\pi}{2}\right) = 0$

B, C, D

For non–negative integers n, let

$$f(n)=\frac{\sum_{k=0}^{n}\sin\left(\frac{k+1}{n+2}\pi\right)\sin\left(\frac{k+2}{n+2}\pi\right)}{\sum_{k=0}^{n}\sin^2\left(\frac{k+1}{n+2}\pi\right)}$$

Assuming $\cos^{-1}x$ takes values in $[0, \pi]$, which of the following options is/are correct?

A. $f(4)=\frac{\sqrt{3}}{2}$

B. If $\alpha = \tan(\cos^{-1}f(6))$, then $\alpha^2 + 2\alpha - 1 = 0$

C. $\sin(7\cos^{-1}f(5))| = 0$

D. $\lim_{n\to\infty} f(n)=\frac{1}{2}$

A, B, C

For $a \in \mathbb{R}\ |a| > 1$, let

$$\lim_{n\to\infty}\left(\frac{1+\sqrt[3]{2}+....+\sqrt[3]{n}}{n^{7/3}\left(\frac{1}{(an+1)^2}+\frac{1}{(an+2)^2}+...+\frac{1}{(an+n)^2}\right)}\right)=54$$

Then the possible value(s) of a is/are

A. 8

B. –6

C. 7

D. –9

A, D

Let $f: \mathbb{R} \to \mathbb{R}$ be a function. We say that f has

PROPERTY 1 if $\lim_{h\to 0}\frac{f(h)-f(0)}{\sqrt{|h|}}$ exists and is finite, and

PROPERTY 2 if $\lim_{h\to 0}\frac{f(h)-f(0)}{h^2}$ exists and if finite.

Then which of the following options is/are correct?

A. $f(x) = x|x|$ has PROPERTY 2

B. $f(x) = \sin x$ has PROPERTY 2

C. $f(x) = |x|$ has PROPERTY 1

D. $f(x) = x^{2/3}$ has PROPERTY 1

C, D

Let e denote the base of the natural logarithm. The value of the real number a for which the right hand limit

$$\lim_{x\to 0^+}\frac{(1-x)^{1/x}-e^{-1}}{x^a}$$

is equal to a nonzero real number, is _____

1

Let the functions $f:(-1, 1) \to \mathbb{R}$ and $g: (-1, 1) \to (-1, 1)$ be defined by

$f(x) = |2x - 1| + |2x + 1|$ and $g(x) = x - [x]$,

where $[x]$ denotes the greatest integer less than or equal to x. Let $f \circ g : (-1, 1) \to \mathbb{R}$ be the composite function defined by $(f \circ g)(x) = f(g(x))$. Suppose c is the number of points in the interval $(-1,1)$ at which fog is **NOT** continuous, and suppose d is the number of points in the interval $(-1,1)$ at which fog is **NOT** differentiable. Then the value of $c + d$ is _____

4

The value of the limit

$$\lim_{x \to \frac{\pi}{2}} \frac{4\sqrt{2}(\sin 3x + \sin x)}{\left(2\sin 2x \sin\frac{3x}{2} + \cos\frac{5x}{2}\right) - \left(\sqrt{2} + \sqrt{2}\cos 2x + \cos\frac{3x}{2}\right)}$$

is _____

8

For any positive integer n, let $S_n : (0, \infty) \to R$ be defined by

$$S_n(x) = \sum_{k=1}^{n} \cot^{-1}\left(\frac{1 + k(k+1)x^2}{x}\right),$$

where for any $x \in R$, $\cot^{-1}(x) \in (0, \pi)$ and $\tan^{-1}(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then which of the following statements is(are) **TRUE**?

(A) $S_{10}(x) = \frac{\pi}{2} - \tan^{-1}\left(\frac{1 + 11x^2}{10x}\right)$, for all $x > 0$

(B) $\lim_{n \to \infty} \cot(S_n(x)) = x$, for all $x > 0$

(C) The equation $S_3(x) = \frac{\pi}{4}$ has a root in $(0, \infty)$

(D) $\tan(S_n(x)) \le \frac{1}{2}$, for all $n \ge 1$ and $x > 0$

Let $f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to R$ be a continuous function such that $f(0) = 1$ and $\int_0^{\frac{\pi}{3}} f(t)\,dt = 0$. Then which of the following statements is(are) **TRUE**?

(A) The equation $f(x) - 3\cos 3x = 0$ has at least one solution in $\left(0, \frac{\pi}{3}\right)$

(B) The equation $f(x) - 3\sin 3x = -\frac{6}{\pi}$ has at least one solution in $\left(0, \frac{\pi}{3}\right)$

(C) $\lim_{x \to 0} \frac{x\int_0^x f(t)\,dt}{1 - e^{x^2}} = -1$

(D) $\lim_{x \to 0} \frac{\sin x \int_0^x f(t)\,dt}{x^2} = -1$

A, B, C

$$\lim_{x\to\frac{3\pi}{4}} \frac{1+\sqrt[3]{\tan x}}{1-2\cos^2 x}$$

(a) $\lim_{x\to 0} \tan^{-1}\frac{a}{x^2}$ where $a \in R$

(b) Plot the graph of the function $f(x) = \lim_{t\to 0}\left(\frac{2x}{\pi}\tan^{-1}\frac{x}{t^2}\right)$

Find the sum of an infinite geometric series whose first term is the limit of the function

$f(x) = \frac{\tan x - \sin x}{\sin^3 x}$ as $x \to 0$ and whose common ratio is the limit of the function

$g(x) = \frac{1-\sqrt{x}}{(\cos^{-1}x)^2}$ as $x \to 1$. (Use of series expansion or L' Hospital's rule is not allowed.)

$$\lim_{x\to 0} \frac{8}{x^8}\left[1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}\cos\frac{x^2}{4}\right] \qquad \lim_{x\to 1} \frac{(\ln(1+x)-\ln 2)(3.4^{x-1}-3x)}{[(7+x)^{\frac{1}{3}}-(1+3x)^{\frac{1}{2}}].\sin(x-1)}$$

Let $f(x) = \frac{x}{\sin x},\ x>0$ and $g(x) = x+3,\quad x<1$

$\quad\; = 2-x,\ x \le 0 \qquad\qquad\quad = x^2-2x-2,\ 1 \le x < 2$

$\qquad\qquad\qquad\qquad\qquad\quad\;\; = x-5,\quad x \ge 2$

find LHL and RHL of $g(f(x))$ at $x = 0$ and hence find $\lim_{x\to 0} g(f(x))$.

Let $P_n = a^{P_{n-1}} - 1$, $\forall\, n = 2, 3, \ldots\ldots$ and Let $P_1 = a^x - 1$ where $a \in R^+$ then evaluate $\lim_{x\to 0}\frac{P_n}{x}$.

$$\lim_{x\to -\infty} \frac{(3x^4+2x^2)\sin\frac{1}{x}+|x|^3+5}{|x|^3+|x|^2+|x|+1}$$

If $f(x) = \ln \operatorname{cosec}(x\pi)\quad 0<x<1$ and $g(x) = \frac{2^{f(x)}+1}{3^{f(x)}+1}$ then

$\qquad = \ln \sin(2x\pi)\quad 1<x<3/2$

find $\tan^{-1}(g(1^-))$ and $\sec^{-1}(g(1^+))$.

At the end-points and the midpoint of a circular arc AB tangent lines are drawn, and the points A and B are joined with a chord. Prove that the ratio of the areas of the two triangles thus formed tends to 4 as the arc AB decreases indefinitely.

$$\lim_{x\to 0}\left[\frac{(1+x)^{1/x}}{e}\right]^{1/x} \qquad \lim_{x\to 0}\left[\sin^2\left(\frac{\pi}{2-ax}\right)\right]^{\sec^2\left(\frac{\pi}{2-bx}\right)} \qquad \lim_{x\to\infty} x^2\sin\left(\ln\sqrt{\cos\frac{\pi}{x}}\right)$$

$$\lim_{x\to\infty}\left[\cos\left(2\pi\left(\frac{x}{1+x}\right)^a\right)\right]^{x^2}\quad a \in R \qquad \left(\frac{a_1^{\frac{1}{x}}+a_2^{\frac{1}{x}}+a_3^{\frac{1}{x}}+\ldots\ldots+a_n^{\frac{1}{x}}}{n}\right)^{nx} \text{ where } a_1, a_2, a_3, \ldots\ldots a_n > 0$$

Let $f(x) = \frac{\sin^{-1}(1-\{x\}).\cos^{-1}(1-\{x\})}{\sqrt{2\{x\}}\,.\,(1-\{x\})}$ then find $\underset{x\to0^+}{\text{Lim}}\ f(x)$ and $\underset{x\to0^-}{\text{Lim}}\ f(x)$, where $\{x\}$ denotes the fractional part function.

Find the values of a, b & c so that $\underset{x\to0}{\text{Lim}}\ \frac{ae^x - b\cos x + ce^{-x}}{x.\sin x} = 2$

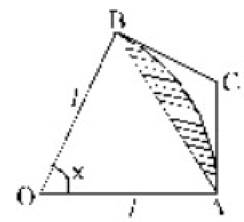
$\underset{x\to a}{\text{Lim}}\ \frac{1}{(a^2-x^2)^2}\left(\frac{a^2+x^2}{ax} - 2\sin\left(\frac{a\pi}{2}\right)\sin\left(\frac{\pi x}{2}\right)\right)$ where a is an odd integer

$\underset{y\to0}{\text{Lim}}\left[\underset{x\to\infty}{\text{Limit}}\frac{\exp\left(x\ln(1+\frac{ay}{x})\right) - \exp\left(x\ln(1+\frac{by}{x})\right)}{y}\right]$

If s_n be the sum of n terms of the series, $\sin x + \sin 2x + \sin 3x + \ldots + \sin nx$ then show that $\underset{n\to\infty}{\text{Limit}}\ \frac{s_1 + s_2 + \ldots + s_n}{n} = \frac{1}{2}\cot\frac{x}{2}\ \ (x \ne 2k\pi,\ k \in I)$

$\underset{x\to0}{\text{Lim}}\left[\frac{\ell n\,(1+x)^{1+x}}{x^2} - \frac{1}{x}\right]$ Let $P_n = \frac{2^3-1}{2^3+1}.\frac{3^3-1}{3^3+1}.\frac{4^3-1}{4^3+1}\ldots\ldots\frac{n^3-1}{n^3+1}$. Evaluate $\underset{n\to\infty}{\text{Lim}}\ P_n$

A circular arc of radius 1 subtends an angle of x radians, $0 < x < \frac{\pi}{2}$ as shown in the figure. The point C is the intersection of the two tangent lines at A & B. Let T(x) be the area of triangle ABC & let S(x) be the area of the shaded region. Compute :



(a) T(x) (b) S(x) & (c) the limit of $\frac{T(x)}{S(x)}$ as $x \to 0$.

(a) $\underset{x\to\infty}{\text{Lim}}\ \frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}}$ **(b)** $\underset{x\to\infty}{\text{Lim}}\left[\sqrt{x+\sqrt{x+\sqrt{x}}} - \sqrt{x}\right]$

If $f(n,\theta) = \prod_{r=1}^{n}\left(1-\tan^2\frac{\theta}{2^r}\right)$, then compute $\underset{n\to\infty}{\text{Lim}}\ f(n,\theta)$ $\underset{x\to0}{\text{Lim}}\ \frac{2(\tan x - \sin x) - x^3}{x^5}$

Through a point A on a circle, a chord AP is drawn & on the tangent at A a point T is taken such that AT = AP. If TP produced meet the diameter through A at Q, prove that the limiting value of AQ when P moves upto A is double the diameter of the circle.

Find a & b if : **(i)** $\underset{x\to\infty}{\text{Lim}}\left[\frac{x^2+1}{x+1} - ax - b\right] = 0$ **(ii)** $\underset{x\to-\infty}{\text{Lim}}\left[\sqrt{x^2-x+1} - ax - b\right] = 0$

$\underset{x\to0}{\text{Lim}}\ \frac{\sin(\pi\cos^2 x)}{x^2}$ equals

(A) $-\pi$ (B) π (C) $\frac{\pi}{2}$ (D) 1

Evaluate $\lim_{x\to 0} \frac{a^{\tan x} - a^{\sin x}}{\tan x - \sin x}$, $a > 0$.

The integer n for which $\lim_{x\to 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n}$ is a finite non-zero number is

(A) 1 (B) 2 (C) 3 (D) 4

If $\lim_{x\to 0} \frac{\sin(nx)[(a-n)nx - \tan x]}{x^2} = 0$ $(n > 0)$ then the value of 'a' is equal to

(A) $\frac{1}{n}$ (B) $n^2 + 1$ (C) $\frac{n^2+1}{n}$ (D) None

Find the value of $\lim_{n\to\infty} \left[\frac{2}{\pi}(n+1)\cos^{-1}\left(\frac{1}{n}\right) - n\right]$.

If α, β are the roots of the quadratic equation $ax^2 + bx + c = 0$ then $\lim_{x\to\alpha} \frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$ equals

(A) 0 (B) $\frac{1}{2}(\alpha-\beta)^2$ (C) $\frac{a^2}{2}(\alpha-\beta)^2$ (D) $-\frac{a^2}{2}(\alpha-\beta)^2$

ABC is an isosceles triangle inscribed in a circle of radius r. If $AB = AC$ & h is the altitude from A to BC and P be the perimeter of ABC then $\lim_{h\to 0} \frac{\Delta}{P^3}$ equals (where Δ is the area of the triangle)

(A) $\frac{1}{32r}$ (B) $\frac{1}{64r}$ (C) $\frac{1}{128r}$ (D) none

If [x] denotes the greatest integer $\le x$, then $\text{Limit}_{n\to\infty} \frac{1}{n^4}\left([1^3x] + [2^3x] + \ldots\ldots + [n^3x]\right)$ equals

(A) x/2 (B) x/3 (C) x/6 (D) x/4

Let the function f, g and h be defined as follows :

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & \text{for } -1 \le x \le 1 \text{ and } x \ne 0 \\ 0 & \text{for } \quad x = 0 \end{cases}$$

$$g(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{for } -1 \le x \le 1 \text{ and } x \ne 0 \\ 0 & \text{for } \quad x = 0 \end{cases}$$

$$h(x) = |x|^3 \qquad \text{for } -1 \le x \le 1$$

Which of these functions are differentiable at x = 0?

(A) f and g only (B) f and h only (C) g and h only (D) none

$\displaystyle \underset{x\to\frac{\pi}{2}}{\text{Limit}} \frac{\sin x}{\cos^{-1}\left[\frac{1}{4}(3\sin x - \sin 3x)\right]}$ where [ ] denotes greatest integer function , is

(A) $\frac{2}{\pi}$ (B) 1 (C) $\frac{4}{\pi}$ (D) does not exist

The limiting value of the function $f(x) = \dfrac{2\sqrt{2} - (\cos x + \sin x)^3}{1 - \sin 2x}$ when $x \to \frac{\pi}{4}$ is

(A) $\sqrt{2}$ (B) $\frac{1}{\sqrt{2}}$ (C) $3\sqrt{2}$ (D) $\frac{3}{\sqrt{2}}$

$\displaystyle \lim_{n\to\infty} \frac{1^2 n + 2^2(n-1) + 3^2(n-2) + \ldots + n^2 .1}{1^3 + 2^3 + 3^3 + \ldots + n^3}$ is equal to :

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{1}{2}$ (D) $\frac{1}{6}$

For the function $f(x) = \displaystyle \lim_{n\to\infty} \frac{1}{1 + n\sin^2(\pi x)}$, which of the following holds?

(A) The range of f is a singleton set (B) f is continuous on R

(C) f is discontinuous for all x ∈ I (D) f is discontinuous for some x ∈ R

$\displaystyle \lim_{x\to 1/\sqrt{2}} \frac{x - \cos(\sin^{-1} x)}{1 - \tan(\sin^{-1} x)}$ is

(A) $\frac{1}{\sqrt{2}}$ (B) $-\frac{1}{\sqrt{2}}$ (C) $\sqrt{2}$ (D) $-\sqrt{2}$

$\displaystyle \underset{x\to\infty}{\text{Limit}} \frac{\cot^{-1}\left(\sqrt{x+1}-\sqrt{x}\right)}{\sec^{-1}\left\{\left(\frac{2x+1}{x-1}\right)^{x}\right\}}$ is equal to

(A) 1 (B) 0 (C) $\pi/2$ (D) non existent

Let $f(x) = \dfrac{\ln(x^2+e^x)}{\ln(x^4+e^{2x})}$. If $\underset{x\to\infty}{\text{Limit}} f(x) = l$ and $\underset{x\to-\infty}{\text{Limit}} f(x) = m$ then :

(A) $l = m$ (B) $l = 2m$ (C) $2l = m$ (D) $l + m = 0$

$\underset{n\to\infty}{\text{Lim}} \cos\left(\pi\sqrt{n^2+n}\right)$ when n is an integer :

(A) is equal to 1 (B) is equal to -1 (C) is equal to zero (D) does not exist

$\displaystyle \underset{x\to 0}{\text{Limit}} \frac{(\sin x-\tan x)^2-(1-\cos 2x)^4+x^5}{7.(\tan^{-1}x)^7+(\sin^{-1}x)^6+3\sin^5 x}$ is equal to

(A) 0 (B) $\dfrac{1}{7}$ (C) $\dfrac{1}{3}$ (D) 1

The value of $\displaystyle \underset{x\to 0}{\text{Limit}} \frac{(\tan(\{x\}-1))\ \sin\{x\}}{\{x\}(\{x\}-1)}$ where $\{x\}$ denotes the fractional part function:

(A) is 1 (B) is tan 1 (C) is sin 1 (D) is non existent

The value of $\lim_{x\to 0} (\cos ax)^{\operatorname{cosec}^2 bx}$ is

(A) $e^{\left(-\frac{8b^2}{a^2}\right)}$ (B) $e^{\left(-\frac{8a^2}{b^2}\right)}$ (C) $e^{\left(-\frac{a^2}{2b^2}\right)}$ (D) $e^{\left(-\frac{b^2}{2a^2}\right)}$

$\displaystyle \underset{x\to\infty}{\text{Lim}} \frac{2+2x+\sin 2x}{(2x+\sin 2x)e^{\sin x}}$ is :

(A) equal to zero (B) equal to 1 (C) equal to -1 (D) non existent

Let $f(x) = \begin{cases} \dfrac{\tan^2\{x\}}{x^2 - [x]^2} & \text{for } x > 0 \\ 1 & \text{for } x = 0 \\ \sqrt{\{x\}\cot\{x\}} & \text{for } x < 0 \end{cases}$ where $[x]$ is the step up function and $\{x\}$ is the fractional part function of x, then :

(A) $\underset{x \to 0^+}{\text{Limit}}\ f(x) = 1$ (B) $\underset{x \to 0^-}{\text{Limit}}\ f(x) = 1$

(C) $\cot^{-1}\left(\underset{x \to 0^-}{\text{Limit}}\ f(x)\right)^2 = 1$ (D) f is continuous at $x = 1$.

$\underset{x \to 0}{\text{Limit}}\ \dfrac{\sin(6x^2)}{\ell n \cos(2x^2 - x)} =$

(A) 12 (B*) – 12 (C) 6 (D) –6

$\lim\limits_{x \to \pi/2} \left[\dfrac{x - \frac{\pi}{2}}{\cos x}\right]$ is : (where [.] represents greatest integer function.

(A) – 1 (B) 0 (C*) – 2 (D) does not exist

If $f(x) = \begin{cases} x^2 + 2 & , x \geq 2 \\ 1 - x & , x < 2 \end{cases}$ and $g(x) = \begin{cases} 2x & , x > 1 \\ 3 - x & , x \leq 1 \end{cases}$, evaluate $\lim\limits_{x \to 1} f(g(x))$.

Evaluate $\lim\limits_{n \to \infty} \dfrac{[1.\,2x] + [2\,.\,3x] + \ldots\ldots + [n\,.\,(n+1)\,x]}{n^3}$ where [.] denotes the greatest integer function

If $f(x) = \begin{cases} x - 1, & x \geq 1 \\ 2x^2 - 2, & x < 1 \end{cases}$, $g(x) = \begin{cases} x + 1, & x > 0 \\ -x^2 + 1, & x \leq 0 \end{cases}$ and $h(x) = |x|$

then find $\lim\limits_{x \to 0} f(g(h(x)))$

(A) 1 (B*) 0 (C) –1 (D) does not exists

$\lim\limits_{x \to a^-} \left(\dfrac{|x|^3}{a} - \left[\dfrac{x}{a}\right]^3\right)$ (a > 0), where [x] denotes the greatest integer less than or equal to x is

(A) $a^2 + 1$ (B) $a^2 - 1$ (C*) a^2 (D) $-a^2$

If α and β be the roots of $ax^2 + bx + c = 0$, then $\lim\limits_{x \to \alpha} \left(1 + ax^2 + bx + c\right)^{\frac{1}{x - \alpha}}$ is

(A) $a(\alpha - \beta)$ (B) $\ln|a(\alpha - \beta)|$ (C*) $e^{a(\alpha - \beta)}$ (D) $e^{a|\alpha - \beta|}$

$\lim_{x\to 0}\left[(1-e^{x})\dfrac{\sin x}{|x|}\right]$ is (where [·] represents greatest integral part function)

(A*) – 1 (B) 1 (C) 0 (D) does not exist

$\lim_{x\to 0^{+}} \log_{\sin(x/2)} \sin x$ is equal to

(A*) 1 (B) 0 (C) 4 (D) $\frac{1}{4}$

If $\ell = \lim_{x\to\infty} (\sin\sqrt{x+1} - \sin\sqrt{x})$ and $m = \lim_{x\to-\infty} [\sin\sqrt{x+1} - \sin\sqrt{x}]$ where [.] denotes the greatest integer function then :

(A) $\ell = m = 0$ (B*) $\ell = 0$; m is undefined

(C) ℓ, m both do not exist (D) $\ell = 0$, $m \neq 0$ (although m exist)

Given a real valued function f such that $f(x) = \begin{cases} \dfrac{\tan^2[x]}{(x^2-[x]^2)} & x>0 \\ 1 & x=0 \\ \sqrt{\{x\}\cot\{x\}} & x<0 \end{cases}$

where, [x] is the integral part and {x} is the fractional part of x, then

(A) $\lim_{x\to 0} f(x) = 1$ (B) $\lim_{x\to 0^-} f(x) = \cot 1$

(C*) $\cot^{-1}\left(\lim_{x\to 0^-} f(x)\right)^2 = 1$ (D) f is continuous at x = 0

The limit $\lim_{\theta\to 0}\left(\left[\dfrac{n\sin\theta}{\theta}\right]+\left[\dfrac{n\tan\theta}{\theta}\right]\right)$, where [x] is the greatest integer function and $n \in N$, is

(A) 2n (B) 2n + 1 (C*) 2n – 1 (D) does not exist

The limit $\lim_{x\to 0} \dfrac{1-\cos x\sqrt{\cos 2x}}{x^2}$ is equal to

(A) 1/2 (B) –1/2 (C*) 3/2 (D) 1

$\lim_{n\to\infty}\left(\dfrac{1}{\sqrt{n^2}}+\dfrac{1}{\sqrt{n^2+1}}+\dfrac{1}{\sqrt{n^2+2}}+\ldots\ldots\ldots\ldots+\dfrac{1}{\sqrt{n^2+2n}}\right)$

(A) 1 (B) 1/2 (C) 0 (D*) 2

The value of $\underset{x \to 0}{\text{Limit}} \dfrac{\cos(\sin x) - \cos x}{x^4}$ is equal to:

(A) 1/5 (B*) 1/6 (C) 1/4 (D) 1/2

$\underset{x \to \infty}{\text{Limit}} \dfrac{e^x \left(\left(2^{x^n}\right)^{\frac{1}{e^x}} - \left(3^{x^n}\right)^{\frac{1}{e^x}} \right)}{x^n}$, $n \in N$ is equal to :

(A) 0 (B*) ln(2/3) (C) ln(3/2) (D) none

Show that $\lim\limits_{n \to \infty} \dfrac{1}{n^4} \left(1\left(\sum_{k=1}^{n} k\right) + 3\left(\sum_{k=1}^{n-1} k\right) + 5\left(\sum_{k=1}^{n-2} k\right) + \ldots\ldots + (2n-1).1 \right) = \dfrac{1}{12}$

Let $f(x) = \dfrac{\sin^{-1}(1 - \{x\}) . \cos^{-1}(1 - \{x\})}{\sqrt{2\{x\}}\ (1 - \{x\})}$ then find $\underset{x \to 0^+}{\text{Lim}} f(x)$ and $\underset{x \to 0^-}{\text{Lim}} f(x)$.

(where {x} denotes the fractional part of x.)

Let $f(x) = \underset{m \to \infty}{\text{Limit}} \left\{ \underset{n \to \infty}{\text{Limit}} \left(\cos^{2m}(n!\pi x) \right) \right\}$ where $x \in R$. Prove that

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$$

Evaluate $\underset{x \to 0^+}{\text{Limit}} \left\{ \underset{n \to \infty}{\text{Limit}} \left(\dfrac{[1^2 (\sin x)^x] + [2^2 (\sin x)^x] + \ldots\ldots + [n^2 (\sin x)^x]}{n^3} \right) \right\}$,

where [.] denotes the greatest integer function.

Consider two functions f(x) = $\lim\limits_{n\to\infty}\left(\cos\frac{x}{\sqrt{n}}\right)^n$ and g(x) = $-x^{4b}$ where b = $\lim\limits_{x\to\infty}\left(\sqrt{x^2+x+1}-\sqrt{x^2+1}\right)$

Then

f(x) is

(A) e^{-x^2} (B*) $e^{\frac{-x^2}{2}}$ (C) e^{x^2} (D) $e^{\frac{x^2}{2}}$

g(x) is
(A*) $-x^2$ (B) x^2 (C) x^4 (D) $-x^4$

Number of solutions of f(x) + g(x) = 0 is
(A*) 2 (B) 4 (C) 0 (D) 1

For x > 0, $\lim\limits_{x\to 0}(\sin x)^{1/x}+(1/x)^{\sin x}$ is

(A) 0 (B) – 1 (C*) 1 (D) 2

f(x) is defined as f : (–1, 1) → R and is differentiable on (–1, 1). It is given that $f'(0)=\lim\limits_{n\to\infty} n\left(f\left(\frac{1}{n}\right)\right)$ also f(0) = 0 . Find the value of $\lim\limits_{n\to\infty}\left[\frac{2}{\pi}(1+n)\cos^{-1}\left(\frac{1}{n}\right)-n\right]$ given that $\left|\cos^{-1}\left(\frac{1}{n}\right)\right|\le\frac{\pi}{2}$.

$$\operatorname*{Limit}_{x\to\pi/2}\frac{\left(1-\tan\frac{x}{2}\right)(1-\sin x)}{\left(1+\tan\frac{x}{2}\right)(\pi-2x)^3}\text{ is :}$$

(A) 1/16 (B) – 1/16 (C*) 1/32 (D) – 1/32

$\lim\limits_{x\to\infty}\frac{\log^{x^n}-[x]}{[x]}$, n ∈ N, ([x] denotes greatest integer less thaen or equal to x).

(A*) has value – 1 (B) has value 0 (C) has value 1 (D) does not exists

If

$$f(x) = \begin{cases} \dfrac{[x]^2 + \sin [x]}{[x]} & \text{for } [x] \neq 0 \\ 0 & \text{for } [x] = 0 \end{cases}$$

where $[x]$ denotes the greatest integer less than or equal to x, then $\lim_{x \to 0} f(x)$ equals

(a) 1 (b) 0

(c) –1 (d) $\lim\limits_{x \to 0+} f(x) = 0$

Ans. (d)

$$\lim_{x \to a} \left\{ \left[\left(\frac{a^{1/2} + x^{1/2}}{a^{1/4} - x^{1/4}} \right)^{-1} - \frac{2(ax)^{1/4}}{x^{3/4} - a^{1/4} x^{1/2} + a^{1/2} x^{1/4} - a^{3/4}} \right]^{-1} - \sqrt{2}^{\log_4 a} \right\}^8$$

is

(a) a (b) $a^{3/4}$

(c) a^2 (d) none of these

Ans. (c)

Let

$$f(x) = \lim_{n\to\infty} \frac{\log(2+x) - x^{2n}\sin x}{1 + x^{2n}}.$$

Then

(a) f is continuous at $x = 1$

(b) $\lim_{x\to 1+} f(x) \neq \lim_{x\to 1-} f(x)$

(c) $\lim_{x\to 1+} f(x) = \sin 1$

(d) $\lim_{x\to 1-} f(x)$ doesn't exist

Ans. (b)

The value of $\lim_{n\to\infty} \cos\frac{x}{2}\cos\frac{x}{4}\ldots\cos\frac{x}{2^n}$ is

(a) 1 (b) $\frac{\sin x}{x}$

(c) $\frac{x}{\sin x}$ (d) none of these

Ans. (b)

Let f be a continuous function on **R** such that $f\left(\frac{1}{4n}\right) = (\sin e^n)\, e^{-n^2} + \frac{n^2}{n^2+1}$. Then the value of $f(0)$ is

(a) 1 (b) 1/2

(c) 0 (d) none of these

Ans. (a)

If $\lim_{x \to a} (f(x)\, g(x))$ exists for any functions f and g then

(a) $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist

(b) $\lim_{x \to a} f(x)$ exist but $\lim_{x \to a} g(x)$ may not exist

(c) $\lim_{x \to a} f(x)$ mayn't exist but $\lim_{x \to a} g(x)$ exist

(d) $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ may not exist

Ans. (d)

The $\lim_{x \to 0} x^8 \left[\frac{1}{x^3}\right]$ (where [x] is greatest integer function) is

(a) a nonzero real number

(b) a rational number

(c) an integer (d) zero

Ans. (b), (c) and (d)

$$\Delta(x) = \begin{vmatrix} \tan x & \tan(x+h) & \tan(x+2h) \\ \tan(x+2h) & \tan x & \tan(x+h) \\ \tan(x+h) & \tan(x+2h) & \tan x \end{vmatrix}$$

Find the value of $\lim_{h \to 0} \frac{\sqrt{3}\,\Delta(\pi/3)}{h^2}$

(a) $f(x) = x \text{ sgn } (x - 1)$ (p) $\lim_{x \to 1} f(x)$ doesn't exist

(b) $f(x) = \dfrac{\sin\left(\sin(\tan(x^2/2))\right)}{\log \cos 3x}$ (q) $\lim_{x \to 0} f(x)$ doesn't exist

(c) $f(x) = \dfrac{\sqrt[3]{1+\tan^{-1} 3x} - \sqrt[3]{1-\sin^{-1} 3x}}{\sqrt{1-\sin^{-1} 2x} - \sqrt{1+\tan^{-1} 2x}}$ (r) $\lim_{x \to 0} f(x) = -1/9$

(d) $f(x) = \dfrac{e^{1/x} - 1}{e^{1/x} + 1}$ (s) $\lim_{x \to 0} f(x) = -1$

Statement-1 If a and b are positive and $[x]$ denotes the greatest integer $\leq x$, then $\lim_{x \to 0+} \frac{x}{a}\left[\frac{b}{x}\right] = \frac{b}{a}$.

Statement-2 $\lim_{x \to \infty} \frac{\{x\}}{x} = 0$, where $\{x\}$ denotes fractional part of x.

Ans. (a)

Let $L = \lim_{x \to 0} \frac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}$, $a > 0$

If L is finite, then

(a) $a = 2$

(b) $a = 1$

(c) $L = \frac{1}{64}$

(d) $L = \frac{1}{32}$

Evaluate $\lim_{x \to 1^-} \frac{1 - \sqrt{x}}{(\cos^{-1} x)^2}$.

Evaluate $\lim_{x \to -1^+} \frac{\sin^{-1}\left(\sqrt{\pi} - \sqrt{\cos^{-1} x}\right)}{\sqrt{1 - x^2}}$.

Solve $\lim_{x \to \frac{\pi}{2}} \sqrt{\frac{\tan x - \sin\{\tan^{-1}(\tan x)\}}{\tan x + \cos^2(\tan x)}}$

Evaluate $\lim\limits_{x\to 0} \dfrac{\sin x - x^2 - \{x\}.\{-x\}}{x\cos x - x^2 - \{x\}.\{-x\}}$

where $\{.\}$ denotes the fractional part

Evaluate $\lim\limits_{x \to 0} \dfrac{x + \ell n\left(\sqrt{1+x^2} - x\right)}{x^3}$

1 If $\lim\limits_{x\to\infty}\left(\dfrac{x^2-1}{x+1} - ax - b\right) = 2$, find the values of a and b.

2 Find a and b if

$\lim\limits_{x\to-\infty}\left\{\sqrt{(x^2-x+1)} - ax - b\right\} = 0$

Find a polynomial of the least degree, such that $\lim\limits_{x\to 0}\left(1+\dfrac{x^2+f(x)}{x^2}\right)^{1/x} = e^2$.

Evaluate $\lim\limits_{x\to 0}\lim\limits_{n\to\infty} \dfrac{\tan x^2 + (x+1)^n.\sin x}{x^2 + (x+1)^n}$

$$\lim_{n\to\infty}\prod_{r=3}^{n}\frac{r^3-8}{r^3+8}$$

Evaluate $\lim_{x\to 0}\left(\frac{(1+[x])^{1/\{x\}}}{e}\right)^{1/\{x\}}$

if it exist, where $\{x\}$ denotes the fractional part of x.

. $\lim_{x\to 0}\left(\frac{(1+x)^{\frac{2}{x}}}{e^2}\right)^{\frac{4}{\sin x}}$ is :

(a) e^4 (b) e^{-4} (c) e^8 (d) e^{-8}

$\lim_{x\to\infty}\frac{3}{x}\left[\frac{x}{4}\right]=\frac{p}{q}$ (where [·] denotes greatest integer function), then $p+q$ (where p, q are relative prime) is :

(a) 2 (b) 7 (c) 5 (d) 6

$\lim_{x\to\frac{\pi}{2}}\frac{\sin x}{\cos^{-1}\left[\frac{1}{4}(3\sin x-\sin 3x)\right]}$, (where [·] denotes greatest integer function) is :

(a) $\frac{2}{\pi}$ (b) 1 (c) $\frac{4}{\pi}$ (d) does not exist

Let f be a continuous function on R such that $f\left(\frac{1}{4^n}\right)=(\sin e^n)e^{-n^2}+\frac{n^2}{n^2+1}$, then $f(0)=$

(a) 1 (b) 0 (c) -1 (d) $\frac{1}{4}$

For $n \in N$, let $f_n(x) = \tan\frac{x}{2}(1+\sec x)(1+\sec 2x)(1+\sec 4x)\ldots\ldots(1+\sec 2^n x)$, the $\lim\limits_{x\to 0}\frac{f_n(x)}{2x}$ is equal to :

(a) 0 (b) 2^n (c) 2^{n-1} (d) 2^{n+1}

The value of $\lim\limits_{x\to\frac{\pi}{4}}(1+[x])^{\frac{1}{ln(\tan x)}}$ is :

(where [.] denotes greatest integer function).

(a) 0 (b) 1 (c) e (d) $\frac{1}{e}$

The value of $\lim\limits_{n\to\infty}\left(\frac{n!}{n^n}\right)^{\frac{3n^3+4}{4n^4-1}}$, $n \in N$ is equal to :

(a) $\left(\frac{1}{e}\right)^{3/4}$ (b) $e^{3/4}$ (c) e^{-1} (d) 0

If $f(x) = \lim\limits_{n\to\infty} x\left(\frac{3}{2}+[\cos x]\left(\sqrt{n^2+1}-\sqrt{n^2-3n+1}\right)\right)$ where $[y]$ denotes largest integer $\le y$, then identify the correct statement(s).

(a) $\lim\limits_{x\to 0} f(x) = 0$

(b) $\lim\limits_{x\to\frac{\pi}{2}} f(x) = \frac{3\pi}{4}$

(c) $f(x) = \frac{3x}{2}\ \forall\, x \in \left[0, \frac{\pi}{2}\right]$

(d) $f(x) = 0\ \forall\, x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

The value of $\lim\limits_{n\to\infty}\cos^2(\pi(\sqrt[3]{n^3+n^2+2n}))$ (where $n \in N$):

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{9}$

If $\alpha, \beta \in \left(-\frac{\pi}{2}, 0\right)$ such that $(\sin\alpha+\sin\beta)+\frac{\sin\alpha}{\sin\beta}=0$ and $(\sin\alpha+\sin\beta)\frac{\sin\alpha}{\sin\beta}=-1$ and $\lambda = \lim\limits_{n\to\infty}\frac{1+(2\sin\alpha)^{2n}}{(2\sin\beta)^{2n}}$ then :

(a) $\alpha = -\frac{\pi}{6}$ (b) $\lambda = 2$ (c) $\alpha = -\frac{\pi}{3}$ (d) $\lambda = 1$

Let $f(x)=\begin{cases}|x-2|+a^2-6a+9 & , \quad x<2\\ 5-2x & , \quad x\geq 2\end{cases}$

If $\lim_{x\to 2}[f(x)]$ exists, the possible values a can take is/are (where $[\cdot]$ represents the greatest integer function)

(a) 2 (b) $\frac{5}{2}$ (c) 3 (d) $\frac{7}{2}$

Let $f(x)=\begin{bmatrix}x+3 & ; & -2<x<0\\ 4 & ; & x=0\\ 2x+5 & ; & 0<x<1\end{bmatrix}$, then

$\lim_{x\to 0^-} f([x-\tan x])$ is : ($[\cdot]$ denotes greatest integer fucntion)

(a) 2 (b) 4 (c) 5 (d) None of these

$\lim_{x\to 0^+} f\left(\left\{\frac{x}{\tan x}\right\}\right)$ is : ($\{\cdot\}$ denotes fractional part of function)

(a) 4 (b) 5 (c) 7 (d) None of these

If $\lim_{x\to 0}\frac{\ln\cot\left(\frac{\pi}{4}-\beta x\right)}{\tan\alpha x}=1$, then $\frac{\alpha}{\beta}=$......

If $\lim_{x\to 0}\frac{f(x)}{\sin^2 x}=8$, $\lim_{x\to 0}\frac{g(x)}{2\cos x-xe^x+x^3+x-2}=\lambda$ and $\lim_{x\to 0}(1+2f(x))^{\frac{1}{g(x)}}=\frac{1}{e}$, then $\lambda=$

If α,β are two distinct real roots of the equation $ax^3+x-1-a=0, (a\neq -1, 0)$, none of which is equal to unity, then the value of $\lim_{x\to(1/\alpha)}\frac{(1+a)x^3-x^2-a}{(e^{1-\alpha x}-1)(x-1)}$ is $\frac{al(k\alpha-\beta)}{\alpha}$. Find the value of kl.

Find $\lim_{x\to\alpha^+}\left[\frac{\min(\sin x,\{x\})}{x-1}\right]$ where α is root of equation $\sin x+1=x$ (here $[\cdot]$ represent greatest integer and $\{\cdot\}$ represent fractional part function)

Let $f(x)=\cos 2x\cdot\cos 4x\cdot\cos 6x\cdot\cos 8x\cdot\cos 10x$, then $\lim_{x\to 0}\frac{1-(f(x))^3}{5\sin^2 x}$ equals :

(a) 660 (b) 135 (c) 132 (d) 66

If $\lim_{x\to 0}\left(\left[\frac{\sin^{-1}x}{x}\right]+\left[\frac{2^2\sin^{-1}2x}{x}\right]+\left[\frac{3^2\sin^{-1}3x}{x}\right]+\ldots\ldots+\left[\frac{n^2\sin^{-1}nx}{x}\right]\right)=100$, then the value of n, is :

. Let $f : R \to (0, \infty)$ be such that $f(x) + \dfrac{e^{x+x^2}}{f(x)} \le e^x + e^{x^2}\ \forall\ x > 0$, then $\lim\limits_{x\to 1} f(x)$ is

(a) 1 (b) $\dfrac{1}{e}$ (c) e (d) $2e$

Let $f(x) = \begin{cases} \lim\limits_{n\to\infty}\left(\dfrac{px}{n}\displaystyle\sum_{r=1}^{n}\dfrac{[r^2 - e^{-x} + r - 1]}{r(r+1)}\right) + \lambda, & x > 0 \\ q, & x = 0 \\ \lim\limits_{n\to\infty}\displaystyle\sum_{r=1}^{n}\dfrac{\{r^2 + r + e^x - 1\}}{r(r+1)}, & x < 0 \end{cases}$

is differentiable in R (the set of all real numbers)

[Note : [y] and {y} denote greatest integer and fractional part function of y.]

. The value of $p + q + \lambda$ is equal to :

(a) -2 (b) 2 (c) 1 (d) 3

. The value of $f'(\ln 2) + f'\left(\ln\dfrac{1}{2}\right) + f'\left(\ln\dfrac{3}{2^2}\right) + f'\left(\ln\dfrac{5}{2^3}\right) + \ldots\ \infty$ is equal to :

(a) 3 (b) 4 (c) 6 (d) 2

If g is the inverse of f then $g'\left(\dfrac{1}{2}\right)$ is equal to :

(a) $\ln 2$ (b) $-\ln 2$ (c) 2 (d) $\dfrac{1}{2}$

The possible value(s) of k for which $\lim\limits_{x\to\infty}\dfrac{2x^3 - (\tan^{-1}x)^3}{\dfrac{8}{\pi}x^3\cot^{-1}|kx| + k^2x^6\sin\dfrac{1}{x^3} - 3kx^3} = \dfrac{1}{2}$, is

(a) 0 (b) -1 (c) 2 (d) 4

Let $f(x) = \dfrac{\left(\dfrac{\pi}{2} - \cos^{-1}(1 - \{x\}^2)\right)\cdot(\cos^{-1}(1 - \{x\}))^2}{2(\{x\} - \{x\}^3)}$. If $f(0^+) = p$ and $f(0^{-1}) = q$, then find the value of $\left(\dfrac{p\pi}{q}\right)$.

[Note : $\{k\}$ denotes the fractional part of k.]

If α and β ($\alpha < \beta$) are the roots of the equation

$$\lim_{t\to\infty} \cos^{-1}\left[\sin\left(\tan^{-1}\left(\frac{\sqrt{tx}}{\sqrt{tx^2-3tx+t-1}-x}\right)\right)\right] = \frac{\pi}{6}$$

then find the value of $(8^{\alpha} + 2^{\beta} - \alpha\beta)$.

Let $f(x) = \lim_{n\to\infty} \ln\left(\sqrt{e^{\cos x}\sqrt{e^{3\cos x}\sqrt{e^{5\cos x}\,.....\,\sqrt{e^{(2n+1)\cos x}}}}}\right)$. If $g(x) = \left[\frac{1}{3}f(x)\right]$, then find the number of points in $[0, 2\pi]$ where $g(x)$ is discontinuous.

[**Note:** $[y]$ denotes greatest integer function less than or equal to y.]

Let $f(x) = \text{sgn}\,(x^2 - 4x + 4 + k^2)$, $x \in R$. If $f(x)$ is discontinuous at exactly one point then the value of $(\tan^{-1} k + \cos^{-1} k + \text{cosec}^{-1}(2k-1))$ is equal to $\frac{m\pi}{2}$ where m is a whole number. Find the value of m.

If $S_n = \sum_{r=1}^{n} \tan^{-1}\left[\frac{2(2r-1)}{4+r^2(r^2-2r+1)}\right]$ then find the value of

$$\lim_{n\to\infty} \sum_{n=2}^{n} (\cot(S_{n-1}) - \cot(S_n)).$$

Let $\lim_{n\to\infty} \frac{1+cn^2}{(2n+3+2\sin n)^2} = \frac{1}{2}$. If $c \le \alpha \le \beta$ where α and β are the roots of the quadratic equation $x^2 - 2px + p^2 - 1 = 0$, then find the minimum integral value of p.

If $f(x) = \lim_{n\to\infty} \frac{\left(1-\cos\left\{1-\tan\left(\frac{\pi}{4}-x\right)\right\}\right)(x+1)^n + \lambda\sin\left((n-\sqrt{n^2-8n})x\right)}{x^2(x+1)^n + x}$, $x \ne 0$ is continuous at $x = 0$, then find the value of $(f(0) + 2\lambda)$.

If $f:(0,\infty)\to N$ and

$$f(x)=\left[\frac{x^2+x+1}{x^2+1}\right]+\left[\frac{4x^2+x+2}{2x^2+1}\right]+\left[\frac{9x^2+x+3}{3x^2+1}\right]+\dots+\left[\frac{n^2x^2+x+n}{nx^2+1}\right],\ n\in N \text{ then}$$

find the value of $\lim\limits_{n\to\infty}\left[\dfrac{f(x)-n}{(f(x))^2-\dfrac{n^3(n+2)}{4}}\right]$.

[Note : [y] denotes the greatest integer less than or equal to y.]

$\lim\limits_{x\to 0}\left[\min(y^2-4y+11).\dfrac{\sin x}{x}\right]$ (where [.] denotes the greatest integer function) is

1) 5 2) 6 3) 7 4) Does not exist

Let $f:R\to R$ be a positive increasing function with $\lim\limits_{x\to\infty}\dfrac{f(3x)}{f(x)}=1$ then $\lim\limits_{x\to\infty}\dfrac{f(2x)}{f(x)}=$

1) $\dfrac{2}{3}$ 2) $\dfrac{3}{2}$ 3) 3 4) 1