$A(\vec{a})$; $B(\vec{b})$; $C(\vec{c})$ are the vertices of the triangle ABC such that $\vec{a} = \frac{1}{2}(2\hat{i} - \vec{r} - 7\hat{k})$; $\vec{b} = 3\vec{r} + \hat{j} - 4\hat{k}$; $\vec{c} = 22\hat{i} - 11\hat{j} - 9\vec{r}$. A vector $\vec{p} = 2\hat{j} - \hat{k}$ is such that $(\vec{r} + \vec{p})$ is parallel to $\hat{i}$ and $(\vec{r} - 2\hat{i})$ is parallel to $\vec{p}$. Show that there exists a point D $(\vec{d})$ on the line AB with $\vec{d} = 2t\hat{i} + (1-2t)\hat{j} + (t-4)\hat{k}$. Also find the shortest distance C from AB.

The pv's of the four angular points of a tetrahedron are: $A(\hat{j} + 2\hat{k})$; $B(3\hat{i} + \hat{k})$; $C(4\hat{i} + 3\hat{j} + 6\hat{k})$ & $D(2\hat{i} + 3\hat{j} + 2\hat{k})$. Find:

(i) the perpendicular distance from A to the line BC.
(ii) the volume of the tetrahedron ABCD.
(iii) the perpendicular distance from D to the plane ABC.
(iv) the shortest distance between the lines AB & CD.

The length of an edge of a cube $ABCDA_1B_1C_1D_1$ is equal to unity. A point E taken on the edge $\overrightarrow{AA_1}$ is such that $\left|\overrightarrow{AE}\right| = \frac{1}{3}$. A point F is taken on the edge $\overrightarrow{BC}$ such that $\left|\overrightarrow{BF}\right| = \frac{1}{4}$. If O_1 is the centre of the cube, find the shortest distance of the vertex B_1 from the plane of the $\Delta\ O_1EF$.

Find the point R in which the line AB cuts the plane CDE where
$\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} + 2\hat{k}$, $\vec{c} = -4\hat{j} + 4\hat{k}$, $\vec{d} = 2\hat{i} - 2\hat{j} + 2\hat{k}$ & $\vec{e} = 4\hat{i} + \hat{j} + 2\hat{k}$.

Given the points P (1, 1, –1), Q (1, 2, 0) and R (–2, 2, 2). Find

(a) $\overrightarrow{PQ} \times \overrightarrow{PR}$
(b) Equation of the plane in
 (i) scalar dot product form
 (ii) parametric form
 (iii) cartesian form
 (iv) if the plane through PQR cuts the coordinate axes at A, B, C then the area of the ΔABC

Find the angle between the two straight lines whose direction cosines l, m, n are given by $2l + 2m - n = 0$ and $mn + nl + lm = 0$.

If two straight line having direction cosines l, m, n satisfy $al + bm + cn = 0$ and $fmn + gnl + hlm = 0$ are perpendicular, then show that $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$.

P is any point on the plane $lx + my + nz = p$. A point Q taken on the line OP (where O is the origin) such that OP. OQ = p^2. Show that the locus of Q is $p(lx + my + nz) = x^2 + y^2 + z^2$.

Find the equation of the plane through the points (2, 2, 1), (1, –2, 3) and parallel to the x-axis.

Through a point P (f, g, h), a plane is drawn at right angles to OP where 'O' is the origin, to meet the coordinate axes in A, B, C. Prove that the area of the triangle ABC is $\frac{r^5}{2fgh}$ where OP = r.

The plane $lx + my = 0$ is rotated about its line of intersection with the plane $z = 0$ through an angle θ. Prove that the equation to the plane in new position is $lx + my \pm z\sqrt{l^2 + m^2}\tan\theta = 0$

Find the equations of the straight line passing through the point (1, 2, 3) to intersect the straight line $x + 1 = 2(y - 2) = z + 4$ and parallel to the plane $x + 5y + 4z = 0$.

Find the equations of the two lines through the origin which intersect the line $\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1}$ at an angle of $\frac{\pi}{3}$.

A variable plane is at a constant distance p from the origin and meets the coordinate axes in points A, B and C respectively. Through these points, planes are drawn parallel to the coordinates planes. Find the locus of their point of intersection.

Find the distance of the point P (– 2, 3, – 4) from the line $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ measured parallel to the plane $4x + 12y - 3z + 1 = 0$.

Find the equation to the line passing through the point (1, –2, –3) and parallel to the line

$$2x + 3y - 3z + 2 = 0 = 3x - 4y + 2z - 4.$$

Find the equation of the line passing through the point (4, –14, 4) and intersecting the line of intersection of the planes : $3x + 2y - z = 5$ and $x - 2y - 2z = -1$ at right angles.

Let P = (1, 0, – 1) ; Q = (1, 1, 1) and R = (2, 1, 3) are three points.

(a) Find the area of the triangle having P, Q and R as its vertices.

(b) Give the equation of the plane through P, Q and R in the form $ax + by + cz = 1$.

(c) Where does the plane in part (b) intersect the y-axis.

(d) Give parametric equations for the line through R that is perpendicular to the plane in part (b).

Find the point where the line of intersection of the planes $x - 2y + z = 1$ and $x + 2y - 2z = 5$, intersects the plane $2x + 2y + z + 6 = 0$.

Feet of the perpendicular drawn from the point P (2, 3, –5) on the axes of coordinates are A, B and C. Find the equation of the plane passing through their feet and the area of ΔABC.

Find the equations to the line which can be drawn from the point (2, –1, 3) perpendicular to the lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \quad \text{and} \quad \frac{x-4}{4} = \frac{y}{5} = \frac{z+3}{3} \quad \text{at right angles.}$$

Find the equation of the plane containing the straight line $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z}{5}$ and perpendicular to the plane $x - y + z + 2 = 0$.

Find the value of p so that the lines $\frac{x+1}{-3} = \frac{y-p}{2} = \frac{z+2}{1}$ and $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$ are in the same plane. For this value of p, find the coordinates of their point of intersection and the equation of the plane containing them.

Find the equations to the line of greatest slope through the point (7, 2 , −1) in the plane $x - 2y + 3z = 0$ assuming that the axes are so placed that the plane $2x + 3y - 4z = 0$ is horizontal.

Let ABCD be a tetrahedron such that the edges AB, AC and AD are mutually perpendicular. Let the area of triangles ABC, ACD and ADB be denoted by x, y and z sq. units respectively. Find the area of the triangle BCD.

The position vectors of the four angular points of a tetrahedron OABC are (0, 0, 0); (0, 0, 2); (0, 4, 0) and (6, 0, 0) respectively. A point P inside the tetrahedron is at the same distance 'r' from the four plane faces of the tetrahedron. Find the value of 'r'.

The line $\frac{x+6}{5} = \frac{y+10}{3} = \frac{z+14}{8}$ is the hypotenuse of an isosceles right angled triangle whose opposite vertex is (7, 2, 4). Find the equation of the remaining sides.

Find the foot and hence the length of the perpendicular from the point (5, 7, 3) to the line $\frac{x-15}{3} = \frac{y-29}{8} = \frac{5-z}{5}$. Also find the equation of the plane in which the perpendicular and the given straight line lie.

Find the equation of the line which is reflection of the line $\frac{x-1}{9} = \frac{y-2}{-1} = \frac{z+3}{-3}$ in the plane $3x - 3y + 10z = 26$.

Find the equation of the plane containing the line $\frac{x-1}{2} = \frac{y}{3} = \frac{z}{2}$ and parallel to the line $\frac{x-3}{2} = \frac{y}{5} = \frac{z-2}{4}$.

Find also the S.D. between the two lines.

The position vectors of the points P & Q are $5\hat{i}+7\hat{j}-2\hat{k}$ and $-3\hat{i}+3\hat{j}+6\hat{k}$ respectively. The vector $\vec{A}=3\hat{i}-\hat{j}+\hat{k}$ passes through the point P & the vector $\vec{B}=-3\hat{i}+2\hat{j}+4\hat{k}$ passes through the point Q. A third vector $2\hat{i}+7\hat{j}-5\hat{k}$ intersects vectors $\vec{A}$ & $\vec{B}$. Find the position vectors of the points of intersection.

Vectors $\vec{AB} = 3\hat{i} - \hat{j} + \hat{k}$ & $\vec{CD} = -3\hat{i} + 2\hat{j} + 4\hat{k}$ are not coplanar. The position vectors of points A and C are $6\hat{i}+7\hat{j}+4\hat{k}$ and $-9\hat{j}+2\hat{k}$ respectively. Find the position vectors of a point P on the line AB & a point Q on the line CD such that $\vec{PQ}$ is perpendicular to $\vec{AB}$ and $\vec{CD}$ both.

Let the vectors $\vec{a}, \vec{b}, \vec{c}$ & $\vec{d}$ be such that $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = \vec{0}$. Let P_1 & P_2 be planes determined by the pairs of vectors $\vec{a}, \vec{b}$ & $\vec{c}, \vec{d}$ respectively. Then the angle between P_1 and P_2 is :

(A) 0　　(B) π/4　　(C) π/3　　(4) π/2

) Find the equation of the plane passing through the points (2, 1, 0) , (5, 0, 1) and (4, 1, 1).

If P is the point (2, 1, 6) then find the point Q such that PQ is perpendicular to the plane in (i) and the mid point of PQ lies on it.

If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then k =

(A) $\frac{2}{9}$ (B) $\frac{9}{2}$ (C) 0 (D) – 1

Let P be the plane passing through (1, 1, 1) and parallel to the lines L_1 and L_2 having direction ratios 1, 0, −1 and −1, 1, 0 respectively. If A, B and C are the points at which P intersects the coordinate axes, find the volume of the tetrahedron whose vertices are A, B, C and the origin.

(b) A variable plane at a distance of 1 unit from the origin cuts the co-ordinate axes at A, B and C. If the centroid D (x, y, z) of triangle ABC satisfies the relation $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=k$, then the value of k is

(A) 3 (B) 1 (C) 1/3 (D) 9

Find the equation of the plane containing the line $2x - y + z - 3 = 0$, $3x + y + z = 5$ and at a distance of $\frac{1}{\sqrt{6}}$ from the point (2, 1, – 1).

(a) A plane passes through (1, −2, 1) and is perpendicular to two planes $2x - 2y + z = 0$ and $x - y + 2z = 4$. The distance of the plane from the point (1, 2, 2) is

(A) 0 (B) 1 (C) $\sqrt{2}$ (D) $2\sqrt{2}$

Let $\vec{A}$ be vector parallel to line of intersection of planes P_1 and P_2 through origin. P_1 is parallel to the vectors $2\hat{j} + 3\hat{k}$ and $4\hat{j} - 3\hat{k}$ and P_2 is parallel to $\hat{j} - \hat{k}$ and $3\hat{i} + 3\hat{j}$, then the angle between vector $\vec{A}$ and $2\hat{i} + \hat{j} - 2\hat{k}$ is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{3\pi}{4}$

A line makes the same angle θ, with each of the *x* and *z* axis. If the angle β which it makes with *y*-axis, is such that $\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equals

[1] 3/5 [2] 1/5 [3] 2/3 [4] 2/5

A line with direction cosines proportional to 2, 1, 2 meets each of the lines $x = y + a = z$ and $x + a = 2y = 2z$. The co-ordinates of each of the points of intersection are given by

[1] (3*a*, 2*a*, 3*a*) (*a*, *a*, 2*a*) [2] (3*a*, 2*a*, 3*a*) (*a*, *a*, *a*)

[3] (3*a*, 3*a*, 3*a*) (*a*, *a*, *a*) [4] (2*a*, 3*a*, 3*a*) (2*a*, *a*, *a*)

Equation of the plane passing through the point (4, 3, 7) and through the line $\frac{x-1}{5}=\frac{y+2}{6}=\frac{z-3}{4}$ will be -

[1] 4x + 8y + 7z = 41 [2] 4x – 8y + 7z = 41 [3] 4x – 8y – 7z = 41 [4] 4x – 8y + 7z = 39

The two lines x = ay + b, z = cy + d; and x = a′y + b′, z = c′ + d′ are perpendicular to each other of

[1] $aa' + cc' = 1$ [2] $\frac{a}{a'}+\frac{c}{c'} = -1$ [3] $\frac{a}{a'}+\frac{c}{c'} = 1$ [4] $aa' + cc' = -1$

A variable plane passes through a fixed point (1, 2, 3). The locus of the foot of the perpendicular drawn from origin to this plane is:

(A*) $x^2 + y^2 + z^2 - x - 2y - 3z = 0$
(B) $x^2 + 2y^2 + 3z^2 - x - 2y - 3z = 0$
(C) $x^2 + 4y^2 + 9z^2 + x + 2y + 3 = 0$
(D) $x^2 + y^2 + z^2 + x + 2y + 3z = 0$

The reflection of the point (2, –1, 3) in the plane 3x – 2y – z = 9 is :

(A) $\left(\frac{26}{7},\frac{15}{7},\frac{17}{7}\right)$ (B*) $\left(\frac{26}{7},\frac{-15}{7},\frac{17}{7}\right)$ (C) $\left(\frac{15}{7},\frac{26}{7},\frac{-17}{7}\right)$ (D) $\left(\frac{26}{7},\frac{17}{7},\frac{-15}{7}\right)$

Equation of the angle bisector of the angle between the lines $\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{1}$ & $\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{-1}$ is :

(A*) $\frac{x-1}{2} = \frac{y-2}{2}$; $z - 3 = 0$
(B) $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$
(C) $x - 1 = 0$; $\frac{y-2}{1} = \frac{z-3}{1}$
(D) None of these

The distance of the point, (–1, –5, –10) from the point of intersection of the line, $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane, x – y + z = 5, is:

(A) 10 (B) 11 (C) 12 (D*) 13

The distance of the point (1, –2, 3) from the plane x – y + z = 5 measured parallel to the line, $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$, is:

(A*) 1 (B) 6/7 (C) 7/6 (D) None of these

A plane meets the coordinate axes in A, B, C and (α, β, γ) is the centroid of the triangle ABC. Then the equation of the plane is

(A*) $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma} = 3$ (B) $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma} = 1$ (C) $\frac{3x}{\alpha}+\frac{3y}{\beta}+\frac{3z}{\gamma} = 1$ (D) $\alpha x + \beta y + \gamma z = 1$

A variable plane passes through a fixed point (a, b, c) and meets the coordinate axes in A, B, C. Locus of the point common to the planes through A, B, C and parallel to coordinate plane, is

(A*) $\frac{a}{x}+\frac{b}{y}+\frac{c}{z} = 1$ (B) $\frac{x}{a}+\frac{y}{b}+\frac{z}{c} = 1$ (C) ax + by + cz = 1 (D) None of these

Two systems of rectangular axes have the same origin. If a plane cuts them at distances a, b, c and a_1, b_1, c_1 from the origin, then

(A*) $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{a_1^2}+\frac{1}{b_1^2}+\frac{1}{c_1^2}$ (B) $\frac{1}{a^2}-\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{a_1^2}-\frac{1}{b_1^2}+\frac{1}{c_1^2}$

(C) $a^2+b^2+c^2=a_1^2+b_1^2+c_1^2$ (D) $a^2-b^2+c^2=a_1^2-b_1^2+c_1^2$

Equation of plane which passes through the point of intersection of lines $\frac{x-1}{3}=\frac{y-2}{1}=\frac{z-3}{2}$ ar

$\frac{x-3}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ and at greatest distance from the point (0, 0, 0) is:

(A) $4x + 3y + 5z = 25$ (B*) $4x + 3y + 5z = 50$
(C) $3x + 4y + 5z = 49$ (D) $x + 7y - 5z = 2$

The non zero value of 'a' for which the lines $2x - y + 3z + 4 = 0 = ax + y - z + 2$ and $x - 3y + z = 0 = x + 2y + z + 1$ are co-planar is :

(A*) -2 (B) 4 (C) 6 (D) 0

If the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$, $\frac{x-1}{3}=\frac{y-2}{-1}=\frac{z-3}{4}$ and $\frac{x+k}{3}=\frac{y-1}{2}=\frac{z-2}{h}$ are concurrent then

(A) $h = -2$, $k = -6$ (B) $h=\frac{1}{2}$, $k = 2$ (C) $h = 6$, $k = 2$ (D*) $h = 2$, $k=\frac{1}{2}$

The direction ratios .of a normal to the plane through (1, 0, 0), (0,1, 0), which makes an angle of $\pi/4$ with the plane $x + y = 3$ are :

(A) $(1, \sqrt{2}, 1)$ (B*) $(1, 1, \sqrt{2})$ (C) (1, 1, 2) (D) $(\sqrt{2}, 1, 1)$

The equation of the line $x + y + z - 1 = 0$, $4x + y - 2z + 2 = 0$ written in the symmetrical form is

(A*) $\frac{x+1}{1}=\frac{y-2}{-2}=\frac{z-0}{1}$ (B*) $\frac{x}{1}=\frac{y}{-2}=\frac{z-1}{1}$

(C*) $\frac{x+1/2}{1}=\frac{y-1}{-2}=\frac{z-1/2}{1}$ (D) $\frac{x-1}{2}=\frac{y+2}{-1}=\frac{z-2}{2}$

The equations of the planes through the origin which are parallel to the line

$\frac{x-1}{2}=\frac{y+3}{-1}=\frac{z+1}{-2}$ and distant $\frac{5}{3}$ from it are

(A*) $2x + 2y + z = 0$ (B) $x + 2y + 2z = 0$ (C) $2x - 2y + z = 0$ (D*) $x - 2y + 2z = 0$

The direction cosines of the lines bisecting the angle between the lines whose direction cosines are ℓ_1, m_1, n_1 and ℓ_2, m_2, n_2 and the angle between theses lines is θ, are

(A) $\frac{\ell_1+\ell_2}{\cos\frac{\theta}{2}}, \frac{m_1+m_2}{\cos\frac{\theta}{2}}, \frac{n_1+n_2}{\cos\frac{\theta}{2}}$

(B*) $\frac{\ell_1+\ell_2}{2\cos\frac{\theta}{2}}, \frac{m_1+m_2}{2\cos\frac{\theta}{2}}, \frac{n_1+n_2}{2\cos\frac{\theta}{2}}$

(C) $\frac{\ell_1+\ell_2}{\sin\frac{\theta}{2}}, \frac{m_1+m_2}{\sin\frac{\theta}{2}}, \frac{n_1+n_2}{\sin\frac{\theta}{2}}$

(D*) $\frac{\ell_1-\ell_2}{2\sin\frac{\theta}{2}}, \frac{m_1-m_2}{2\sin\frac{\theta}{2}}, \frac{n_1-n_2}{2\sin\frac{\theta}{2}}$

The equation of line AB is $\frac{x}{2} = \frac{y}{-3} = \frac{z}{6}$. Through a point P(1, 2, 5), line PN is drawn perpendicular to AB and line PQ is drawn parallel to the plane $3x + 4y + 5z = 0$ to meet AB is Q. Then

(A*) coordinate of N is $\left(\frac{52}{49}, -\frac{78}{49}, \frac{156}{49}\right)$

(B*) the coordinates of Q is $\left(3, -\frac{9}{2}, 9\right)$

(C*) the equation of PN is $\frac{x-1}{3} = \frac{y-2}{-176} = \frac{z-5}{-89}$

(D*) the equation of PQ is $\frac{x-1}{4} = \frac{y-2}{-13} = \frac{z-5}{8}$

The planes $2x - 3y - 7z = 0$, $3x - 14y - 13z = 0$ and $8x - 31y - 33z = 0$

(A*) pass through origin
(B*) intersect in a common line
(C) from a triangular prism
(D) none of these

Column – I		Column – II	
(A)	The distance of the point (1, 3, 4) from the plane $2x - y + z = 3$ measured parallel to the line $\frac{x}{2} = \frac{y}{-1} = \frac{z}{-1}$ is	(p)	0
(B)	The shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is	(q)	$\frac{1}{\sqrt{6}}$
(C)	The points (0, –1, –1), (4, 5, 1), (3, 9, 4) and (–4, 4, k) are coplanar then k =	(r)	4
(D)	The volume of tetrahedron included between the plane $2x - 3y + 4z - 12 = 0$ and three co-ordinate planes is	(s)	12

Ans. (A) → (p), (B) → (q), (C) → (r), (D) → (s)

Statement 1 : The locus represented by $xy + yz = 0$ is A pair of perpendicular planes.
Statement 2 : If $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are perpendicular then $a_1a_2 + b_1b_2 + c_1c_2 = 1$.
(C*) Statement-1 is True, Statement-2 is False

Statement 1 : The equation $2x^2 - 6y^2 + 4z^2 + 18yz + 2zx + xy = 0$ represents a pair of perpendicular planes.
Statement 2 : A pair of planes given by $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$ are perpendicular. if $a + b + c = 0$
(D*) Statement-1 is False, Statement-2 is True

Comprehension Let L_1 and L_2 be the lines whose equation are $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ respectively. A and B are two points on L_1 and L_2 respectively such that AB is perpendicular both the lines L_1 and L_2.

1 Shortest distance between the lines L_1 and L_2 is
(A) $\sqrt{30}$ (B) $2\sqrt{30}$ (C*) $3\sqrt{30}$ (D) none of these

2 Co-ordinates of the point A are
(A) (1, 8, 2) (B*) (3, 8, 3) (C) (–3, 8, 3) (D) none of these

3 Co-ordinates of the point B are
(A*) (–3, –7, 6) (B) (2, 7, 6) (C) (1, 6, 3) (D) none of these

The value of k such that $\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2}$ lies in the plane $2x - 4y + z = 7$, is
(A) 7 (B) –7 (C) no real value (D) 4

A plane passes through (1, −2, 1) and is perpendicular to two planes $2x - 2y + z = 0$, $x - y + 2z = 4$. The distance of the plane from the point (1, 2, 2) is

(A) 0 (B) 1 (C) $\sqrt{2}$ (D) $2\sqrt{2}$

Consider the following linear equations **[IIT–2007]**

$$ax + by + cz = 0$$
$$bx + cy + az = 0$$
$$cx + ay + bz = 0$$

Match the conditions/expressions in **Column I** with statements in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the ORS.

	Column I		**Column II**
(A)	$a + b + c \neq 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	(P)	the equation represent planes meeting only at a single point
(B)	$a + b + c = 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$	(q)	the equations represent the line $x = y = z$
(C)	$a + b + c \neq 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$	(r)	the equations represent identical planes
(D)	$a + b + c = 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	(s)	the equations represent the whole of three dimensional space

Equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is

(A) $x + 2y - 2z = 0$ (B) $3x + 2y - 2z = 0$ (C) $x - 2y + z = 0$ (D) $5x + 2y - 4z = 0$

The point P is the intersection of the straight line joining the points $Q(2,3,5)$ and $R(1, -1, 4)$ with the plane $5x - 4y - z = 1$. If S is the foot of the perpendicular drawn from the point $T(2, 1, 4)$ to QR, then the length of the line segment PS is

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) 2 (D) $2\sqrt{2}$

The equation of a plane passing through the line of intersection of the planes $x + 2y + 3z = 2$ and $x - y + z = 3$ and at a distance $\frac{2}{\sqrt{3}}$ from the point $(3, 1, -1)$ is

(A) $5x - 11y + z = 17$ (B) $\sqrt{2}x + y = 3\sqrt{2} - 1$

(C) $x + y + z = \sqrt{3}$ (D) $x - \sqrt{2}y = 1 - \sqrt{2}$

If the straight lines $\frac{x-1}{2}=\frac{y+1}{k}=\frac{z}{2}$ and $\frac{x+1}{5}=\frac{y+1}{2}=\frac{z}{k}$ are coplanar, then the plane(s) containing these two lines is(are)

(A) $y+2z=-1$ (B) $y+z=-1$ (C) $y-z=-1$ (D) $y-2z=-1$

Let a, b and c be three real numbers satisfying

$$[a \quad b \quad c]\begin{bmatrix}1 & 9 & 7\\ 8 & 2 & 7\\ 7 & 3 & 7\end{bmatrix}=[0 \quad 0 \quad 0] \qquad \text{...........(E)}$$

58. If the point $P(a, b, c)$, with reference to (E), lies on the plane $2x+y+z=1$, then the value of $7a+b+c$ is

(A) 0 (B) 12 (C) 7 (D) 6

A plane P passes through the intersection of $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})+3=0$ and $\vec{r}\cdot(\hat{i}-\hat{j}+\hat{k})=2$. If p divide the line segment joining (3, 0, 2) and (0, 3, –1) in the ratio 2 : 1 internally and the equation of plane is given by $\vec{r}\cdot(a\hat{i}-\hat{j}+b\hat{k})=c$ when a, b, c $\in$ N than a + b + c is/are __________.

Match the following column–I with column–II.

	Column – I		Column – II
(A)	The equation of the right bisector plane of the segment joining (2, 3, 4) and (6, 7, 8) is	(p)	$\frac{13}{\sqrt{21}}$
(B)	The equation of the plane through the point (1, 2, –3) which is parallel to the plane 3x – 5y + 2z = 11 is given by	(q)	$\frac{7}{\sqrt{21}}$
(C)	The distance of the point (2, 1, –1) from the plane x – 2y + 4z = 9 is	(r)	$\frac{20}{\sqrt{21}}$
(D)	The line $\frac{x-4}{1}=\frac{y-2}{1}=\frac{z-k}{2}$ lies completely on the plane 2x – 4y + z = 7, then the value of $\frac{k}{\sqrt{21}}$ is	(s)	x + y + z – 15 = 0
		(t)	3x – 5y + 2z + 13 = 0

If the line $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-4}{6}$ intersect the xy, yz and zx planes at A, B and C respectively, and if volume of the tetrahedron OABD is V, where 'O' is origin and D is the image of C in the x–axis, then the value of [V] is __________. (Where [.] denote greatest integer function).

Show that the equation to the plane containing the line $\frac{y}{b}+\frac{z}{c}=1$, $x=0$ and parallel to the line $\frac{x}{a}-\frac{z}{c}=1$, $y=0$ is $\frac{x}{a}-\frac{y}{b}-\frac{z}{c}+1=0$. Hence show that, if 2d be the shortest distance between the lines, then $\frac{1}{d^2}=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$

A point P moves on a fixed plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$. The plane through P perpendicular to OP meets the axes in A, B, C. The planes through A, B, C parallel to co-ordinate planes intersect in Q. Show that the locus of Q is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{ax}+\frac{1}{by}+\frac{1}{cz}$

If P′ (α′, β′, γ′) be refection of point P (α, β, γ) with respect to a line passing through origin and whose direction cosine are l, m, n, then prove that $\frac{\alpha+\alpha'}{l}=\frac{\beta+\beta'}{m}=\frac{\gamma+\gamma'}{n}=2\,(l\alpha+\beta m+n\gamma)$

If the plane $x=y=z$ intersect the plane $b^2x+(2-4a)y+z=1$, $a^2x+(1-2b)y+z=-1$, then the all possible values of a and b are

(A) a = 1, b = 1
(B) a = 1, b = 2
(C) a = 2, b = 1
(D) a = 2, b = 2

The lines $r = a + \lambda(b \times c)$ and $r = b + \mu(c \times a)$ will intersect if

(A) $a \times c = b \times c$
(B) $a \cdot c = b \cdot c$
(C) $b \times a = c \times a$
(D) none of these

The straight lines whose direction cosines are given by the equations $al + bm + cn = 0$ and $ul^2 + vm^2 + wn^2 = 0$ are parallel if

(A) $\frac{a^2}{u}+\frac{b^2}{v}+\frac{c^2}{w}=1$
(B) $\frac{a^2}{u}+\frac{b^2}{v}+\frac{c^2}{w}=0$
(C) $\frac{a^2}{u}+\frac{b^2}{w}=\frac{2b^2}{v}$
(D) $\frac{a^2}{u}+\frac{c^2}{w}+\frac{b^2}{v}=-1$

A variable plane cuts off intercepts from the co-ordinate axes which are equal to the roots of the equation $x^3 + 5x = p - qx^2$ (p, q are real numbers). The locus of the foot of the perpendicular from the origin to the plane is

(A) $(x^2 + y^2 + z^2)^2\,(xy + yz + zx) = 5$
(B) $(x^2 + y^2 + z^2)^4\,(1/xy + 1/yz + 1/zx) = 5$
(C) $(x^2 + y^2 + z^2)^2\,(1/xy + 1/yz + 1/zx) = 5$
(D) $(x^2 + y^2 + z^2)^4\,(xy + yz + zx) = 5$

Given the planes $2x + 3y - 4z + 7 = 0$ and $x - 2y + 32 - 5 = 0$, if a point P is (1, −2, 3), then

(A) O and P both lie in obtuse angle
(B) O and P both lie in acute angle
(C) O lies in acute angle, P lies in obtuse angle
(D) O lies in obtuse angle, P lies in acute angle

The equations of two straight lines are $\frac{x-1}{2}=\frac{y+3}{1}=\frac{z-2}{-3}$ and $\frac{x-2}{1}=\frac{y-1}{-3}=\frac{z+3}{2}$

Statement-1: The given lines are coplanar

and

Statement-2: The equations $2r-s=1$, $r+3s=4$, $3r+2s=5$ are consistent

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 is **NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

Read the paragraph carefully and answer the following questions:

A ray of light emancipating from the point source P(1, −3, 2) and traveling parallel to the line $\frac{x-2}{1}=\frac{y}{3}=\frac{z+1}{2}$ is incident on the plane $x+y-3z=0$ at the point Q. After reflecting from the plane the ray travels along the line QR. It is also known that the reflected ray, the incident ray and the normal to the plane at the point of incident are in the same plane. Then

325. The equation of the line QR is

(A) $\frac{x-12}{15}=\frac{y-22}{37}=\frac{z-4}{10}$

(B) $\frac{x-3}{3}=\frac{y-15}{7}=\frac{z-6}{2}$

(C) $\frac{x-3}{15}=\frac{y-6}{37}=\frac{z-3}{10}$

(D) $\frac{x+3}{3}=\frac{y+6}{7}=\frac{z+3}{2}$

326. The equation of the plane PAR is

(A) $5x+2y-z+3=0$

(B) $11x-5y+2z=30$

(C) $5x-y-z=6$

(D) $x-y+z=6$

Read the paragraph carefully and answer the following questions:

Suppose inverse of a matrix is defined as $A^{-1}=\frac{A^T}{f(A)}$ and A^{-1} exists, where $f(A)=|A^T|$. Let A be an n^{th} order matrix then

343. Which of the following statement is correct?

(A) $n=2$, $A=\text{adj}(A)$

(B) $n=3$, $A=\text{adj}(A)$

(C) $n=2$, $A\neq\text{adj}(A)$

(D) $n=3$, $A\neq\text{adj}(A)$

344. If $|A|\ \{\neq f(A)\}$ is the probability that the equation $x^{n+1}+(2n-1)\frac{x^n}{n!}+(2n+3)x^{n-1}+n=0$ has exactly one real root then $f(A)^n=x^{n+2}$ has

(A) two real roots and two imaginary roots

(B) two real roots and one imaginary roots

(C) one real root and two imaginary roots

(D) one real root and three imaginary roots

The plane denoted by $\pi_1 : 4x+7y+4z+81=0$ is rotated through a right angle about its line of intersection with the plane $\pi_2 : 5x+3y+10z=25$. If the plane in its new position be denoted as π, and the distance of this plane from origin is $\sqrt{k}$, where $k\in N$, then $(k-12)/100$ is _____

A rubber cube $C = \{(x, y, z) \mid 0 \le x, y, z \le 1\}$ is cut by a sharp knife along the plane $x = y$, $y = z$ and $z = x$. If no piece is moved until all three cuts are made, then the number of pieces is _____

P is a point on the plane $ax + by + cz = d$. A point Q is taken on the line OP such OP. $OQ = d^2$, then the locus of Q satisfies $\frac{d(ax+by+cz)}{x^2+y^2+z^2}$ is equal to _____

A variable plane cuts the x-axis, y-axis and z-axis at the points A, B and C respectively such that the volume of the tetrahedron OABC remains constant equal to 32 cubic unit and O is the origin of the coordinate system

	Column I		Column II
(A)	The locus of the centroid of the tetrahedron is	(p)	$xyz = 24$
(B)	the locus of the point equidistant from O, A, B and C is	(q)	$(x^2+y^2+z^2)^3 = 192xyz$
(C)	The length of the foot of perpendicular from origin to the plane is	(r)	$xyz = 30$
(D)	If PA, PB and PC are mutually perpendicular then the locus of P is	(s)	$(x^2+y^2+z^2)^3 = 1536xyz$
		(t)	$xyz = 3$

The distance of the point having position vector $-\hat{i} + 2\hat{j} + 6\hat{k}$ from the straight line passing through the point $(2, 3, -4)$ and parallel to the vector, $6\hat{i} + 3\hat{j} - 4\hat{k}$ is

(a) $2\sqrt{13}$ (b) $4\sqrt{3}$ (c) 6 (d) 7

The vertices B and C of a ΔABC lie on the line, $\frac{x+2}{3} = \frac{y-1}{0} = \frac{z}{4}$ such that $BC = 5$ units. Then, the area (in sq units) of this triangle, given that the point $A(1, -1, 2)$ is

(a) $\sqrt{34}$ (b) $2\sqrt{34}$
(c) $5\sqrt{17}$ (d) 6

Let $\sqrt{3}\,\hat{i} + \hat{j}$, $\hat{i} + \sqrt{3}\,\hat{j}$ and $\beta\hat{i} + (1-\beta)\hat{j}$ respectively be the position vectors of the points A, B and C with respect to the origin O. If the distance of C from the bisector of the acute angle between OA and OB is $\frac{3}{\sqrt{2}}$, then the sum of all possible values of β is

(a) 1 (b) 3
(c) 4 (d) 2

The length of the perpendicular drawn from the point (2, 1, 4) to the plane containing the lines $\mathbf{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and $\mathbf{r} = (\hat{i} + \hat{j}) + \mu(-\hat{i} + \hat{j} - 2\hat{k})$ is

(a) 3 (b) $\frac{1}{3}$ (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

A perpendicular is drawn from a point on the line $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z}{1}$ to the plane $x+y+z=3$ such that the foot of the perpendicular Q also lies on the plane $x-y+z=3$. Then, the coordinates of Q are

(a) $(-1, 0, 4)$ (b) $(4, 0, -1)$
(c) $(2, 0, 1)$ (d) $(1, 0, 2)$

If $Q(0,-1,-3)$ is the image of the point P in the plane $3x-y+4z=2$ and R is the point $(3,-1,-2)$, then the area (in sq units) of ΔPQR is

(a) $\frac{\sqrt{91}}{2}$ (b) $2\sqrt{13}$
(c) $\frac{\sqrt{91}}{4}$ (d) $\frac{\sqrt{65}}{2}$

The magnitude of the projection of the vector $2\hat{i}+3\hat{j}+\hat{k}$ on the vector perpendicular to the plane containing the vectors $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+3\hat{k}$, is

(a) $\frac{\sqrt{3}}{2}$ (b) $\sqrt{6}$ (c) $3\sqrt{6}$ (d) $\sqrt{\frac{3}{2}}$

The image of the line $\frac{x-1}{3}=\frac{y-3}{1}=\frac{z-4}{-5}$ in the plane $2x-y+z+3=0$ is the line

(a) $\frac{x+3}{3}=\frac{y-5}{1}=\frac{z-2}{-5}$ (b) $\frac{x+3}{-3}=\frac{y-5}{-1}=\frac{z+2}{5}$
(c) $\frac{x-3}{3}=\frac{y+5}{1}=\frac{z-2}{-5}$ (d) $\frac{x-3}{-3}=\frac{y+5}{-1}=\frac{z-2}{5}$

If the distance of the point $P(1,-2,1)$ from the plane $x+2y-2z=\alpha$, where $\alpha>0$, is 5, then the foot of the perpendicular form P to the plane is

(a) $\left(\frac{8}{3},\frac{4}{3},-\frac{7}{3}\right)$ (b) $\left(\frac{4}{3},\frac{4}{3},\frac{1}{3}\right)$
(c) $\left(\frac{1}{3},\frac{2}{3},\frac{10}{3}\right)$ (d) $\left(\frac{2}{3},\frac{1}{3},\frac{5}{2}\right)$

A line with positive direction cosines passes through the point $P(2,-1,2)$ and makes equal angles with the coordinate axes. The line meets the plane $2x+y+z=9$ at point Q. The length of the line segment PQ equals

(a) 1 (b) $\sqrt{2}$ (c) $\sqrt{3}$ (d) 2

56. The distance of the point (1, 1, 1) from the plane passing through the point (–1, –2, –1) and whose normal is perpendicular to both the lines L_1 and L_2, is

(a) $2/\sqrt{75}$ unit (b) $7/\sqrt{75}$ unit

(c) $13/\sqrt{75}$ units (d) $23/\sqrt{75}$ units

57. The shortest distance between L_1 and L_2 is

(a) 0 unit (b) $17/\sqrt{3}$ units

(c) $41/5\sqrt{3}$ unit (d) $17/5\sqrt{3}$ units

58. The unit vector perpendicular to both L_1 and L_2 is

(a) $\dfrac{-\hat{i}+7\hat{j}+7\hat{k}}{\sqrt{99}}$ (b) $\dfrac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$

(c) $\dfrac{-\hat{i}+7\hat{j}+5\hat{k}}{5\sqrt{3}}$ (d) $\dfrac{7\hat{i}-7\hat{j}-\hat{k}}{\sqrt{99}}$

59. Consider three planes $P_1 : x - y + z = 1$

$P_2 : x + y - z = -1$

and $P_3 : x - 3y + 3z = 2$

Let L_1, L_2, L_3 be the lines of intersection of the planes P_2 and P_3, P_3 and P_1, P_1 and P_2, respectively.

Statement I Atleast two of the lines L_1, L_2 and L_3 are non-parallel.

Statement II The three planes do not have a common point.

Consider the planes $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$.

Statement I The parametric equations of the line of intersection of the given planes are $x = 3 + 14t$, $y = 1 + 2t$, $z = 15\,t$.

Statement II The vectors $14\hat{i} + 2\hat{j} + 15\hat{k}$ is parallel to the line of intersection of the given planes.

Consider the lines

$L_1 : \dfrac{x-1}{2} = \dfrac{y}{-1} = \dfrac{z+3}{1}$, $L_2 : \dfrac{x-4}{1} = \dfrac{y+3}{1} = \dfrac{z+3}{2}$ and the planes $P_1 : 7x + y + 2z = 3$, $P_2 : 3x + 5y - 6z = 4$. Let $ax + by + cz = d$ the equation of the plane passing through the point of intersection of lines L_1 and L_2 and perpendicular to planes P_1 and P_2.

Match List I with List II and select the correct answer using the code given below the lists.

	List I		List II
P.	$a =$	1.	13
Q.	$b =$	2.	-3
R.	$c =$	3.	1
S.	$d =$	4.	-2

Codes

	P	Q	R	S		P	Q	R	S
(a)	3	2	4	1	(b)	1	3	4	2
(c)	3	2	1	4	(d)	2	4	1	3

If the distance between the plane $Ax - 2y + z = d$ and the plane containing the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ is $\sqrt{6}$, then $|d|$ is equal to....

41. Perpendiculars are drawn from points on the line $\frac{x+2}{2} = \frac{y+1}{-1} = \frac{z}{3}$ to the plane x + y + z = 3. The feet of perpendiculars lie on the line

(A) $\frac{x}{5} = \frac{y-1}{8} = \frac{z-2}{-13}$ (B) $\frac{x}{2} = \frac{y-1}{3} = \frac{z-2}{-5}$

(C) $\frac{x}{4} = \frac{y-1}{3} = \frac{z-2}{-7}$ (D) $\frac{x}{2} = \frac{y-1}{-7} = \frac{z-2}{5}$

Sol. **(D)**

A line l passing through the origin is perpendicular to the lines

$l_1 : (3+t)\hat{i} + (-1+2t)\hat{j} + (4+2t)\hat{k}, -\infty < t < \infty$, $l_2 : (3+2s)\hat{i} + (3+2s)\hat{j} + (2+s)\hat{k}, -\infty < s < \infty$

Then, the coordinate(s) of the point(s) on l_2 at a distance of $\sqrt{17}$ from the point of intersection of l and l_1 is (are)

(A) $\left(\frac{7}{3}, \frac{7}{3}, \frac{5}{3}\right)$ (B) $(-1, -1, 0)$

(C) $(1, 1, 1)$ (D) $\left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right)$

(B, D)

43. Two lines $L_1 : x = 5, \frac{y}{3-\alpha} = \frac{z}{-2}$ and $L_2 : x = \alpha, \frac{y}{-1} = \frac{z}{2-\alpha}$ are coplanar. Then α can take value(s)

(A) 1 (B) 2

(C) 3 (D) 4

Sol. **(A, D)**

From a point $P(\lambda, \lambda, \lambda)$, perpendiculars PQ and PR are drawn respectively on the lines $y = x, z = 1$ and $y = -x, z = -1$. If P is such that $\angle QPR$ is a right angle, then the possible value(s) of λ is(are)

(A) $\sqrt{2}$ (B) 1

(C) -1 (D) $-\sqrt{2}$

Suppose that $\vec{p}, \vec{q}$ and $\vec{r}$ are three non-coplanar vectors in R^3. Let the components of a vector $\vec{s}$ along $\vec{p}, \vec{q}$ and $\vec{r}$ be 4, 3 and 5, respectively. If the components of this vector $\vec{s}$ along $(-\vec{p}+\vec{q}+\vec{r}), (\vec{p}-\vec{q}+\vec{r})$ and $(-\vec{p}-\vec{q}+\vec{r})$ are x, y and z, respectively, then the value of $2x + y + z$ is

In $\mathbb{R}^3$, consider the planes $P_1 : y = 0$ and $P_2 : x + z = 1$. Let P_3 be a plane, different from P_1 and P_2, which passes through the intersection of P_1 and P_2. If the distance of the point (0, 1, 0) from P_3 is 1 and the distance of a point (α, β, γ) from P_3 is 2, then which of the following relations is (are) true ?

(A) $2\alpha + \beta + 2\gamma + 2 = 0$
(B) $2\alpha - \beta + 2\gamma + 4 = 0$
(C) $2\alpha + \beta - 2\gamma - 10 = 0$
(D) $2\alpha - \beta + 2\gamma - 8 = 0$

*42. Consider a pyramid OPQRS located in the first octant ($x \geq 0, y \geq 0, z \geq 0$) with O as origin, and OP and OR along the x-axis and the y-axis, respectively. The bases OPQR of the pyramid is a square with OP = 3. The point S is directly above the mid-point T of diagonal OQ such that TS = 3. Then

(A) the acute angle between OQ and OS is $\frac{\pi}{3}$
(B) the equation of the plane containing the triangle OQS is $x - y = 0$
(C) the length of the perpendicular from P to the plane containing the triangle OQS is $\frac{3}{\sqrt{2}}$
(D) the perpendicular distance from O to the straight line containing RS is $\sqrt{\frac{15}{2}}$

Sol. **(B, C, D)**

42. Let P be the image of the point (3, 1, 7) with respect to the plane $x - y + z = 3$. Then the equation of the plane passing through P and containing the straight line $\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$ is

(A) $x + y - 3z = 0$
(B) $3x + z = 0$
(C) $x - 4y + 7z = 0$
(D) $2x - y = 0$

Sol. **(C)**

The equation of the plane passing through the point (1, 1, 1) and perpendicular to the planes $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$, is

[A] $14x + 2y - 15z = 1$
[B] $14x - 2y + 15z = 27$
[C] $14x + 2y + 15z = 31$
[D] $-14x + 2y + 15z = 3$

C

Let $P_1 : 2x + y - z = 3$ and $P_2 : x + 2y + z = 2$ be two planes. Then, which of the following statement(s) is (are) TRUE ?

(A) The line of intersection of P_1 and P_2 has direction ratios 1, 2, −1
(B) The line $\frac{3x-4}{9} = \frac{1-3y}{9} = \frac{z}{3}$ is perpendicular to the line of intersection of P_1 and P_2
(C) The acute angle between P_1 and P_2 is 60°
(D) If P_3 is the plane passing through the point (4, 2, −2) and perpendicular to the line of intersection of P_1 and P_2, then the distance of the point (2, 1, 1) from the plane P_3 is $\frac{2}{\sqrt{3}}$

C, D

Let L_1 and L_2 denote the lines $\vec{r} = \hat{i} + \lambda\left(-\hat{i} + 2\hat{j} + 2\hat{k}\right), \lambda \in R$ and $\vec{r} = \mu\left(2\hat{i} - \hat{j} + 2\hat{k}\right), \mu \in R$ respectively. If L_3 is a line which is perpendicular to both L_1 and L_2 and cuts both of them, then which of the following options describe (s) L_3?

A. $\vec{r} = \frac{2}{9}\left(4\hat{i} + \hat{j} + \hat{k}\right) + t\left(2\hat{i} + 2\hat{j} - \hat{k}\right), t \in R$

B. $\vec{r} = \frac{1}{3}\left(2\hat{i} + \hat{k}\right) + t\left(2\hat{i} + 2\hat{j} - \hat{k}\right), t \in R$

C. $\vec{r} = \frac{2}{9}\left(2\hat{i} - \hat{j} + 2\hat{k}\right) + t\left(2\hat{i} + 2\hat{j} - \hat{k}\right), t \in R$

D. $\vec{r} = t\left(2\hat{i} + 2\hat{j} - \hat{k}\right), t \in R$

A, B, C

Three lines are given by $\vec{r} = \lambda\hat{i}, \lambda \in R$, $\vec{r} = \mu\left(\hat{i} + \hat{j}\right), \mu \in R$ and $\vec{r} = v\left(\hat{i} + \hat{j} + \hat{k}\right), v \in R$. Let the lines cut the plane $x + y + z = 1$ at the points A, B and C respectively. If the area of the triangle ABC is Δ then the value of $(6\Delta)^2$ equals

Three lines

$L_1 : \vec{r} = \lambda\hat{i}, \lambda \in \mathbb{R}$

$L_2 : \vec{r} = \hat{k} + \mu\hat{j}, \mu \in \mathbb{R}$ and

$L_3 : \vec{r} = \hat{i} + \hat{j} + v\hat{k}, v \in \mathbb{R}$

are given. For which point(s) Q on L_2 can we find a point P on L_1 and a point R on L_3 so that P, Q and R are collinear?

A. $\hat{k} - \frac{1}{2}\hat{j}$

B. $\hat{k} + \frac{1}{2}\hat{j}$

C. $\hat{k}$

D. $\hat{k} + \hat{j}$

A, B

Let L_1 and L_2 be the following straight lines.

$$L_1 : \frac{x-1}{1} = \frac{y}{-1} = \frac{z-1}{3} \text{ and } L_2 : \frac{x-1}{-3} = \frac{y}{-1} = \frac{z-1}{1}$$

Suppose the straight line

$$L : \frac{x-\alpha}{l} = \frac{y-1}{m} = \frac{z-\gamma}{-2}$$

lies in the plane containing L_1 and L_2, and passes through the point of intersection of L_1 and L_2. If the line L bisects the acute angle between the lines L_1 and L_2, then which of the following statements is/are TRUE?

(A) $\alpha - \gamma = 3$ (B) $l + m = 2$ (C) $\alpha - \gamma = 1$ (D) $l + m = 0$

Let $\alpha, \beta, \gamma, \delta$ be real numbers such that $\alpha^2 + \beta^2 + \gamma^2 \neq 0$ and $\alpha + \gamma = 1$. Suppose the point $(3, 2, -1)$ is the mirror image of the point $(1, 0, -1)$ with respect to the plane $\alpha x + \beta y + \gamma z = \delta$. Then which of the following statements is/are TRUE?

(A) $\alpha + \beta = 2$

(B) $\delta - \gamma = 3$

(C) $\delta + \beta = 4$

(D) $\alpha + \beta + \gamma = \delta$

A, B, C

The equation of the plane which passes through the line of intersection of the planes $\mathbf{r} \cdot \mathbf{n}_1 = q_1$, $\mathbf{r} \cdot \mathbf{n}_2 = q_2$ and is parallel to the line of intersection of the line of intersection of the planes $\mathbf{r} \cdot \mathbf{n}_3 = q_2$ and $\mathbf{r} \cdot \mathbf{n}_4 = q_4$ given that $[\mathbf{n}_1\ \mathbf{n}_3\ \mathbf{n}_4\} = \{\mathbf{n}_2\ \mathbf{n}_3\ \mathbf{n}_4]$ is

(a) $\mathbf{r} \cdot \mathbf{n}_2 - q_2 = \mathbf{r} \cdot \mathbf{n}_3 - q_3$

(b) $\mathbf{r} \cdot \mathbf{n}_1 - q_1 = [\mathbf{n}_1\ \mathbf{n}_2\ \mathbf{n}_3]\ (\mathbf{r} \cdot \mathbf{n}_2 - q_2)$

(c) $\mathbf{r} \cdot \mathbf{n}_1 - q_1 = \mathbf{r} \cdot \mathbf{n}_2 - q_2$

(d) $\mathbf{r} \cdot \mathbf{n}_3 - q_3 = \mathbf{r} \cdot \mathbf{n}_1 - q_1$,

Ans. (c)

Consider the lines $L_1 : \frac{x-1}{2} = \frac{y}{-1} = \frac{z+3}{1}, L_2 \frac{x-4}{1} = \frac{y+3}{1} = \frac{z+3}{2}$ and the planes $P_1 : 7x + y + 2z = 3, P_2 = 3x + 5y - 6z = 4$ $ax + by + cz = d$ be the equation of the plane passing through the point of intersection of lines L_1 and L_2, and perpendicu planes P_1 and P_2.

Match List I with List II and select the correct answer using the code given below the lists :

List I		List II	
P.	$a =$	1.	13
Q.	$b =$	2.	−3
R.	$c =$	3.	1
S.	$d =$	4.	−2

Codes:

	P	Q	R	S
(a)	3	2	4	1
(b)	1	3	4	2
(c)	3	2	1	4
(d)	2	4	1	3

If the distance between the plane $Ax - 2y + z = d$ and the plane containing the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ is $\sqrt{6}$, then find $|d|$.

1. A straight line L intersects perpendicularly both the lines :

$$\frac{x+2}{2}=\frac{y+6}{3}=\frac{z-34}{-10} \text{ and } \frac{x+6}{4}=\frac{y-7}{-3}=\frac{z-7}{-2},$$

then the square of perpendicular distance of origin from L is

Let OA, OB, OC be coterminous edges of a cuboid. If l, m, n be the shortest dist sides OA, OB, OC and their respective skew body diagonals to them, respe

$$\frac{\left(\frac{1}{l^2}+\frac{1}{m^2}+\frac{1}{n^2}\right)}{\left(\frac{1}{OA^2}+\frac{1}{OB^2}+\frac{1}{OC^2}\right)}.$$

If A is the matrix $\begin{bmatrix} 1 & -3 \\ -1 & 1 \end{bmatrix}$, then $A-\frac{1}{3}A^2+\frac{1}{9}A^3 \ldots\ldots+\left(-\frac{1}{3}\right)^n A^{n+1}+\ldots\ldots$

Find $\left|\frac{a}{b}\right|$.

The ortho centre of triangle formed by the points (2,1,5),(3,2,3), (4,0,4) is

1) (2,1,5) 2) (3,2,3) 3) (4,0,4) 4) (3,1,4)

If origin is the orthocentre of a triangle formed bythe points $(\cos\alpha,\sin\alpha,0),(\cos\beta,\sin\beta,0),(\cos\gamma,\sin\gamma,0)$ then $\Sigma\cos(2\alpha-\beta-\gamma)=$

1) 0 2) 1 3) 2 4) 3

4

28. The angle between any two diagonals of a cube is

1) $\cos^{-1}\left(\frac{1}{3}\right)$ 2) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ 3) $\cos^{-1}\sqrt{\frac{2}{3}}$ 4) $\cos^{-1}\frac{2}{3}$

29. The angle between a diagonal of a cube and the diagonal of a face of the cube is

1) $\cos^{-1}\left(\frac{1}{3}\right)$ 2) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ 3) $\cos^{-1}\sqrt{\frac{2}{3}}$ 4) $\cos^{-1}\frac{2}{3}$

30. Angle between a diagonal of a cube and edge of cube is

1) $\cos^{-1}\left(\frac{1}{3}\right)$ 2) $\cos^{-1}\left(\frac{1}{2}\right)$ 3) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ 4) $\cos^{-1}\left(\sqrt{3}\right)$

31. A line makes angles $\alpha,\beta,\gamma,\delta$ with the four diagonal of a cube, then $\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta=$

1) $\frac{1}{3}$ 2) $\frac{2}{3}$ 3) $\frac{1}{5}$ 4) $\frac{4}{3}$

32. If a line makes angles $\alpha,\beta,\gamma,\delta$ with the 4 diagonals of a cube then $\sin^2\alpha+\sin^2\beta+\sin^2\gamma+\sin^2\delta=$

1) 5/3 2) 4/3 3) 8/3 4) 2/3

If θ is the angle between two lines whose d.c's are (l_1,m_1,n_1) and (l_2,m_2,n_2). Then find the value of $\Sigma\frac{(l_1+l_2)^2}{4\cos^2\frac{\theta}{2}}+\Sigma\frac{(l_1-l_2)^2}{4\sin^2\frac{\theta}{2}}$

If α,β,γ are be angles which a line makes with positive direction of the axes then find the value of $\cos^2\alpha+\cos(\beta+\gamma)\cos(\beta-\gamma)$

Suppose the direction cosines of two lines are given by $al+bm+cn=0$ ***and*** $fmn+gln+hlm=0$ ***where f, g, h, a, b, c are arbitrary constants and l, m, n are direction cosines of the lines. On the basis of the above information answer the following***

For $f = g = h = 1$ both lines satisfy the relation

a) $a\left(\frac{l}{m}\right)^2 + (a+b-c)\left(\frac{l}{m}\right) + b = 0$

b) $b\left(\frac{m}{n}\right)^2 + (b+c-a)\left(\frac{m}{n}\right) + c = 0$

c) $c\left(\frac{n}{l}\right)^2 + (c+a-b)\frac{n}{l} + a = 0$

d) All the above

The given lines will be perpendicualr if

a) $\frac{f}{a} - \frac{g}{b} + \frac{h}{c} = 0$ b) $\frac{f}{a} + \frac{g}{b} - \frac{h}{c} = 0$ c) $-\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$ d) $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$

The lines are parallel if

a) $a^2f^2 + b^2g^2 + c^2h^2 = 2(bcgh + achf + abfg)$

b) $a^2f^2 + b^2g^2 + c^2h^2 = 2(bcgh - achf + abfg)$

c) $a^2f^2 - b^2g^2 + c^2h^2 = 2(bcgh + achf + abfg)$

d) none

The reflection of the plane $2x - 3y + 4z - 3 = 0$ in the plane $x - y + z - 3 = 0$ is the plane

1) $4x - 3y + 2z - 15 = 0$

2) $x - 3y + 2z - 15 = 0$

3) $4x + 3y - 2z + 15 = 0$

4) $4x + 3y + 2z + 15 = 0$

Consider a pyramid *OPQRS* located in the first octant $(x \geq 0, y \geq 0 z \geq 0)$ with *O* as origin, and *OP* and *OR* along the x–axis and the y-axis, respectively. The base *OPQR* of the pyramid is a square with *OP*=3. The point S is directly above the midpoint *T* of diagonal *OQ* such that *TS* = 3. Then

a) the acute angle between *OQ* and *OS* is $\pi/3$

b) the equation of the plane containing the triangle *OQS* is $x-y = 0$

c) the length of the perpendicular from P to the plane containing the triangle OQS is $\frac{3}{\sqrt{3}}$

d) the perpendicualr distance from O to the straight line containing RS is $\sqrt{\frac{15}{2}}$

Column-I	Column-II
A) The shortest distance between the two straight lines $\frac{x-\frac{3}{4}}{2}=\frac{y+\frac{1}{2}}{3}=\frac{z-\frac{6}{7}}{4}$ and $\frac{4x-3}{5}=\frac{2y+1}{7}=\frac{7z-6}{6}$ is	P) $\frac{9}{2}$
B) Two lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=z$ intersect at a point then $k=$	Q) 0
C) The sphere $x^2+y^2+z^2=25$ intersects the plane $3x-4z+5=0$ in a circle then its radius is	R) 2
D) The line $x=y=z$ meets the plane $x+y+z=1$ at P and the sphere $x^2+y^2+z^2=1$ a the points R and S then $PR+PS=$	S) $2\sqrt{6}$

Passage - I :

Let the planes $P_1 : 2x-y+z=2$ and $P_2 : x+2y-z=3$ are given. On the basis of the above information, Answer the following questions

5. Equation of the plane which passes through the point (–1, 3, 2) and is perpendicular to each of the planes P_1 and P_2 is

a) $x+3y-5z+2=0$ b) $x+3y+5z-18=0$

c) $x-3y-5z+20=0$ d) $x-3y+5z=0$

6. The equation of the acute angle bisector of planes P_1 and P_2 is

a) $x-3y+2z+1=0$ b) $3x+y-5=0$ c) $x+3y-2z+1=0$ d) $3x+z+7=0$

7. The equation of the bisector of angle of the planes P_1 and P_2 which is not containing origin, is

a) $x-3y+2z+1=0$ b) $x+3y=5$ c) $x+3y+2z+2=0$ d) $3x+y=5$

A ray of light comes along the line $L=0$ and strikes the plane mirror kept along the plane $P=0$ at B. A(2, 1, 6) is a point on the line $L=0$ whose image about $P=0$ is A'. It is given that $L=0$ is $\frac{x-2}{3}=\frac{y-1}{4}=\frac{z-6}{5}$ and $P=0$ is $x+y-2z=3$

The coordinates of A' are

a) (6, 5, 2) b) (6, 5, –2) c) (6, –5, 2) d) (–6,5,2)

The coordinates of B are

a) (5, 10, 6) b) (10, 15, 11) c) (–10, –15, –14) d) (10,5,6)

If $L_1 = 0$is the reflected ray, then its equation is

a) $\frac{x+10}{4}=\frac{y-5}{4}=\frac{z+2}{3}$ b) $\frac{x+10}{3}=\frac{y+15}{5}=\frac{z+14}{5}$

c) $\frac{x+10}{4}=\frac{y+15}{5}=\frac{z+14}{3}$ d) $\frac{x+10}{5}=\frac{y+15}{4}=\frac{z+14}{3}$

In R^3 let L be a striaht line passing through the origin . Suppose that al the points on L are at a constant distacne from the two planes $P_1 : x + 2y - z + 1 = 0$ and $P_2: 2x - y + z - 1 = 0$. Let M be the locus of the feet of the perpendiculars drawn from the points on L to the plane P_1 Which of the following points lie (s) on M ?

a) $\left(0,-\frac{5}{6},-\frac{2}{3}\right)$ b) $\left(-\frac{1}{6},-\frac{1}{3},\frac{1}{6}\right)$ c) $\left(-\frac{5}{6},0,\frac{1}{6}\right)$ d) $\left(-\frac{1}{3},0,\frac{2}{3}\right)$

Column -I	Column -II
A) The distance between the line $\vec{r}=(2\hat{i}-2\hat{j}+3\hat{k})+\lambda(\hat{i}-\hat{j}+4\hat{k})$ and plane $\vec{r}.(\hat{i}+5\hat{j}+\hat{k})=5$	p) $\frac{25}{3\sqrt{14}}$
B) The distance between parallel planes $\vec{r}.(2\hat{i}-\hat{j}+3\hat{k})=4$ and $\vec{r}.(6\hat{i}-3\hat{j}+9\hat{k})+13=0$ is	q) 13 / 7
C) The distace of a point (2,5, – 3) from the plane $\vec{r}.(6\hat{i}-3\hat{j}+2\hat{k})=4$ is	r) $\frac{10}{3\sqrt{3}}$
D) The distance of the point (1,0, – 3) from the plane $x-y-2-9=0$ measured parallel to line $\frac{x-2}{2}=\frac{y+2}{3}=\frac{z-6}{-6}$	s) 7

Column I	Column II
A) Let $\vec{a}=2\hat{i}+3\hat{j}-5\hat{k},\ \vec{b}=3\hat{i}-6\hat{j}+7\hat{k},\ \vec{c}=12\hat{i}+5\hat{j}$. A vector in the plane of $\overline{a}$ and $\overline{b}$ whose projection on $\vec{c}$ is $\frac{1}{13}$ is	p) a null vector
B) Let $\overline{a}$ be a vector parallel to the line of intersection of the planes π_1 and π_2, through the origin. π_1 is parallel to the vectors $3\hat{i}+2\hat{j}$ and $3\hat{j}-4\hat{k}$. π_2 is parallel to $-\hat{j}+\hat{k}$ and $5\hat{i}+5\hat{j}$. Then $\vec{A}$ is	q) $-\frac{\hat{j}}{\sqrt{2}}+\frac{\hat{k}}{\sqrt{2}}$
C) Point (a, b, c) lies on the plane, $x+y+z=2$. Let $\vec{p}=a\hat{i}+b\hat{j}+c\hat{k}$ and $\hat{j}\times(\hat{j}\times\vec{p})=\vec{0}$ and $\hat{k}\times(\hat{k}\times\vec{p})=\vec{0}$ Then $\vec{p}$ is	r) $-\frac{1}{3}(51\hat{i}-123\hat{j}+148\hat{k})$
D) A unit vector coplanar with and perpendicular to $\hat{i}+\hat{j}+\hat{k}$ is	s) $15\lambda\hat{i}-5\lambda\hat{j}+20\lambda\hat{k}$

Two vertices of ΔABC are $A(1,0,0)B(2,0,0)$. Third vertex C lies on the line $\frac{x-1}{1}=\frac{y}{2}=\frac{z-1}{2}$ and the orthocenter of ΔABC lies on $x^2+2xy+2zx-3x-2y-z+K=0$ then K equals

If the three plane $x = y\sin\psi + z\sin\phi$, $y = z\sin\theta + x\sin\psi$, $z = x\sin\phi + y\sin\theta$, intersect in a l

then where $\theta,\phi \in \left(0, \frac{\pi}{2}\right)$

θ, ϕ & ψ satisfy

a) $\sin^2\theta + \sin^2\phi + \sin^2\psi = 1$

b) $\sin^2\theta + \sin^2\phi + \sin^2\psi + 2\sin\theta\sin\phi\sin\psi = 1$

c) $\cos^2\theta + \cos^2\phi + \cos^2\psi = 1$

d) $\sin^2\theta + \sin^2\phi + \sin^2\psi = 1$

$\theta + \phi + \psi =$

a) 90^0 b) 120^0 c) 150^0 d) 180^0

Equation of their common line is

a) $\frac{x}{\sin\theta} = \frac{y}{\sin\phi} = \frac{z}{\sin\psi}$

b) $\frac{x}{\cos\theta} = \frac{y}{\cos\phi} = \frac{z}{\cos\psi}$

c) $\frac{x}{\tan\theta} = \frac{y}{\tan\phi} = \frac{z}{\tan\psi}$

d) $\frac{x}{\cot\theta} = \frac{y}{\cot\phi} = \frac{z}{\cot\psi}$

Find the equation of a straight line in the plane $\vec{r}\cdot\vec{n} = d$ which is parallel to $\vec{r} = \vec{a} + \lambda\vec{b}$ and passes through the foot of the perpendicular drawn from point $P(\vec{a})$ to $\vec{r}\cdot\vec{n} = d$ (where $\vec{n}\cdot\vec{b} = 0$).

a) $\vec{r} = \vec{a} + \left(\frac{d - \vec{a}\cdot\vec{n}}{n^2}\right)\vec{n} + \lambda\vec{b}$

b) $\vec{r} = \vec{a} + \left(\frac{d - \vec{a}\cdot\vec{n}}{n}\right)\vec{n} + \lambda\vec{b}$

c) $\vec{r} = \vec{a} + \left(\frac{\vec{a}\cdot\vec{n} - d}{n^2}\right)\vec{n} + \lambda\vec{b}$

d) $\vec{r} = \vec{a} + \left(\frac{\vec{a}\cdot\vec{n} - d}{n}\right)\vec{n} + \lambda\vec{b}$

Let S be the set of all column matrices $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ such that $b_1, b_2, b_3 \in \mathbb{R}$ and the system of equations (in real variables)

$$-x + 2y + 5z = b_1$$
$$2x - 4y + 3z = b_2$$
$$x - 2y + 2z = b_3$$

has at least one solution. Then, which of the following system(s) (in real variables) has (have) at least one solution for each $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \in S$?

(A) $x + 2y + 3z = b_1$, $4y + 5z = b_2$ and $x + 2y + 6z = b_3$
(B) $x + y + 3z = b_1$, $5x + 2y + 6z = b_2$ and $-2x - y - 3z = b_3$
(C) $-x + 2y - 5z = b_1$, $2x - 4y + 10z = b_2$ and $x - 2y + 5z = b_3$
(D) $x + 2y + 5z = b_1$, $2x + 3z = b_2$ and $x + 4y - 5z = b_3$

A, D