Let $A=[a_{ij}]_{3\times3}$ be such that $a_{ij}=\begin{bmatrix} 3; \text{ when } \hat{i}=\hat{j} \\ 0; \quad \hat{i}\neq\hat{j} \end{bmatrix}$, then $\left\{\frac{\det(\text{adj}(\text{adj } A))}{5}\right\}$ equals :

(where $\{\cdot\}$ denotes fractional part function)

(a) $\frac{2}{5}$ (b) $\frac{1}{5}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

A square matrix P satisfies $P^2=I-P$, where I is identity matrix. If $P^n=5I-8P$, then n is :

(a) 4 (b) 5 (c) 6 (d) 7

Let matrix $A=\begin{bmatrix} x & y & -z \\ 1 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$ where $x, y, z \in N$. If det. (adj. (adj. A)) $=2^8\cdot3^4$ then the number of such matrices A is :

[Note : adj. A denotes adjoint of square matrix A.]

(a) 220 (b) 45 (c) 55 (d) 110

If A is matrix of order 3 such that $|A|=5$ and $B=adj\ A$, then the value of $\left||A^{-1}|(AB)^T\right|$ is equal to

(where $|A|$ denotes determinant of matrix A. A^T denotes transpose of matrix A, A^{-1} denotes inverse of matrix A. $adj\ A$ denotes adjoint of matrix A)

(a) 5 (b) 1 (c) 25 (d) $\frac{1}{25}$

Let M be a 3×3 matrix satisfying $M^3=0$. Then which of the following statement(s) are true:

(a) $\left|\frac{1}{2}M^2+M+I\right|\neq0$ (b) $\left|\frac{1}{2}M^2-M+I\right|=0$

(c) $\left|\frac{1}{2}M^2+M+I\right|=0$ (d) $\left|\frac{1}{2}M^2-M+I\right|\neq0$

Consider a square matrix A of order 2 which has its elements as 0, 1, 2 and 4. Let N denotes the number of such matrices.

	Column-I		Column-II
(A)	Possible non-negative value of $\det(A)$ is	(P)	2
(B)	Sum of values of determinants corresponding to N matrices is	(Q)	4
(C)	If absolute value of $(\det(A))$ is least, then possible value of $\lvert\text{adj}(\text{adj}(\text{adj } A))\rvert$	(R)	−2
(D)	If $\det(A)$ is least, then possible value of $\det(4A^{-1})$ is	(S)	0
		(T)	8

	Column-I		Column-II
(A)	**If A is an idempotent matrix and I is an identify matrix of the same order, then the value of n, such that $(A+I)^n = I + 127A$ is**	**(P)**	9
(B)	**If $(I-A)^{-1} = I + A + A^2 + ...A^7$, then $A^n = O$ where n is**	**(Q)**	10
(C)	**If A is matrix such that $a_{ij} = (i+j)(i-j)$, then A is singular if order of matrix is**	**(R)**	7
(D)	**If a non-singular matrix A is symmetic, such that A^{-1} is also symmetric, then order of A can be**	**(S)**	8

A and B are two square matrices. Such that $A^2B = BA$ and if $(AB)^{10} = A^k \cdot B^{10}$. Find the value of $k - 1020$.

Let A_n and B_n be square matrices of order 3, which are defined as :

$A_n = [a_{ij}]$ and $B_n = [b_{ij}]$ where $a_{ij} = \dfrac{2i+j}{3^{2n}}$ and $b_{ij} = \dfrac{3i-j}{2^{2n}}$ for all i and j, $1 \le i, j \le 3$.

If $l = \lim\limits_{n\to\infty} \text{Tr.}\ (3A_1 + 3^2A_2 + 3^3A_3 + ... + 3^nA_n)$ and

$m = \lim\limits_{n\to\infty} \text{Tr.}\ (2B_1 + 2^2B_2 + 2^3B_3 + ...+2^nB_n)$, then find the value of $\dfrac{(l+m)}{3}$

Let A be a 2×3 matrix whereas B be a 3×2 matrix. If det. $(AB) = 4$, then the value of det. (BA), is :

Find the maximum value of the determinant of an arbitrary 3×3 matrix A, each of whose entries $a_{ij} \in \{-1, 1\}$.

The set of natural numbers is divided into array of rows and columns in the form of matrices as $A_1 = [1], A_2 = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}, A_3 = \begin{bmatrix} 6 & 7 & 8 \\ 9 & 10 & 11 \\ 12 & 13 & 14 \end{bmatrix}$ and so on. Let the trace of A_{10} be λ. Find unit digit of λ?

If AB = A and BA = B, then both of A and B are idempotent.

If A is idempotent, then $B = I - A$ is idempotent and $AB = BA = O$.

Prove that if A and B are two matrices such that $AB = A$ and $BA = B$, then A^T, B^T are idempotent.

Use Cayley–Hamilton Theorem to express $2A^5 - 3A^4 + A^2 - 4I$ as a linear polynomial

A, when $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

If $(I - A)(I + A)^{-1}$ is orthogonal, prove that A is skew-symmetric.

If $\begin{bmatrix} 1 & 2 & a \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}^n = \begin{bmatrix} 1 & 18 & 2007 \\ 0 & 1 & 36 \\ 0 & 0 & 1 \end{bmatrix}$

then find the value of $(n + a)$.

$$\text{Let } A = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}^n = (a_{ij}(n)).$$

If $\lim_{n \to \infty} \frac{a_{12}(n)}{a_{22}(n)} = l$ where $l^2 = \sqrt{a} + \sqrt{b}$ $(a, b \in N)$, find the value of $(a + b)$.

If l_r, m_r, n_r, $r = 1, 2, 3$ be the direction cosines of three mutually perpendicular lines referred to an orthogonal cartesian co–ordinate system, then prove that

$$\begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}$$ is an orthogonal matrix.

If A is a non-singular matrix of size 3×3, then adj (adj A) is equal to

(a) $|A|\, A$ (b) $|A|^2\, A$

(c) $|A|^{-1}\, A$ (d) none of these.

Ans. (a)

Let A and B be two matrices different from I such that $AB = BA$ and $A^n - B^n$ is invertible for some positive integer n. If $A^n - B^n = A^{n+1} - B^{n+1} = A^{n+2} - B^{n+2}$, then

(a) $I - A$ is singular

(b) $I - B$ is singular

(c) $A + B = AB + I$

(d) $(I - A)(I - B)$ is non-singular

Ans. (a), (b), (c)

If A is matrix of size $n \times n$ such that $A^2 + A + 2I = O$, then

(a) A is non-singular

(b) $A \neq O$

(c) $|A| \neq 0$

(d) $A^{-1} = -\frac{1}{2}(A + I)$.

Ans. (a), (b), (c), (d)

Let $\alpha = \pi/5$ and

$$A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}, \text{ then}$$

$$B = A + A^2 + A^3 + A^4$$

is

(a) singular (b) non-singular
(c) skew-symmetric (d) $|B| = 1$

Ans. (a), (b)

Suppose $a_1, a_2, \ldots$ real numbers, with $a_1 \neq 0$. If $a_1, a_2, a_3, \ldots$ are in A.P. then

(a) $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_5 & a_6 & a_7 \end{bmatrix}$ is singular

(b) the system of equations $a_1x + a_2y + a_3z = 0$, $a_4x + a_5y + a_6z = 0$, $a_7x + a_8y + a_9z = 0$ has infinite number of solutions

(c) $B = \begin{bmatrix} a_1 & ia_2 \\ ia_2 & a_1 \end{bmatrix}$ is non singular

(d) none of these

Ans. (a), (b), (c)

Let A be a square matrix of order $n \times n$. A constant λ is said to be characteristic root of A if there exists a $n \times 1$ matrix X such that

$$AX = \lambda X$$

Example 1 If λ is a characteristic root of A, then

(a) $A - \lambda I = O$ (b) $A - \lambda I$ is singular

(c) $A - \lambda I$ is non-singular (d) none of these

Example 2 If a constant λ is such that $A - \lambda I$ is non-singular, then

(a) $\lambda = 0$ (b) $\lambda \neq 0$

(c) λ is not a characteristic root of A

(d) none of these

Example 3 If 0 is a characteristic root of A, then

(a) A is non-singular (b) A is singular

(c) $A = O$ (d) $A = I_n$

Example 4 If λ is a characteristic root of A and $n \in \mathbf{N}$, then λ^n is a characteristic root of

(a) A^n (b) A^{n-1}

(c) A^{-n} (d) $A - A^n + A^{-n}$

Example 5 Let P be a non-singular matrix, then which of the following matrices have the same characteristic roots.

(a) A and AP (b) A and PA

(c) A and $P^{-1}AP$ (d) none of these

Ans. 51. (b), 52. (c), 53. (b), 54. (a), 55. (c)

Let $\mathcal{A}$ be the set of all 3×3 symmetric matrices all whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0.

Example 56 The number of matrices in $\mathcal{A}$ is

(a) 12 (b) 6

(c) 9 (d) 3

Example 57 The number of elements in $\mathcal{A}$ for which the system of linear equations

$$A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

has a unique solution, is

(a) less than 4

(b) at least 4 but less than 7

(c) at least 7 but less than 10

(d) at least 10.

Example 58 The number of matrices A in $\mathcal{A}$ for which the system of linear equations

$$A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

is inconsistent is

(a) 0 (b) more than 2

(c) 2 (d) 1

Example 59 The number of matrices A in $\mathcal{A}$ such that $|A| = 0$, is

(a) 4 (b) 6

(c) 8 (d) none of these

Ans. 56 (a) 57. (b) 58. (b) 59. 6

Example 66 If A and B are two distinct matrices such that $A^3 = B^3$ and $A^2B = B^2A$, find $\det(A^3 + B^3)$.

Ans. 0

Example 67 Suppose a matrix A satisfies $A^2 - 5A + 7I = O$. If $A^8 = aA + bI$, find a.

Ans. 1265

Example 68 Suppose A and B are two non singular matrices such that $B \neq I$, $A^6 = I$ and $AB^2 = BA$. Find the least value of k for $B^k = I$.

Ans. 127

Example 70 Let $A = \begin{bmatrix} 0 & -\tan\alpha \\ \tan\alpha & 0 \end{bmatrix}$,

$$B(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

(a) $(I + A)(I - A)^{-1}$	(p) $A - B(\alpha)$
(b) $(I - A)(I + A)^{-1}$	(q) $B(2\alpha)$
(c) $B(\alpha)^2$	(r) $B(-2\alpha)$
(d) $B(\alpha)^{-2}$	(s) $AB(-\alpha)$

Example 71 Let A and B be two non-singular matrices such that $(AB)^k = A^k B^k$ for three consecutive positive integral values of k.

(a) ABA^{-1}	(p) A^2
(b) BAB^{-1}	(q) B
(c) AB^2A^{-1}	(r) A
(d) BA^2B^{-1}	(s) B^2

Example 73 Let A be a 2×2 matrix with real entries. Let I be the 2×2 identity matrix. Denote by $tr(A)$, the sum of diagonal entries of A. Assume that $A^2 = I$.

Statement-1: If $A \neq I$ and $A \neq -I$, then $\det(A) = -1$.

Statement-2: If $A \neq I$ and $A \neq -I$, then $tr(A) \neq 0$.

Ans. (c)

Example 75 Suppose $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ satisfies the equation $X^2 - 4X + 3I = O$.

Statement-1: If $a + d \neq 4$, then there are just two such matrix X.

Statement-2: There are infinite number of matrices X, satisfying $X^2 - 4X + 3I = O$.

Ans. (b)

69. Let p, q, r be the roots of $x^3 + x^2 + k \neq 0$, with $k \neq 0$. Denote the matrix on the left side by A.

(a) $\begin{bmatrix} 0 & (p-q)^3 & (p-r)^3 \\ (q-p)^3 & 0 & (q-r)^3 \\ (r-p)^3 & (r-q)^3 & 0 \end{bmatrix}$ (p) A is singular

(b) $\begin{bmatrix} p & q & r \\ q & r & p \\ r & p & q \end{bmatrix}$ (q) A is non-singular

(c) $\begin{bmatrix} p & 0 & 0 \\ 0 & q & 0 \\ 0 & 0 & r \end{bmatrix}$ (r) $|A| = 0$

(d) $\begin{bmatrix} 0 & 0 & p \\ 0 & q & 0 \\ r & 0 & 0 \end{bmatrix}$ (s) $|A|^2 = k^2$

A square matrix A is said to be orthogonal if $A'A = AA' = I_n$

54. If A and B are orthogonal matrices, of the same size, then which one of the following is an orthogonal matrix
(a) AB
(b) $A + B$
(c) $A + iB$
(d) $i(A + B)$

55. If $k \in \mathbf{C}$ and both $A = (a_{ij})_{n \times n}$ and kA are orthogonal matrices, then
(a) k is nth root of unity
(b) k is imaginary
(c) k must be a positive real number
(d) none of these

56. If A is real skew-symmetric matrix is such that $A^2 + I = 0$, then
(a) A is orthogonal matrix
(b) A is orthogonal matrix of odd order
(c) A is orthogonal matrix of even order
(d) none of these

57. If both $A - \frac{1}{2} I$ and $A + \frac{1}{2} I$ are orthogonal matrices, then
(a) A is skew-symmetric matrix of even order
(b) A is orthogonal
(c) $A^2 = \frac{3}{4} I$
(d) none of these

6. Let A be a square matrix such that $A^2 = I$, then at least one of $I - A$ and $I + A$ is
(a) singular
(b) symmetric
(c) skew symmetric
(d) none-singular

7. Let A be a square matrix such that $A^2 = A$ and $|A| \neq 0$, then

(a) $A = A'$ (b) $A = -A'$

(c) $A' = -I$ (d) $A = -I$

8. Let A be as in problem 8 and $B = 2A - I$ where I is the identity matrix, then

(a) $B^2 = 2B$ (b) $B^2 = I$

(c) $B^2 = O$ (d) $B^2 + B = O$

9. Suppose A and B are two nonsingular matrices such that $AB = BA^2$ and $B^5 = I$, then

(a) $A^{32} = I$ (b) $A^{31} = I$

(c) $A^{30} = I$ (d) $A^{50} = I$

10. Suppose A is a skew symmetric matrix, λ is a scalar and $X \neq O$ is a column matrix such that $AX = \lambda X$, then

(a) $\lambda \neq 0$ (b) λ is purely real

(c) λ is purely imaginary (d) none of these

11. Suppose A and B two matrices. Let C, D, E be the matrices obtained by interchanging ith and jth rows of A, B and AB respectively then

(a) $E = CD$ (b) $E = CB$

(c) $E = AD$ (d) none of these

12. Suppose A and B are two matrices. Let C, D and E be the matrices obtained by interchange ith and jth columns of A, B and AB respectively, then

(a) $E = CD$ (b) $E = CB$

(c) $E = AD$ (d) none of these

Let A be a symmetric matrix such that $A^5 = O$ and $B = I + A + A^2 + A^4$, then B is

(a) symmetric (b) singular

(c) non-singular (d) skew symmetric

Let $A = (a_{ij})_{n \times n}$ be a matrix such that $a_{ij} \in \mathbf{C}$. Define trace of A, denoted by $tr(A)$ as follows:

$$tr(A) = \sum_{i=1}^{n} a_{ii}$$

Let A, B and C be three $n \times n$ matrices such that $C'C = I_n$ and $\lambda, \mu \in \mathbf{C}$.

26. $\text{tr}(\lambda A + \mu B)$ equals
   (a) $\lambda\ \text{tr}(A) + \mu\ \text{tr}(B)$ (b) $(\lambda + \mu)\ \text{tr}(A + B)$
   (c) $\lambda\mu\ \text{tr}(A + B)$ (d) $(\lambda + \mu)\ \text{tr}(AB)$

27. $\text{tr}(AB)$ equals
   (a) $\text{tr}(A)\ \text{tr}(B)$ (b) $\text{tr}(BA)$
   (c) $\text{tr}(A) + \text{tr}(B)$ (d) none of these

28. $\text{tr}(AA')$ is
   (a) positive (b) negative
   (c) non-negative (d) non-positive

29. $\text{tr}(C'A\,C)$ equals
   (a) $\text{tr}(A)$ (b) $\text{tr}(CA)$
   (c) $\text{tr}(AC')$ (d) $\text{tr}(C)$

30. $\text{tr}(ABC)$ equals
   (a) $\text{tr}(CBA)$ (b) $\text{tr}(BAC)$
   (c) $\text{tr}(ACB)$ (d) $\text{tr}(ABC)$

31. If matrix A is such that $A^2 = A$ and $(I + A)^3 = I + kA$, find k.

32. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and

$$A^{-1} = \frac{1}{6}\,(A^2 + \alpha A + \beta I), \text{ find } \beta.$$

36. Let A be a non-singular matrix and $X \neq O$, $Y \neq O$ be two column matrices such that

$$AX = (1/100)X \text{ and } A^{-1}Y = \lambda Y,$$

find one of the possible values of λ.

Let $A = (a_{ij})_{3\times3}$ be such that $\det(A) = 5$, find $\det(A\ \text{adj}(A))$.

6. If $P = \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP'$

and $X = P' Q^{2005} P$ then X equals

(a) $\frac{1}{4}\begin{bmatrix} 2+\sqrt{3} & 1 \\ -1 & -2-\sqrt{3} \end{bmatrix}$

(b) $\frac{1}{4}\begin{bmatrix} 2005 & 2-\sqrt{3} \\ 2+\sqrt{3} & 2005 \end{bmatrix}$

(c) $\begin{bmatrix} 4+2005\sqrt{3} & 6015 \\ 2005 & 4-2005\sqrt{3} \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

Let a, b and c be three distinct real numbers and $f(x)$ be a quadratic polynomial satisfying the equation

$$\begin{bmatrix} 4a^2 & 4a & 1 \\ 4b^2 & 4b & 1 \\ 4c^2 & 4c & 1 \end{bmatrix} \begin{bmatrix} f(-1) \\ f(1) \\ f(2) \end{bmatrix} = \begin{bmatrix} 3a^2 + 3a \\ 3b^2 + 3b \\ 3c^2 + 3c \end{bmatrix}$$

Let V be the point of local maxima of $y = f(x)$ and A be the point where $y = f(x)$ meets the x-axis and B be a point on $y = f(x)$ such that AB subtends a right angle at V. Find the area of the region lying between the curve and chord AB.

Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$ and X_1, X_2, X_3 be three column matrices such that

$AX_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $AX_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$ and $AX_3 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ and let X be a 3×3 matrix whose columns are X_1, X_2, X_3.

8. Value of det (X) is
   (a) −2 (b) −1
   (c) 3 (d) 0
9. Sum of the elements of X^{-1} is
   (a) −1 (b) 0
   (c) 4 (d) 3/4

10. If $[a] = [3\ 2\ 0]\ X \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$, then a equals

(a) 5
(b) 4
(c) 3/2
(d) 5/2

Let $\mathcal{A}$ be the set of all 3 × 3 symmetric matrices all of whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0.

11. The number of matrices in $\mathcal{A}$ is

(a) 12
(b) 6
(c) 9
(d) 3

12. The number of matrices A in $\mathcal{A}$ for which the system of linear equations

$$A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

has a unique solution, is

(a) less than 4
(b) at least 4 but less than 7
(c) at least 7 but less than 10
(d) at lcast 10

13. The number of matrices A in $\mathcal{A}$ for which the system of linear equations

$$A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

is inconsistent, is

(a) 0
(b) more than 2
(c) 2
(d) 1

The number of 3×3 matrices A whose entries are either 0 or 1 and for which the system $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has exactly two distinct solutions, is

(a) 0 (b) 2^9-1 (c) 168 (d) 2

19. Let $P = [a_{ij}]$ be a 3×3 matrix and let $Q = [b_{ij}]$, where $b_{ij} = 2^{i+j} a_{ij}$ for $1 \le i, j \le 3$. If the determinant of P is 2, then the determinant of the matrix Q is

(a) 2^{10} (b) 2^{11} (c) 2^{12} (d) 2^{13}

20. If P is a 3×3 matrix such that $P^T = 2P + I$, where P^T is the transpose of P and I is the 3×3 identity matrix, then there exists a column matrix $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \ne \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ such that

(a) $PX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ (b) $PX = X$

(c) $PX = 2X$ (d) $PX = -X$

21. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix}$ and I be the identity matrix of order 3.

If $Q = [q_{ij}]$ is a matrix such that $P^{50} - Q = I$, then $\dfrac{q_{31} + q_{32}}{q_{21}}$ equals

(a) 52 (b) 103

(c) 201 (d) 205

5. For 3 × 3 matrices M and N, which of the following statement(s) is (are) **NOT** correct?

(a) N^TMN is symmetric or skew symmetric, according as M is symmetric or skew symmetric

(b) MN – NM is skew symmetric for all symmetric matrices M and N

(c) MN is symmetric for all symmetric matrices M and N

(d) (adj M) (adj N) – adj (MN) for all invertible matrices M and N

Let ω be a complex cube root of unity with $\omega \neq 1$ and $P = [p_{ij}]$ be a $n \times n$ matrix with $p_{ij} = \omega^{i+j}$. Then $p^2 \neq 0$, when n = ***(JEE Adv. 2013)***

(a) 57 (b) 55 (c) 58 (d) 56

7. Let M be a 2 × 2 symmetric matrix with integer entries. Then M is invertible if ***(JEE Adv. 2014)***

(a) The first column of M is the transpose of the second row of M

(b) The second row of M is the transpose of the first column of M

(c) M is a diagonal matrix with non-zero entries in the main diagonal

(d) The product of entries in the main diagonal of M is not the square of an integer

Let M and N be two 3 × 3 matrices such that $MN = NM$. Further, if $M \neq N^2$ and $M^2 = N^4$, then ***(JEE Adv. 2014)***

(a) determinant of $(M^2 + MN^2)$ is 0

(b) there is 3 × 3 non-zero matrix U such that $(M^2 + MN^2)U$ is the zero matrix

(c) determinant of $(M^2 + MN^2) \geq 1$

(d) for a 3 × 3 matrix U, if $(M^2 + MN^2)U$ equals the zero matrix then U is the zero matrix

10. Let X and Y be two arbitrary, 3 × 3, non-zero, skew-symmetric matrices and Z be an arbitrary 3 × 3, non zero, symmetric matrix. Then which of the following matrices is (are) skew symmetric? ***(JEE Adv. 2015)***

(a) $Y^3Z^4 - Z^4Y^3$ (b) $X^{44} + Y^{44}$

(c) $X^4Z^3 - Z^3X^4$ (d) $X^{23} + Y^{23}$

11. Let $P=\begin{bmatrix}3 & -1 & -2\\ 2 & 0 & \alpha\\ 3 & -5 & 0\end{bmatrix}$, where $\alpha \in \mathbb{R}$. Suppose $Q=[q_{ij}]$ is a matrix such that $PQ = kI$, where $k \in \mathbb{R}$, $k \neq 0$ and I is the identity matrix of order 3. If $q_{23} = -\frac{k}{8}$ and $\det(Q) = \frac{k^2}{2}$, then *(JEE Adv. 2016)*

(a) $a = 0, k = 8$ (b) $4a - k + 8 = 0$

(c) $\det(P\,\text{adj}(Q)) = 2^9$ (d) $\det(Q\,\text{adj}(P)) = 2^{13}$

2. Let k be a positive real number and let

$$A=\begin{bmatrix}2k-1 & 2\sqrt{k} & 2\sqrt{k}\\ 2\sqrt{k} & 1 & -2k\\ -2\sqrt{k} & 2k & -1\end{bmatrix} \text{ and } B=\begin{bmatrix}0 & 2k-1 & \sqrt{k}\\ 1-2k & 0 & 2\sqrt{k}\\ -\sqrt{k} & -2\sqrt{k} & 0\end{bmatrix}$$

If $\det(\text{adj}\,A) + \det(\text{adj}\,B) = 10^6$. then [k] is equal to

[**Note** : adj M denotes the adjoint of square matrix M and $[k]$ denotes the largest integer less than or equal k. *(2010)*

3. Let M be a 3×3 matrix satisfying

$$M\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}-1\\2\\3\end{bmatrix},\ M\begin{bmatrix}1\\-1\\0\end{bmatrix}=\begin{bmatrix}1\\1\\-1\end{bmatrix}, \text{ and } M\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}0\\0\\12\end{bmatrix}.$$ Then the

sum of the diagonal entries of M is *(2011)*

I have the following $n \times n$ matrix:

$$A = \begin{bmatrix} a & b & \dots & b \\ b & a & \dots & b \\ \vdots & \vdots & \ddots & \vdots \\ b & b & \dots & a \end{bmatrix}$$

where $0 < b < a$.

I am interested in the expression for the determinant $\det[A]$ in terms of a, b and n.

Proof of this result related to Fibonacci numbers: $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^n = \begin{pmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{pmatrix}$?

Prove that $A + I$ is invertible if A is nilpotent

$I_m - AB$ is invertible if and only if $I_n - BA$ is invertible.

If A and $AB - BA$ commute, show that $AB - BA$ is nilpotent

If $A = AB - BA$, is A nilpotent?

If $A + B = AB$ then $AB = BA$

$AB - BA = I$ having no solutions

54. For 3×3 matrices M and N, which of the following statement(s) is (are) NOT correct ?
(A) $N^T MN$ is symmetric or skew symmetric, according as M is symmetric or skew symmetric
(B) MN – NM is skew symmetric for all symmetric matrices M and N
(C) MN is symmetric for all symmetric matrices M and N
(D) (adj M) (adj N) = adj(MN) for all invertible matrices M and N

Sol. **(C, D)**

56. Consider the set of eight vectors $V = \{a\hat{i} + b\hat{j} + c\hat{k} \,;\, a, b, c \in \{-1, 1\}\}$. Three non-coplanar vectors can be chosen from V in 2^p ways. Then p is ________

Sol. **(5)**

47. Let M be a 2×2 symmetric matrix with integer entries. Then M is invertible if
(A) the first column of M is the transpose of the second row of M
(B) the second row of M is the transpose of the first column of M
(C) M is a diagonal matrix with non-zero entries in the main diagonal
(D) the product of entries in the main diagonal of M is not the square of an integer

48. Let M and N be two 3×3 matrices such that MN = NM. Further, if $M \neq N^2$ and $M^2 = N^4$, then
(A) determinant of $(M^2 + MN^2)$ is 0
(B) there is a 3×3 non-zero matrix U such that $(M^2 + MN^2)U$ is the zero matrix
(C) determinant of $(M^2 + MN^2) \geq 1$
(D) for a 3×3 matrix U, if $(M^2 + MN^2)U$ equals the zero matrix then U is the zero matrix

50. Let X and Y be two arbitrary, 3×3, non-zero, skew-symmetric matrices and Z be an arbitrary 3×3, non-zero, symmetric matrix. Then which of the following matrices is (are) skew symmetric ?
(A) $Y^3Z^4 - Z^4Y^3$ (B) $X^{44} + Y^{44}$
(C) $X^4Z^3 - Z^3X^4$ (B) $X^{23} + Y^{23}$

44. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in \mathbb{R}$. Suppose $Q = [q_{ij}]$ is a matrix such that PQ = kI, where $k \in \mathbb{R}, k \neq 0$ and I is the identity matrix of order 3. If $q_{23} = -\frac{k}{8}$ and $\det(Q) = \frac{k^2}{2}$, then
(A) $\alpha = 0, k = 8$ (B) $4\alpha - k + 8 = 0$
(C) $\det(P\,\text{adj}(Q)) = 2^9$ (D) $\det(Q\,\text{adj}(P)) = 2^{13}$

Sol. **(B, C)**

37. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix}$ and I be the identity matrix of order 3. If $Q = [q_{ij}]$ is a matrix such that $P^{50} - Q = I$, then $\frac{q_{31} + q_{32}}{q_{21}}$ equals
(A) 52 (B) 103
(C) 201 (D) 205

Sol. **(B)**

Q.37 Which of the following is(are) NOT the square of a 3×3 matrix with real entries ?
[A] $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ [B] $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$
[C] $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ [D] $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

Sol. **A, B**

Q.41 How many 3×3 matrices M with entries from {0, 1, 2} are there, for which the sum of the diagonal entries of M^TM is 5 ?
[A] 126 [B] 198
[C] 162 [D] 135

Sol. **B**

Q.3 Let S be the set of all column matrices $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ such that $b_1, b_2, b_3 \in \mathrm{R}$ and the system of equations (in real variables)

$$\begin{aligned} -x + 2y + 5z &= b_1 \\ 2x - 4y + 3z &= b_2 \\ x - 2y + 2z &= b_3 \end{aligned}$$

has at least one solution. Then, which of the following system(s) (in real variables) has (have) at least one solution for each $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \in \mathrm{S}$?

(A) $x + 2y + 3z = b_1$, $4y + 5z = b_2$ and $x + 2y + 6z = b_3$
(B) $x + y + 3z = b_1$, $5x + 2y + 6z = b_2$ and $-2x - y - 3z = b_3$
(C) $-x + 2y - 5z = b_1$, $2x - 4y + 10z = b_2$ and $x - 2y + 5z = b_3$
(D) $x + 2y + 5z = b_1$, $2x + 3z = b_2$ and $x + 4y - 5z = b_3$

Sol. **A, D**

Q. 4 Let $\mathrm{M} = \begin{bmatrix} 0 & 1 & a \\ 1 & 2 & 3 \\ 3 & b & 1 \end{bmatrix}$ and adj $\mathrm{M} = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$ where a and b area real numbers. Which of the following options is/are correct?

A. $(\text{adj } \mathrm{M})^{-1} + \text{adj } \mathrm{M}^{-1} = -\mathrm{M}$

B. If $\mathrm{M}\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, then $\alpha - \beta + \gamma = 3$

C. $\det(\text{adj } \mathrm{M}^2) = 81$

D. $a + b = 3$

Sol. **A, B, D**

Q.2 Let $x \in \mathbb{R}$ and let

$$\mathrm{P} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}, \mathrm{Q} = \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 6 \end{bmatrix} \text{ and } \mathrm{R} = \mathrm{PQP}^{-1}$$

Then which of the following options is/are correct?

A. For $x = 0$, if $\mathrm{R}\begin{bmatrix} 1 \\ a \\ b \end{bmatrix} = 6\begin{bmatrix} 1 \\ a \\ b \end{bmatrix}$, then $a + b = 5$

B. For $x = 1$, there exists a unit vector $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ for which $\mathrm{R}\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

C. $\det \mathrm{R} = \det\begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 5 \end{bmatrix} + 8$, for all $x \in \mathbb{R}$

D. There exists a real number x such that PQ = QP

Sol. **A, C**

Q.7 Let $P_1 = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $P_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ $P_3 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$P_4 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$, $P_5 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, $P_6 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

and $X = \sum_{k=1}^{6} P_k \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 1 \end{bmatrix} P_k^T$

where P_k^T denotes the transpose of the matrix P_k. Then which of the following options is/are correct?

A. $X - 30I$ is an invertible matrix

B. X is a symmetric matrix

C. The sum of diagonal entries of X is 18

D. If $X\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \alpha\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, then $\alpha = 30$

Sol. **B, C, D**

Q.4 Suppose $\det \begin{bmatrix} \sum_{k=0}^{n} k & \sum_{k=0}^{n} {}^nC_k k^2 \\ \sum_{k=0}^{n} {}^nC_k k & \sum_{k=0}^{n} {}^nC_k 3^k \end{bmatrix} = 0$ holds for some positive integer n. Then $\sum_{k=0}^{n} \frac{{}^nC_k}{k+1}$ equals. ____

Sol. **6.20**

Q. 4 Let $M = \begin{bmatrix} 0 & 1 & a \\ 1 & 2 & 3 \\ 3 & b & 1 \end{bmatrix}$ and $\text{adj } M = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$ where a and b area real numbers. Which of the following options is/are correct?

A. $(\text{adj } M)^{-1} + \text{adj } M^{-1} = -M$

B. If $M\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, then $\alpha - \beta + \gamma = 3$

C. $\det(\text{adj } M^2) = 81$

D. $a + b = 3$

Sol. **A, B, D**

8. Let M be a 3×3 invertible matrix with real entries and let I denote the 3×3 identity matrix. If $M^{-1} = \text{adj}(\text{adj } M)$, then which of the following statements is/are ALWAYS TRUE?

(A) $M = I$ (B) $\det M = 1$ (C) $M^2 = I$ (D) $(\text{adj } M)^2 = I$

Sol. **B, C, D**

Q.4. The trace of a square matrix is defined to be the sum of its diagonal entries. If A is a 2×2 matrix such that the trace of A is 3 and the trace of A^3 is -18, then the value of the determinant of A is _____

Sol. **5**

7. If $A=\begin{bmatrix}2&1\\4&1\end{bmatrix}; B=\begin{bmatrix}3&4\\2&3\end{bmatrix}$ and $C=\begin{bmatrix}3&-4\\-2&3\end{bmatrix}$ then $tr(A)+tr\left(\frac{ABC}{2}\right)+tr\left(\frac{A(BC)^2}{4}\right)+tr\left(\frac{A(BC)^3}{8}\right)+....\infty=$

1) 6 2) 9 3) 12 4) 15

If $AB = A$ and $BA = B$ then

1) A = 2B

2) $A^2 = A$ and $B^2 = B$

3) 2A = B

4) cannot be determined

14. Let $A=\begin{bmatrix}1&2\\3&4\end{bmatrix}$ and $B=\begin{bmatrix}a&0\\0&b\end{bmatrix}, a,b\in N$. Then

1) there connot exists any B such that $AB=BA$
2) there exists more than one but finite number of B's such that $AB=BA$
3) there exists exactly one B such that $AB=BA$.
4) there exists infinitely many b's such that $AB=BA$.

15. If $I=\begin{bmatrix}1&0\\0&1\end{bmatrix}, E=\begin{bmatrix}0&1\\0&0\end{bmatrix}$, then $(aI+bE)^3=$

1) $aI+bE$ 2) a^3I+b^3E 3) a^3I+3ab^2E 4) a^3I+3a^2bE

18. If A,B are two square matrices such that AB=B; BA=A and $n\in N$ then $(A+B)^n=$

1) $2^n(A+B)$ 2) $2^{n-1}(A+B)$ 3) $2^{n+1}(A+B)$ 4) $2^{n/2}(A+B)$

21. Let A be a 2×2 matrix with non-zero entires and let $A^2=I$, where I is a 2×2 identity matrix. Define Tr (A) sum of diagonal elements of A.

Statement 1 : Tr $(A) = 0$

Statement 2 : Tr $(A) \neq 0$

1) Statement 1 is false, statement 2 is true.
2) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1.
3) Statement 1 is ture, statement 2 is true; statement 2 is not a correct explanation for statement 2.
4) Statement 1 is true, statement 2 is false.

30. The maximum number of different possible non-zero entries in a skew-symmetric matrix of order 'n' is

1) $\frac{1}{2}(n^2-n)$ 2) $\frac{1}{2}(n^2+n)$ 3) n^2 4) (n^2-n)

1. Let A and B two symmetric matrices of order 3.

Statement 1 : $A(BA)$ **and** $(AB)A$ **are symmetric matrices**

Statement 2 : AB is symmetic matrix if matrix multiplication of A with B is commutative.

1) Statement 1 is flase, statement 2 is true.

2) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1.

3) Statement 1 is true, statement 2 is true; statement 2 is not a correct explanation for statement 1.

4) Statement 1 is true, statement 2 is flase

A and B are two non - singular matrices such that $A^6 = I$ and $AB^2 = BA(B \neq I)$. A value of k so the $B^k = I$ is

a) 31 b) 32 c) 64 d) 63

4. Let $P=\begin{bmatrix}1 & 0 & 0\\ 4 & 1 & 0\\ 16 & 4 & 1\end{bmatrix}$ and I be the identity matrix of order 3. If $Q=[q_{ij}]$ is a matrix such tha $P^{50}-Q=I$, then $\dfrac{q_{31}+q_{32}}{q_{21}}$ equals

a) 52 b) 103 c) 201 d) 205

5. If P is a 3 × 3 matrrix such that $P^T = 2P + I$ where P^T is the transpose of P and I is 3 × 3 identity matrix then there exists a column matrix $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix} \neq \begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$ such that

a) $PX=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$ b) $PX = X$ c) $PX = 2X$ d) $PX = -X$

If $A=\begin{bmatrix}\alpha_1 & \alpha_2\\ \beta_1 & \beta_2\end{bmatrix}$ (where $\alpha_2 \neq \beta_1$ and $\alpha_1, \alpha_2, \beta_1, \beta_2$ are non-zero) satisfies the equation $x^2 + k = 0$ then

a) Tra (A) = 0 b) $\alpha_1\beta_2 < 0$ c) $\text{Tra}(A) \neq 0$ d) $\alpha_1\beta_2 > 0$

For 3 × 3 matrices M and N which of the following statement(s) is (are) not correct?

a) N^TMN is symmetric or skew symmetric, according as M is symmetric or skew symmetric

b) MN-NM is skew symmetric for all symmetric matrices M and N

c) MN is symmetric for all symmetric matrices M and N

d) (adjM) (adjN) = adj(MN) for all invertible matrices M and N

Passage - II :

If A and B are two square matrices of order 3 × 3 which satisfy AB=A and BA=B, then

12. Which of the following is ture ?

a) If matrix A is singular then matrix B is non-singular

b) If matrix A is non-singular then matrix B is singualr.

c) If matrix A is singualr then matrix B is also singular

d) Cannot say anything.

13. $(A + B)^7$ is equal to

a) 7(A + B) b) $7 \times I_{3\times3}$ c) 64(A + B) d) 8I

14. $(A + I)^5$ is equal to (where is identity matrix)

a) I + 60I b) I + 16A c) I + 31A d) I + 32A

15.

Column-I	Column-II
A) $(I-A)^n$ is if A is idempotent	p) $2^{n-1}(I - A)$
B) $(I-A)^n$ is if A is involuntary	q) $I-nA$
C) $(I-A)^n$ is if A is nilpotent of index 2	r) 0
D) If $A,B,A+I,A+B$ are idempotent matrices, then $AB+BA$ is equal to	s) $I-A$

16. If matrix $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$, where a, b, c are real positive numbers, $abc = 1$ and $A^T A = I$, then find the value of $a^3 + b^3 + c^3$.

17. If the product of n matrices $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}\cdots\cdots\begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$ is equal to the matrix $\begin{pmatrix} 1 & 378 \\ 0 & 1 \end{pmatrix}$ and $n = a^3$ (where $a \in N$) then $a =$

18. If $a_k = k(10_{C_K}), b_k = (10-k)(10_{C_K})$ and $A_k = \begin{pmatrix} a_k & 0 \\ 0 & b_k \end{pmatrix}$ and $A = \sum_{k=1}^{9} A_K = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$ then the sum of digits in $\frac{ab}{2^9 - 1}$ is

19. $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, AA^T = 2I$. If $P(\alpha,\beta)$ divides $Q(1, 2)$ and $R(2, 5)$ in the ratio $c : b$ then $\alpha + \beta =$

20. A be the set of 3 × 3 matrices formed by entries 0, −1 and 1 only. There are three (1), three (−1) and three (0). The number of symmetric matrices with trace $(A) = 0$ is K then $\frac{K}{6}$ is

Let A be a 2×2 matrix with real entires. Let I be the 2×2 identity matrix. Denote by tr(A), the sum of diagonal entires of A. Assume that $A^2 = I$.

Statement 1 : If $A \neq I$ and $A \neq -I$, then det $A = -1$.

Statement 2 : If $A \neq I$ and $A \neq -I$, then tr $(A) \neq 0$.

1) Statement 1 is flase, statement 2 is true.
2) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1.
3) Statement 1 is true, statement 2 is true; statement 2 is not a correct explanation for statement 1.
4) Statement 1 is true, statement 2 is false.

17. If A and B are different matrices satisfying $A^3 = B^3$ and $A^2B = B^2A$, then

1) $\det(A^2 + B^2)$ must be zero
2) $\det(A - B)$ must be zero
3) $\det(A^2 + B^2)$ as well as det $(A-B)$ must be zero.
4) At least one of det $(A^2 + B^2)$ or det $(A-B)$ must be zero.

18. If $f(n) = \alpha^n + \beta^n$ and $\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix} = k(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$, then k is equal to

1) 1
2) –1
3) $\alpha\beta$
4) $\alpha\beta\gamma$

Let A be orthogonal and non-singular matrix of order n, then the determinant of matrix $(A-I_n)$ is equal to

a) $|I_n - A|$
b) $|A||I_n - A|$
c) $|A|$
d) $(-1)^0|A||I_n - A|$

29. If the entries in a 3×3 determinant are either 0 or 1, then the greatest value of their determinants is

1) 1
2) 2
3) 3
4) 9

Let M and N be two 3×3 matrices such that $MN=NM$, further, if $M \neq N^2$ and $M^2 = N^4$ then

a) determinant of $(M^2 + MN^2)$ is 0
b) There is a 3×3 non-zero matrix U such that $(M^2 + MN^2)U$ is zero matrix
c) determinant of $(M^2 + MN^2) \geq 1$
d) for a 3×3 matrix u, if $(M^2 + MN^2)U$ equal to the zero matrix, then U is the zero matrix

Passage - I :

Let α,β be the roots of $ax^2+bx+c=0$ and let $S_n=\alpha^n+\beta^n$, for $n\ge 1$ and $\Delta=\begin{vmatrix}3 & 1+S_1 & 1+S_2\\ 1+S_1 & 1+S_2 & 1+S_3\\ 1+S_2 & 1+S_3 & 1+S_4\end{vmatrix}$

21. If $\Delta_1=\begin{vmatrix}1 & 1 & 1\\ 1 & \alpha & \beta\\ 1 & \alpha^2 & \beta^2\end{vmatrix}$ then $\frac{\Delta}{\Delta_1^2}$ is equal to

a) 1 b) 2 c) 0 d) 3

22. Value of Δ interms of α,β is

a) $((\alpha+1)(\alpha-\beta)(\beta-1))^2$
b) $((\alpha-1)(\alpha-\beta)(\beta+1))^2$
c) $((\alpha-1)(\alpha+\beta)(\beta-1))^2$
d) $((\alpha-1)(\alpha-\beta)(\beta-1))^2$

23. Value of Δ interms of a,b,c is

a) $\frac{(a+b+c)^2(b^2-4ac)}{a^4}$
b) $\frac{(a+b+c)^2(b^2+4ac)}{a^4}$
c) $\frac{(a+b-c)^2(b^2+4ac)}{a^4}$
d) $\frac{(a+b+c)^2(b^2-4ac)}{b^4}$

42. Let k be a positive real number and let $A=\begin{vmatrix}2k-1 & 2\sqrt{k} & 2\sqrt{k}\\ 2\sqrt{k} & 1 & -2k\\ -2\sqrt{k} & 2k & -1\end{vmatrix}$ and $B=\begin{vmatrix}0 & 2k-1 & \sqrt{k}\\ 1-2k & 0 & 2\sqrt{k}\\ -\sqrt{k} & -2\sqrt{k} & 0\end{vmatrix}$

If det (adj A) + det (adj B) = 10^6 then [k] is equal to [Note : adj M denotes the adjoint of a square matrix M and [k] denotes the largest integer less than or equal to k]

If $\Delta(x)=\begin{vmatrix}0 & 2x-2 & 2x+8\\ x-1 & 4 & x^2+7\\ 0 & 0 & x+4\end{vmatrix}$ and $f(x)=\sum_{j=1}^{3}\sum_{i=1}^{3}a_{ij}c_{ij}$, where a_{ij} is the element of i^{th} row and j^{th} column in $\Delta(x)$ and c_{ij} is the cofactor of a_{ij}, for all i,j then greatest value of $f(x)$, for $x\in[-3,18]$ is

10. Let P and Q be 3×3 matrices $P\neq Q$. If $P^3=Q^3$ and $P^2Q=Q^2P$, then determinant of (P^2+Q^2) is equal to

1) -2 2) 1 3) 0 4) – 1

12. If $|A|\neq 0$ and $(A-2I)(A-3I)=0$ then $A^{-1}=$

1) $\frac{A-5I}{6}$ 2) $\frac{5I-A}{5}$ 3) $\frac{5A-I}{6}$ 4) $\frac{5I-A}{6}$

14. Let A be a square matrix all of whose entires are integers. Then which one of the following is true?

1) If det $A = \pm 1$, then A^{-1} exists but all its entries are not necessarily integers.

2) If det $A \neq \pm 1$, then A^{-1} exists and all its entires are non-integers.

3) If det $A = \pm 1$, then A^{-1} exists and all its entries are integers.

4) If det $A = \pm 1$, then A^{-1} need not exist.

17. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then $(AdjA)^{-1} =$

1) I 2) A 3) $-A$ 4) 0

18. Statement-I: There are only finitely many 2 × 2 matrices which commute with the matrix $\begin{bmatrix} 1 & \\ -1 & \end{bmatrix}$

Statement-II: If A is non-singular, then it commutes with I, adj A and A^{-1}

1) Statement 1 is false, statement 2 is true.

2) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1.

3) Statement 1 is true, statement 2 is true; statement 2 is not a correct explanation for statement

4) Statement 1 is true, statement 2 is false.

19. If A is a matrix of order 2 × 2

Statement-I : adj (adjA) = A

Statement-II : $|adjA| = |A|$.

1) only I true 2) only II true 3) I and II true 4) I and II false

2. If A, B, C are three square matrices of same order satisfying $A^2 = A^{-1}$ and let $B = A^{2^n}$, $C = A^{2^{n-2}}$ $(n \in N)$ then det (B-C) =

1) 0 2) 1 3) -1 4) $(-1)^n$

If $A = [a_{ij}]_{4\times 4}$ such that $a_{ij} = \begin{cases} 2 & when\ i = j \\ 0 & when\ i \neq j \end{cases}$ then $\left\{ \frac{\det(adj(adjA))}{7} \right\}$ is (where { } represents fractional part of x) is

1) $\frac{1}{7}$ 2) $2/7$ 3) $3/7$ 4) $9/7$

If A is an 3 × 3 non-singular matrix such that $AA' = A'A$ and $B = A^{-1}A'$, then BB' equals

1) I+B 2) I 3) B^{-1} 4) $(B^{-1})'$

4. Let $A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}, B=\begin{bmatrix}\cos 2\beta & \sin 2\beta\\ \sin 2\beta & -\cos 2\beta\end{bmatrix}$ where $0<\beta<\frac{\pi}{2}$ then which of the following is(are) true

a) $(AB)^2=I$ b) $(AB)^{-1}=AB$ c) $BAB=A^{-1}$

d) The least positive value of α for which $BA^4B=A^{-1}$ is $\frac{2\pi}{3}$

5. Which of the following statements are false

a) If A and B are square matrices of same order such that $ABAB=0$ then it follows that $BABA=0$

b) Let A and B be different nxn matrices with real entries. If $A^3=B^3$ and $A^2B=B^2A$ then A^2+B^2 is invertible

c) If A is square singular and symmetic matrix then $\left(\left(A^{-1}\right)^{-1}\right)^{-1}$ is skew symmetric

d) The matrix of product of two invertible square matrices of same order is also invertible.

6. Which of the following statements is/are true about square matrix A of order n?

a) $(-A)^{-1}$ is equal to $-A^{-1}$ when n is odd only.

b) If $A^n=O$, then $I+A+A^2+.....+A^{n-1}=(I-A)^{-1}$.

c) If A is skew- symmetric matrix of odd order, then its inverse does not exist.

d) $(A^{-1})^T=(A^T)^{-1}$ holds always.

P is a non-singular matrix and A,B are two matrices such that $B=P^{-1}AP$ then the true statements among the following are

a) A is invertible iff B is invertible

b) $B^n=P^{-1}A^nP\,\forall n\in N$

c) $\forall\lambda\in R, B-\lambda I=P^{-1}(A-\lambda I)P$

d) A, B are both singular matrices

Paragraph - II :

Let $A=\begin{bmatrix}1&0&0\\2&1&0\\3&2&1\end{bmatrix}$ *be a square matrix and* C_1, C_2, C_3 *be 3 column matrices s*

$AC_1=\begin{bmatrix}1\\0\\0\end{bmatrix}, AC_2=\begin{bmatrix}2\\3\\0\end{bmatrix}$ *and* $AC_3=\begin{bmatrix}2\\3\\1\end{bmatrix}$ *of matrix B. If the matrix* $C=\frac{1}{3}AB$ *then*

14. The value of sum of elements of B^{-1} is

a) -1 b) 0 c) 4 d) 2

15. The ratio of trace of matrix A to the det of matrix B is

a) 1 : 3 b) 2 :3 c) 1 : 1 d) 3 : 1

16. The value of $\sin^{-1}|A|+\cos^{-1}|C|$ is (where $|A|$ is determinant of A)

a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{4}$ d) 1

17. Let $A=\begin{bmatrix}1 & \tan x\\ -\tan x & 1\end{bmatrix}$

Column-I	Column-II
A) A^{-1}	p) $\begin{bmatrix}1 & \tan x\\ -\tan x & 1\end{bmatrix}$
B) $(adjA)^{-1}$	q) $2\begin{bmatrix}1 & -\tan x\\ \tan x & 1\end{bmatrix}$
C) $adj(adjA)$	r) $\frac{1}{2}\begin{bmatrix}1+\cos 2x & -\sin 2x\\ \sin 2x & 1+\cos 2x\end{bmatrix}$
D) $adj(2A)$	s) $\frac{1}{2}\begin{bmatrix}1+\cos 2x & \sin 2x\\ -\sin 2x & 1+\cos 2x\end{bmatrix}$

18.

Column-I	Column-I
A) If $\lvert A\rvert=2$, then $\lvert 2A^{-1}\rvert=$ (where A is of order 3)	p) 1
B) If $\lvert A\rvert = 1/8$, then $\lvert adj(adj(2A))\rvert =$ (where A is of order 3)	q) 4
C) If $(A+B)^2=A^2+B^2$, and $\lvert A\rvert=2$ then $\lvert B\rvert=$ (Where A and B are of odd order)	r) 24
D) $\lvert A_{2\times2}\rvert = 2, \lvert B_{3\times3}\rvert = 3$ and $\lvert C_{4\times4}\rvert = 4$, then $\lvert ABC\rvert$ is equal to	s) 0

20. If A & B are different matrices satisfying $A^3 = B^3$ *and* $A^2B = B^2A$ Find $\lvert A^2+B^2\rvert =$______

21. If $A=\begin{bmatrix}2 & 1\\ -4 & -2\end{bmatrix}$ and if $I+2A+3A^2+\text{.......}\infty=\begin{bmatrix}\alpha & \beta\\ \gamma & \delta\end{bmatrix}$ then numerical value of $\lvert\alpha+\beta+\gamma+\delta\rvert$ is

22. If A is a square matrix of order 3 such that $\lvert A\rvert = 2$, then $\left\lvert \left(adjA^{-1}\right)^{-1}\right\rvert$ is ______

23. $A=\begin{bmatrix}1 & \tan x\\ -\tan x & 1\end{bmatrix}$ and $f(x)$ is defined as $f(x)=$ det. (A^TA^{-1}) then the value of $\underbrace{f(f(f(f\text{.................}f(x))))}_{\text{n times}}$ is $(n\geq 2)$_________

3. Let $[A_k]_{n\times n}$ be a square matrix of order $n \times n$, such that $a_{ij} = \begin{cases} 0, i \neq j \\ \frac{1}{k+i}, i = j \end{cases}$ and $[B_k]_{n\times n}$ is its inverse matrix, then which is/are true?

a) $\underset{m\to\infty}{Lt}\left(\frac{\sum_{n=1}^{m}\text{trace}(B_k)_{n\times n}}{m^3}\right) = \frac{1}{6}$

b) $\sum_{n=1}^{10}\text{trace}(B_2)_{n\times n} = 320$

c) $\underset{m\to\infty}{Lt}\left(\frac{\sum_{n=1}^{m}\text{trace}(B_k)_{n\times n}}{m^3}\right) = \frac{1}{3}$

d) $\sum_{n=1}^{10}\text{trace}(B_2)_{n\times n} = 330$

If A is symmetic and B is skew symmetric and $A+B$ is non singular and also $C = (A+B)^{-1}(A-B)$ then

a) $C^T(A+B)C = A+B$

b) $C^T(A-B)C = A-B$

c) $C^T AC = A$

d) $C^T AC = O$

Passage - II :

A Pythagorean triplet is triplet of positive integers (a,b,c) such that $a^2 + b^2 = c^2$. Define matrices P,Q,R by $p = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 2 & 2 & 3 \end{bmatrix}, Q = \begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & -2 \\ 2 & 2 & 3 \end{bmatrix}$ ***and*** $R = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & 2 \\ 2 & 2 & 3 \end{bmatrix}$

9. If we write Pythagorean triplet (a,b,c) in matrix form as $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ then which of the following matrix product is not Pythagorean triplet?

a) $Q\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$ b) $P\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$ c) $R\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$ d) $\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$

10. Which one of the following does not hold good?

a) $P^{-1} = adjP$

b) $(PQ)^{-1} = adj(PQ)$

c) $(QR)^{-1} = adj(QR)$

d) $(PQR)^{-1} \neq adj(PQR)$

11. Trace of $(P + Q^T + 2R) =$

a) 17 b) 15 c) 14 d) 18

13. Let $A=\begin{pmatrix}1 & \frac{3}{2}\\ 1 & 2\end{pmatrix}$, $B=\begin{bmatrix}4 & -3\\ -2 & 2\end{bmatrix}$ and $C_r=\begin{pmatrix}r3^r & 2^r\\ 0 & (r-1)3^r\end{pmatrix}$ be given 3 matrices then the value of $\sum_{r=1}^{50} tr\left((AB)^r C_r\right)=3^b a+3$ then $\frac{a+b}{25}=$

14. Let $a_k={}^nC_k\,(0\le k\le n)$ and $A_k=\begin{pmatrix}a_{k-1} & 0\\ 0 & a_k\end{pmatrix}$ $B=\sum_{k=1}^{n-1} A_k.A_{k+1}=\begin{pmatrix}a & 0\\ 0 & b\end{pmatrix}$ then $\frac{a}{b}=$

If $A=\begin{pmatrix}1 & 0\\ 1 & 2\end{pmatrix}$ and $A^8=KA-\ell I$ then $K-\ell=$

15. Consider three matrices $A=\begin{bmatrix}2 & 1\\ 4 & 1\end{bmatrix}$, $B=\begin{bmatrix}3 & 4\\ 2 & 3\end{bmatrix}$ and $C=\begin{bmatrix}3 & -4\\ -2 & 3\end{bmatrix}$ then the absolute value of

$$\det(A)+\det\left(\frac{ABC}{2}\right)+\det\left(\frac{A(BC)^2}{4}\right)+\det\left(\frac{A(BC)^3}{8}\right)+\text{----}\infty \text{ is}$$

Let $A=\begin{bmatrix}1 & 1 & 2\\ 2 & 2 & 1\\ 2 & 1 & 2\end{bmatrix}$ then trace of A^3 is

a) 101 b) 100 c) 99 d) 98

16. Which of the following is/are true?

a) If A is a square matrix such that $A^2=A$ then $(I+A)^3-7A=I$

b) If A is a square matrix such that $A^2=A$ then $(I+A)^3-7A=O$

c) Let B and C be two square matrices such that $BC=CB$ and $C^2=O$. If $A=B+C$ then $A^3-B^3-3B^2C=O$

d) Let B and C be two square matrices such that $BC = CB$ and $C^2=O$. If $A=B+C$ then $A^3+B^3-3B^2C=O$

Passage - I :

Let A be a matrix of order 2×2 such that $A^2=O$

17. $A^2-(a+d)A+(ad-bc)I$ is equal to

a) I b) O c) $-I$ d) $2I$

18. $tr(A)$ is euqal to

a) 1 b) 0 c) -1 d) $2I$

19. $(I+A)^{100}=$

a) 100A b) 100(I+A) c) 100I+A d) I+100A

30 Let $A=\begin{bmatrix}3x^2\\1\\6x\end{bmatrix}$, $B=\begin{bmatrix}a & b & c\end{bmatrix}$, and $C=\begin{bmatrix}(x+2)^2 & 5x^2 & 2x\\5x^2 & 2x & (x+2)^2\\2x & (x+2)^2 & 5x^2\end{bmatrix}$ be three given matrices, where a, b, c and $x \in R$. Given that $tr(AB)=tr(C)\, x \in R$, where *tr(A)* denotes trace of *A*. If $f(x)=ax^2+bx+c$, then the value of $f(1)$ is ______

13. If A is a non zero square matrix of order n with $\det(I+A)\neq 0$ and $A^3=0$, where I, O are unit and null matrices of order nxn respectively then $(I+A)^{-1}=$ [

1) $I-A+A^2$ 2) $I+A+A^2$ 3) $I+A^2$ 4) $I+A$

If A is a matrix such that $A^2+A+2I=0$, then which of the following is/are true?

a) A is nonsingular b) A is symmetric

c) A cannot be skew - symmetric d) $A^{-1}=-\frac{1}{2}(A+I)$

If the matrix A and B are of 3×3 and $(I-AB)$ is invertible then which of the following statements is/are correct?

a) $I-BA$ is not invertible

b) $I-BA$ is invertible

c) $I-BA$ has for its inverse $I+B(I-AB)^{-1}A$

d) $I-BA$ has for its inverse $I+A(I-BA)^{-1}B$

For a given square matrix A, if there exists a matrix B such that AB = BA = I then B is called inverse of A. Every non–singular square matrix possesses inverse and it exists if $|A|\neq 0$, $A^1=\frac{adj(A)}{\det(A)}$ and adjA = |A| (A^{-1}).

Let a matrix $A=\begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}$ then it will satisfy the equation

a) $A^2-4A+I=O$ b) $A^2+4A+I=O$ c) $A^2-4A-5I=O$ d) $A^2-4A+5I=O$

Let a matrix $A=\begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}$ then A^{-1} will be

a) $\begin{bmatrix}2 & -3\\-1 & 2\end{bmatrix}$ b) $\begin{bmatrix}-3 & 2\\2 & -1\end{bmatrix}$ c) $\begin{bmatrix}-1 & 2\\2 & -3\end{bmatrix}$ d) $\begin{bmatrix}-2 & 3\\1 & -2\end{bmatrix}$

Let matrix $A=\begin{bmatrix}3 & 2\\1 & 1\end{bmatrix}$ satisfies the equation $A^2+aA+bI=O$ then the value of $|a+b|=$

a) 1 b) 2 c) 3 d) 4

Match the following

Column I	Column II
A) If A is an idempotent matrix and I is an identity matrix of the same order then the value of 'n', such that $(A+I)^n=I+127A$ is	p) 16
B) If A is matrix such that $a_{ij}=(i+j)(i-j)$, then A issingular if order of matrix is	q) 3
C) If $\lvert A\rvert = 2$ then $\lvert 2A^{-1}\rvert =$ (where A is of order 3)	r) 4
D) If $\lvert A\rvert=\frac{1}{4}$ then $Adj(Adj(2A)) =$ (Where A is of order 3	s) 7

18.

Column I	Column II
A) A is a square matrix such that $A^2=A$. If $(I+A)^8=I+\lambda A$ then $\lambda+1$ is	p) 64
B) If A is a square matrix of order 3 such that $\lvert A\rvert=2$ then $\left\lvert\left(adJA^{-1}\right)^{-1}\right\rvert$ is	q) 1
C) Let $\lvert A\rvert=\lvert a_{ij}\rvert_{3\times3}\neq0$ Each element a_{ij} is multiplied by λ^{i-j}. Let $\lvert B\rvert$ the resulting determinant where $\lvert A\rvert=k\lvert B\rvert$ then k is	r) 256
D) If A is a diagonal matrix of order 3×3 is commutative with every square matrix of order 3×3 under multiplication and trace (A) = 12,then $\lvert A\rvert=$	s) 4

2. If A and B are symmetric matrices of the same order $X=AB+BA$ and $Y=AB-BA$ then $(XY)^T$ is equal to

a) XY　　b) YX　　c) $-YX$　　d) $-XY$

3. Let A_n is a $n\times n$ Matrix in which diagonal elements are 1, 2, 3,..n (ie $a_{11}=1$, $a_{22}=2$, $a_{33}=3$, ... $a_n=i$,... $a_{nn}=n$)and all other elements are equal to n then

a) A_n is singular for all n　　b) A_n is non singular for all n

c) $det.A_n=(-1)^{n+1}n!$　　d) $det.A_n=0$

6. If A and B are respectively a symmetric and a skew symmetric matrix such that $AB = BA$ then

a) $(A-B)^{-1}(A+B)$ is orthogonal matrix when $(A-B)$ is non- singular.

b) $(A+B)^{-1}(A-B)$ is orthogonal matrix when $(A+B)$ is non- singular.

c) $\det\left[(A-B)^{-1}(A+B)\right]=1$ and $\det\left[(A+B)^{-1}(A-B)\right]=1$

d) $\det\left[(A-B)^{-1}(A+B)\right]=-1$ and det $x\left[(A+B)^{-1}(A-B)\right]=1$

7. Let A be an n^{th}- order square matrix and B be adjoint of A then $|AB+KI_n|$ is (where K is scalar quantity)

a) $(|A|+K)^{n-2}$ b) $(|A|+K)^{n}$ c) $(|A|+K)^{n-1}$ d) $(|A|+K)^{n-3}$

Passage - I :

If $A_0 = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ ***and*** $B_0 = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$, $B_n = adj(B_{n-1}), n \in N$ ***and I is an identity matrix of order 3.*** C_1, C_2, C_3 ***represent the column matrix of*** B_0 ***as shown*** $C_1 = \begin{bmatrix} -4 \\ 1 \\ 4 \end{bmatrix}, C_2 = \begin{bmatrix} -3 \\ 0 \\ 4 \end{bmatrix}, C_3 = \begin{bmatrix} -3 \\ 1 \\ 3 \end{bmatrix}$ ***then***

15. det $(A_0 + A_0^2 B_0^2 + A_0^3 + A_0^4 B_0^4 + \ldots\ldots$ up to 10 terms) =

a) 1000 b) –800 c) 0 d) –8000

16. $B_1 + B_2 + \ldots\ldots\ldots + B_{49} =$

a) B_0 b) $7B_0$ c) $49B_0$ d) $51B_0$

Passage - IV :

Let A be the set of all 3 × 3 symmetric matrices all of whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0.

24. The number of matrices in A is

a) 12 b) 6 c) 9 d) 3

25. The number of matrices A in A for which the system of linear equations $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has a unique solution is

a) less than 4
b) at least 4 but less than 7
c) at least 7 but less than 10
d) at least 10

26. The number of matrices A in A for which the system of linear equations $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is inconsistent is

a) 0 b) more than 2 c) 2 d) 1

The entries of 3 × 3 matrix either 1 or –1

List - I	List - II
A) Total number of symmetric matrices	p) 0
B) Maximum value of the determinants so formed	q) 3
C) The maximum value of the trace of the matrix	r) 4
D) No of skew symmetric matrices	s) 64 t) 32

List-I **List-II**

A) If $r>1, M_r = \begin{vmatrix} r-1 & \frac{1}{r} \\ 1 & \frac{1}{(r-1)^2} \end{vmatrix}$ then p) 2

$\underset{n\to\infty}{Lt}\left(|M_2|+|M_3|+|M_4|+.........+|M_n|\right)^{\log_e^n} =$

B) If $\begin{vmatrix} 1 & \cos\alpha & \cos\beta \\ \cos\alpha & 1 & \cos\gamma \\ \cos\beta & \cos\gamma & 1 \end{vmatrix} = \begin{vmatrix} 0 & \cos\alpha & \cos\beta \\ \cos\alpha & 0 & \cos\gamma \\ \cos\beta & \cos\gamma & 0 \end{vmatrix}$ q) 4

then $\sin^2\alpha+\sin^2\beta+\sin^2\gamma$

C) If $A=\begin{pmatrix} 3 & 1 \\ -1 & 1 \end{pmatrix} = C = (BAB^{-1})(B^{-1}A^TB)$ then $\sqrt{\det(C)} =$ r) 1

D) If $A=\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ and $A^4 = -\lambda I$ then $\lambda =$ s) 3

P is a non-singular matrix and A,B are two matrices such that $B=P^{-1}AP$ then the true statements among the following are

a) A is invertible iff B is invertible

b) $B^n = P^{-1}A^nP \forall n \in N$

c) $\forall \lambda \in R, B-\lambda I = P^{-1}(A-\lambda I)P$

d) A,B are both singular matrices

Let A be the 2 ×2 matrix given by $A=(a_{ij})$ where $a_{ij} \in \{0,1,2,3,4\}$ such that $a_{11}+a_{12}+a_{21}+a_{22}=4$ then which of the following statement(s) is /are true?

a) **Number of matrices A such that the trace of A is equal to 4, is 5**

b) **Number of matrices A, such that A is invertible is 18**

c) **Absolute difference between maximum value and minimum value of det(A) is 8**

d) **Number of matrices A such that A is either symmetric (or) skew symmetric and det (A) is divisible by 2, is 5**

Which of the following statements is/are true

a) **If A is a skew symmetric matrix of order n × n, then the maximum number of non-zero elements in A, is n(n-1)**

b) **If A is a symmetric matrix of order n×n, then maximum number of non-zero elements is A, is n^2**

c) **Minimum number of zero elements is an upper triangular matrix of order n × n is $\frac{n(n-1)}{2}$**

d) **total number of matrices that can be formed using all 5 different letters such that no letter is repeated in any matrix is 2.5!**

Which of the following statement (s) is/are true about square matrix A of order n?

a) $(-A)^{-1}$ is equal to $-A^{-1}$ when n is odd only

b) If $A^n=0$, then $I+A+A^2.....+A^{n-1}=(I-A)^{-1}$

c) If A is a skew symmetric matrix of odd order then its inverse does not exist

d) $(A^T)^{-1}=(A^{-1})^T$ holds always

1. If S is a real skew symmetric matrix then which of the following is true

a) $I - S$ is non-singular

b) $(I+S)(I-S)^{-1}$ is orthogonal

c) $(I+S)(I-S)^{-1}$ is non-singular

d) $I+S$ is non- singular

Let A,B be two square matrices such that $A+B = AB$ then

a) $AB = -BA$

b) $AB=BA$

c) $AB = O$

d) $AB = I$

19. Deteminant of A^{50}=

a) 0

b) 1

c) −1

d) 25

20. Trace of A^{50}=

a) 0

b) 1

c) 2

d) 3

21. Sum of product of elements of B_1 with its corresponding cofactors in the adjoint matrix B is equal to

a) Trace of B

b) Det(2B)

c) Trace of A^2

d) Det(B^3)

Paragraph - I :

Let $A=\begin{bmatrix}1&0&0\\1&0&1\\0&1&0\end{bmatrix}$ *satisfies* $A^n=A^{n-2}+A^2-I$ *for* $n\geq 3$. ***Further consider a matrix*** $B_{3\times 3}$ ***with colums***

B_1, B_2, B_3 *such that* $A^{50}B_1=\begin{bmatrix}1\\25\\25\end{bmatrix}, A^{50}B_2=\begin{bmatrix}0\\1\\0\end{bmatrix}, A^{50}B_3=\begin{bmatrix}0\\0\\1\end{bmatrix}$ ***then***

Paragraph - II :

A square matrix A is said to be orthogonal if $A^TA=AA^T=I$ with this information answer the following :

22. $P_{2\times2}$ is orthogonal matrix, $A=\begin{bmatrix}29 & -28\\30 & -29\end{bmatrix}$. If $Q=P^TAP$. Then $PQ^{2014}P^T=$

a) 2012A **b) I** **c) A** **d) A^{2011}**

23. $P_{3\times3}$ orthogonal matrix, α,β,γ are the angles made by a st.line with $\overrightarrow{OX},\overrightarrow{OY},\overrightarrow{OZ}$

$A=\begin{bmatrix}\sin^2\alpha & \sin\alpha\sin\beta & \sin\alpha\sin\gamma\\ \sin\alpha\sin\beta & \sin^2\beta & \sin\beta\sin\gamma\\ \sin\alpha\sin\gamma & \sin\beta\sin\gamma & \sin^2\gamma\end{bmatrix}$ and $Q=P^TAP$ If $PQ^6P^T=2^kA$ then $k=$

a) 5 **b) 7** **c) 6** **d) 0**

24. $A=\begin{bmatrix}0 & 2b & c\\a & b & -c\\a & -b & c\end{bmatrix}$ is orthogonal matrix then $72|abc|=$

a) 4 **b) 12** **c) 9** **d) 1**

Paragraph - II

28. If $\begin{bmatrix}4a^2 & 4a & 1\\4b^2 & 4b & 1\\4c^2 & 4c & 1\end{bmatrix}\begin{bmatrix}f(-1)\\f(1)\\f(2)\end{bmatrix}=\begin{bmatrix}3a^2+3a\\3b^2+3b\\3c^2+3c\end{bmatrix}$ where $f(x)$ is a quadratic function and $f(x)=ax^2+bx+c$ whose maximum value occurs at a point V say (α,β). Let A be the point of intersection of $y=f(x)$ with negative x-axis, say (p,o) and point B is such that the chord AB subtends a right angle at V. Let B be (r, s). Let Δ be the area enclosed by $y=f(x)$ and the chord AB. Then

Column-I	Column-II
A) $\alpha+\beta=$	p) 125/3
B) $p=$	q) -7
C) $r+s=$	r) -2
D) $\Delta=$	s) 1

29.

Column-I	Column-II
A) $(I-A)^n$ is if A idempotent	p) $2^{n-1}(I-A)$
B) $(I-A)^n$ is if A involuntary	q) $I-nA$
C) $(I-A)^n$ is if A nilpotent of index 2	r) A
D) If A is orthogonal, then $(A^T)^{-1}$	s) $I-A$

2. Let M be a 2×2 matrix such that $M\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}-1\\2\end{bmatrix}$ and $M^2\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}$ If x_1 and $x_2\,(x_1 > x_2)$ are the two values of x for which det $(M-XI)=0$ where I is identity matrix of order 2 then the value of $5x_1 + 2x_2$ is

If B is idempotent show that $A = I - B$ is also idempotent and that $AB = BA = O$.

If A and B are idempotent matrices, then show that AB is idempotent if A and B commute.

If A and B are idempotent then $A + B$ will be idempotent if $AB = BA = O$ where O is the null matrix.

If M is a 3×3 matrix, where $M^T\ M = I$ and det $(M) = 1$, then prove that det $(M - I) = 0$.

9. **If α, β, γ are three real numbers then the matrix A given below is** $\begin{bmatrix}1 & \cos(\alpha-\beta) & \cos(\alpha-\gamma)\\ \cos(\beta-\alpha) & 1 & \cos(\beta-\gamma)\\ \cos(\gamma-\alpha) & \cos(\gamma-\beta) & 1\end{bmatrix}$ **is**

(a) singular **(b) symmetric**

(c) invertible **(d) none**

Ans. (a), (b)

Let A be a square matrix satisfying $A^2 + 5A + 5I = 0$. The inverse of $A + 2I$ is equal to :

(a) $A - 2I$ (b) $A + 3I$ (c) $A - 3I$ (d) non-existent

If $A = \begin{pmatrix}1 & 2\\0 & 1\end{pmatrix}, P = \begin{pmatrix}\cos\theta & \sin\theta\\-\sin\theta & \cos\theta\end{pmatrix}, Q = P^T AP$, find $PQ^{2014}P^T$:

(a) $\begin{pmatrix}1 & 2^{2014}\\0 & 1\end{pmatrix}$ (b) $\begin{pmatrix}1 & 4028\\0 & 1\end{pmatrix}$

(c) $(P^T)^{2013}A^{2014}P^{2013}$ (d) $P^TA^{2014}P$

If M be a square matrix of order 3 such that $|M| = 2$, then $\left|adj\left(\frac{M}{2}\right)\right|$ equals to :

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $\frac{1}{16}$

If A and B are two orthogonal matrices of order n and $\det(A) + \det(B) = 0$, then which of the following must be correct ?

(a) $\det(A + B) = \det(A) + \det(B)$

(b) $\det(A + B) = 0$

(c) A and B both are singular matrices

(d) $A + B = 0$

$$\text{Let } A = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}^n = \left(a_{ij}(n)\right).$$

If $\lim_{n \to \infty} \frac{a_{12}(n)}{a_{22}(n)} = l$ where $l^2 = \sqrt{a} + \sqrt{b}$ $(a, b \in N)$, find the value of $(a + b)$.

$A = \begin{bmatrix} a & b \\ b & -a \end{bmatrix}$ and $MA = A^{2m}, m \in N, a, b \in R$, for some matrix M, then which one of the following is correct :

(a) $M = \begin{bmatrix} a^{2m} & b^{2m} \\ b^{2m} & -a^{2m} \end{bmatrix}$

(b) $M = (a^2 + b^2)^m \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(c) $M = (a^m + b^m) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(d) $M = (a^2 + b^2)^{m-1} \begin{bmatrix} a & b \\ b & -a \end{bmatrix}$

Let α be a repeated root of the quadratic equation $f(x) = 0$ and $A(x)$, $B(x)$ and $C(x)$ are polynomials of degree 3, 4 and 5 respectively.

Show that the determinant $\begin{vmatrix} A(x) & B(x) & C(x) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix}$

is divisible by $f(x)$ where $A'(\alpha) = \left(\frac{dA}{dx}\right)_{x=\alpha}$.