7. If $\vec{a}$ and $\vec{b}$ are non-zero, non-collinear vectors such that $|\vec{a}| = 2$, $\vec{a}\cdot\vec{b} = 1$ and angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$. If $\vec{r}$ is any vector such that $\vec{r}\cdot\vec{a} = 2$, $\vec{r}\cdot\vec{b} = 8$, $(\vec{r} + 2\vec{a} - 10\vec{b})\cdot(\vec{a}\times\vec{b}) = 4\sqrt{3}$ and satisfy to $\vec{r} + 2\vec{a} - 10\vec{b} = \lambda(\vec{a}\times\vec{b})$, then λ is equal to :

(a) $\frac{1}{2}$ (b) 2 (c) $\frac{1}{4}$ (d) None of these

Let $\vec{a} = 3\hat{i} + 2\hat{j} + 4\hat{k}$; $\vec{b} = 2(\hat{i} + \hat{k})$ and $\vec{c} = 4\hat{i} + 2\hat{j} + 3\hat{k}$. Sum of the values of '$\alpha$' for which the equation $x\vec{a} + y\vec{b} + z\vec{c} = \alpha(x\hat{i} + y\hat{j} + z\hat{k})$ has non-trivial solution is :

(a) -1 (b) 4 (c) 7 (d) 8

If $\hat{a}, \hat{b}$ are unit vectors and $\vec{c}$ is such that $\vec{c} = \vec{a}\times\vec{c} + \vec{b}$, then the maximum value of $[\vec{a}\ \vec{b}\ \vec{c}]$ is :

(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) $\frac{3}{2}$

If $\vec{a} = \hat{i} + 6\hat{j} + 3\hat{k}$; $\vec{b} = 3\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = (\alpha + 1)\hat{i} + (\beta - 1)\hat{j} + \hat{k}$ are linearly dependent vectors and $|\vec{c}| = \sqrt{6}$; then the possible value(s) of $(\alpha + \beta)$ can be :

(a) 1 (b) 2 (c) 3 (d) 4

Let $OABC$ be a tetrahedron whose edges are of unit length. If $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ and d $\overrightarrow{OC} = \alpha(\vec{a} + \vec{b}) + \beta(\vec{a}\times\vec{b})$, then $(\alpha\beta)^2 = \frac{p}{q}$ where p and q are relatively prime to each other. Find the value of $\left[\frac{q}{2p}\right]$ where [·] denotes greatest integer function.

Let $\vec{v}_0$ be a fixed vector and $\vec{v}_0 = \begin{bmatrix}1\\0\end{bmatrix}$. Then for $n \geq 0$ a sequence is defined $\vec{v}_{n+1} = \vec{v}_n + \left(\frac{1}{2}\right)^{n+1}\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}^{n+1}\vec{v}_0$ then $\lim_{n\to\infty}\vec{v}_n = \begin{bmatrix}\alpha\\\beta\end{bmatrix}$. Find $\frac{\alpha}{\beta}$.

Let $\vec{r} = (\vec{a}\times\vec{b})\sin x + (\vec{b}\times\vec{c})\cos y + 2(\vec{c}\times\vec{a})$, where $\vec{a}, \vec{b}, \vec{c}$ are non-zero and non-coplana vectors. If $\vec{r}$ is orthogonal to $\vec{a} + \vec{b} + \vec{c}$, then find the minimum value of $\frac{4}{\pi^2}(x^2 + y^2)$.

Let P and Q are two points on curve $y = \log_{\frac{1}{2}}\left(x - \frac{1}{2}\right) + \log_2\sqrt{4x^2 - 4x + 1}$ and P is also on $x^2 + y^2 = 10$. Q lies inside the given circle such that its abscissa is integer. Find the smallest possible value of $\overrightarrow{OP}\cdot\overrightarrow{OQ}$ where 'O' being origin.

In above problem find the largest possible value of $|\overrightarrow{PQ}|$.

If $\hat{i}\times[(\vec{a} - \hat{j})\times\hat{i}] + \hat{j}\times[(\vec{a} - \hat{k})\times\hat{j}] + \hat{k}\times[(\vec{a} - \hat{i})\times\hat{k}] = 0$ and $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$, then :

(a) $x + y = 1$ (b) $y + z = \frac{1}{2}$ (c) $x + z = 1$ (d) None of these

The vector $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC. The length of the median through A is :

(a) $\sqrt{288}$ (b) $\sqrt{72}$ (c) $\sqrt{33}$ (d) $\sqrt{18}$

[2008]

The edges of a parallelopiped are of unit lengths and are parallel to non-coplanar unit vectors **a, b, c** such that **a . b** = **b . c** = **c . a** = 1/2. Then the volume of the parallelopiped is

(a) $1/\sqrt{2}$ (b) $1/2\sqrt{2}$ (c) $\sqrt{3}/2$ (d) $1/\sqrt{3}$

Let two non-collinear unit vectors **a** and **b** form an acute angle. A point P moves so that at any time t the position vector **OP** (where O is the origin) is given by **a** cos t + **b** sin t. When P is farthest from origin O, let M be the length of **OP** and **u** be the unit vector along **OP**. Then

(a) $\mathbf{u} = \dfrac{\mathbf{a}+\mathbf{b}}{|\mathbf{a}+\mathbf{b}|}$ and $M = (1 + \mathbf{a} . \mathbf{b})^{1/2}$

(b) $\mathbf{u} = \dfrac{\mathbf{a}-\mathbf{b}}{|\mathbf{a}-\mathbf{b}|}$ and $M = (1 + \mathbf{a} . \mathbf{b})^{1/2}$

(c) $\mathbf{u} = \dfrac{\mathbf{a}+\mathbf{b}}{|\mathbf{a}+\mathbf{b}|}$ and $M = (1 + 2\,\mathbf{a} . \mathbf{b})^{1/2}$

(d) $\mathbf{u} = \dfrac{\mathbf{a}-\mathbf{b}}{|\mathbf{a}-\mathbf{b}|}$ and $M = (1 + 2\,\mathbf{a} . \mathbf{b})^{1/2}$

Let **a, b, c** be three unit non-coplanar vectors and α, β, γ be the angles between **a** and **b**, **b** and **c**,**c** and **a** respectively. Let **x, y, z** be unit vectors along the bisectors of angles α, β, γ respectively. Prove that

$$[\mathbf{x}\times\mathbf{y}\ \ \mathbf{y}\times\mathbf{z}\ \ \mathbf{z}\times\mathbf{x}] = \frac{1}{16}\,[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]\sec^2\frac{\alpha}{2}\sec^2\frac{\beta}{2}\sec^2\frac{\gamma}{2}$$

Let $\mathbf{V} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{W} = \mathbf{i} + 3\mathbf{k}$. If $\mathbf{U}$ is a unit vector, then the maximum value of the scalar triple product $[\mathbf{U}\ \mathbf{V}\ \mathbf{W}]$ is

(a) -1 (b) $\sqrt{10} + \sqrt{6}$

(c) $\sqrt{59}$ (d) $\sqrt{60}$

If the vector $a\mathbf{i} + j + \mathbf{k}$, $\mathbf{i} + b\mathbf{j} + \mathbf{k}$, if $\mathbf{j} + c\mathbf{k}$ $(a \neq b \neq c \neq 1)$ are coplanar, then the value

$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} =$ ______. [1987]

Example 64 If $[\mathbf{b}\ \mathbf{c}\ \mathbf{d}] = 24$ and $(\mathbf{a} \times \mathbf{b}) \times (\mathbf{c} \times \mathbf{d}) + (\mathbf{a} \times \mathbf{c}) \times (\mathbf{d} \times \mathbf{b}) + (\mathbf{a} \times \mathbf{d}) \times (\mathbf{b} \times \mathbf{c}) + k\mathbf{a} = 0$ then k is equal to.

Ans. 48

Example 62 Let $\mathbf{b} = 4\mathbf{i} + 3\mathbf{j}$ and $\mathbf{c}$ be two vectors perpendicular to each other in the *xy*-plane. If $\mathbf{r}_i$, $i = 1, 2 \ldots n$ are the vectors in the same plane having projections 1 and 2 along $\mathbf{b}$ and $\mathbf{c}$ respectively then $\sum_{i=1}^{n} |r_i|^2$ is equal to.

Ans. 20

Let k be the length of any edge of a regular tetrahedron. (A tetrahedron whose edges are all equal in length is called a regular tetrahedron. The angle between a line and a plane is equal to the complement of the angle between the line and the normal to the plane whereas the angle between two planes is equal to the angle between the normals. Let O be the origin of reference and A, B and C vertices with position vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ respectively of the regular tetrahedron.

Example 57 The angle between any edge and a face not containing the edge is

(a) $\cos^{-1}(1/2)$ (b) $\cos^{-1} 1/4$

(c) $\cos^{-1} 1/\sqrt{3}$ (d) $\pi/3$

Example 58 The angle between any two faces is

(a) $\cos^{-1} 1/\sqrt{3}$ (b) $\cos^{-1} 1/4$

(c) $\pi/3$ (d) $\cos^{-1} 1/3$

Example 59 The value of $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]^2$ is

(a) k^2 (b) $(1/2)k^6$

(c) $(1/3)k^2$ (d) k^3

Ans. 57. (c), 58. (d), 59. (b)

$$\mathbf{a}\cdot\mathbf{a}' = \mathbf{b}\cdot\mathbf{b}' = \mathbf{c}\cdot\mathbf{c}' = 1 \text{ and}$$

$$\mathbf{a}\cdot\mathbf{b}' = \mathbf{a}\cdot\mathbf{c}' = \mathbf{b}\cdot\mathbf{a}' = \mathbf{b}\cdot\mathbf{c}' = \mathbf{c}\cdot\mathbf{a}' = \mathbf{c}\cdot\mathbf{b}' = 0$$

is called the reciprocal system to the vectors **a**, **b** and **c**.

Example 51 If $\mathbf{a}', \mathbf{b}', \mathbf{c}'$ is a reciprocal system of **a**, **b**, **c** then

(a) $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]\ [\mathbf{a}'\ \mathbf{b}'\ \mathbf{c}'] = |\mathbf{a}\times\mathbf{a}'|\ |\mathbf{b}\times\mathbf{b}'|\ |\mathbf{c}\times\mathbf{c}'|$

(b) $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]\ [\mathbf{a}'\ \mathbf{b}'\ \mathbf{c}'] = 1$

(c) $\mathbf{a}\times\mathbf{a}' = \mathbf{b}\times\mathbf{b}' = \mathbf{c}\times\mathbf{c}'$

(d) $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]\ [\mathbf{a}'\ \mathbf{b}'\ \mathbf{c}'] < 1$

Example 52 $\mathbf{a}\times\mathbf{a}' + \mathbf{b}\times\mathbf{b}' + \mathbf{c}\times\mathbf{c}'$ is a

(a) zero vector

(b) a nonzero vector

(c) $\dfrac{1}{[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]^3}\ ((\mathbf{a}\times\mathbf{a}')\times\mathbf{b})$

(d) is a scalar multiple of $\mathbf{a}' + \mathbf{b}' + \mathbf{c}'$

Example 53

(a) $\mathbf{a} + \mathbf{b} + \mathbf{c} = \dfrac{1}{[\mathbf{a}'\,\mathbf{b}'\,\mathbf{c}']}\ [\mathbf{b}'\times\mathbf{c}' + \mathbf{c}'\times\mathbf{a}']$

(b) $\mathbf{a} = \dfrac{\mathbf{b}'\times\mathbf{c}'}{[\mathbf{a}'\,\mathbf{b}'\,\mathbf{c}']}$

(c) $\mathbf{b} = \dfrac{\mathbf{a}'\times\mathbf{c}'}{[\mathbf{a}'\,\mathbf{b}'\,\mathbf{c}']}$

(d) $\mathbf{a} + \mathbf{b} + \mathbf{c} = \dfrac{1}{[\mathbf{a}'\,\mathbf{b}'\,\mathbf{c}']}\ [\mathbf{a}'\times\mathbf{b}' + \mathbf{b}'\times\mathbf{c}']$

Ans. 51. (b), 52. (a), 53. (b)

Example 44 Let the unit vectors **A** and **B** be perpendicular and the unit vector **C** be inclined at an angle θ to both **A** and **B**. If $\mathbf{C} = \alpha\mathbf{A} + \beta\mathbf{B} + \gamma(\mathbf{A}\times\mathbf{B})$ then

(a) $\alpha = \beta$ (b) $\gamma^2 = 1 - 2\alpha^2$

(c) $\gamma^2 = -\cos 2\theta$ (d) $\beta^2 = \dfrac{1+\cos 2\theta}{2}$

Ans. (a), (b), (c), (d)

Example 35. If **a**, **b**, **c**, **d** are unit vectors such that $(\mathbf{a} \times \mathbf{b}).(\mathbf{c} \times \mathbf{d}) = 1$ and $\mathbf{a} \cdot \mathbf{c} = 1/2$ then

(a) **a**, **b**, **c** are non-coplanar

(b) **b**, **c**, **d** are non-coplanar

(c) **b**, **d** are non-parallel

(d) **a**, **d** are parallel and **b**, **c** are parallel

Ans. (c)

Example 33 Let two non-collinear unit vectors **a** and **b** form an acute angle. A point P moves so that at any time t the position vector **OP** (where O is the origin) is given by $\mathbf{a}\cos t + \mathbf{b}\sin t$. When P is farthest from origin O, let M be the length of **OP** and **u** be the unit vector along **OP**. Then

(a) $\mathbf{u} = \dfrac{\mathbf{a}+\mathbf{b}}{|\mathbf{a}+\mathbf{b}|}$ and $M = (1 + \mathbf{a}.\mathbf{b})^{1/2}$

(b) $\mathbf{u} = \dfrac{\mathbf{a}-\mathbf{b}}{|\mathbf{a}-\mathbf{b}|}$ and $M = (1 + \mathbf{a}.\mathbf{b})^{1/2}$

(c) $\mathbf{u} = \dfrac{\mathbf{a}+\mathbf{b}}{|\mathbf{a}+\mathbf{b}|}$ and $M = (1 + 2\mathbf{a}.\mathbf{b})^{1/2}$

(d) $\mathbf{u} = \dfrac{\mathbf{a}-\mathbf{b}}{|\mathbf{a}-\mathbf{b}|}$ and $M = (1 + 2\mathbf{a}.\mathbf{b})^{1/2}$

Ans. (a)

Example 30 If $\mathbf{r} \cdot \mathbf{a} = 0$, $\mathbf{r} \cdot \mathbf{b} = 1$ and $[\mathbf{r}\ \mathbf{a}\ \mathbf{b}] = 1$, $\mathbf{a} \cdot \mathbf{b} \neq 0$, $(\mathbf{a} \cdot \mathbf{b})^2 - |\mathbf{a}|^2\,|\mathbf{b}|^2 = 1$ the value of **r** in terms of **a** and **b** is

(a) $\mathbf{a} \times (\mathbf{a} \times \mathbf{b}) + \mathbf{a} \times \mathbf{b}$ (b) $\mathbf{a} \times (\mathbf{a} \times \mathbf{b}) + \dfrac{\mathbf{a} \times \mathbf{b}}{|\mathbf{a} \times \mathbf{b}|^2}$

(c) $\mathbf{a} \times (\mathbf{b} \times \mathbf{a}) + \dfrac{\mathbf{b} \times \mathbf{a}}{|\mathbf{b} \times \mathbf{a}|^2}$ (d) $\dfrac{\mathbf{a} \times (\mathbf{a} \times \mathbf{b})}{|\mathbf{a} \times (\mathbf{a} \times \mathbf{b})|} + \mathbf{a} \times \mathbf{b}$

Ans. (b)

Example 21 If $\mathbf{a}$, $\mathbf{b}$ are nonzero vectors and $\mathbf{a}$ is perpendicular to $\mathbf{b}$ then a nonzero vector $\mathbf{r}$ satisfying $\mathbf{r} \cdot \mathbf{a} = \alpha$, for some scalar α, $\mathbf{a} \times \mathbf{r} = \mathbf{b}$ is

(a) $\dfrac{\alpha\mathbf{a} + (\mathbf{a} + \mathbf{b})}{|\mathbf{a}|^2}$ (b) $\dfrac{\alpha\mathbf{a} + \mathbf{a} \times \mathbf{b}}{|\mathbf{b}|^2}$

(c) $\dfrac{\alpha\mathbf{a} - (\mathbf{a} \times \mathbf{b})}{|\mathbf{a}|^2}$ (d) $\dfrac{\alpha\mathbf{a} - (\mathbf{a} \times \mathbf{b})}{|\mathbf{b}|^2}$

Ans. (c)

Example 18 If $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are unit vectors, then $|\mathbf{a} - \mathbf{b}|^2 + |\mathbf{b} - \mathbf{c}|^2 + |\mathbf{c} - \mathbf{a}|^2$ does not exceed

(a) 4 (b) 9 (c) 8 (d) 6

Example 15 The value of a for which the volume of parallelopiped formed by the vectors $\mathbf{i} + a\mathbf{j} + \mathbf{k}$, $\mathbf{j} + a\mathbf{k}$ and $a\mathbf{i} + \mathbf{k}$ is minimum is

(a) -3 (b) 3 (c) $1/\sqrt{3}$ (d) $-\sqrt{3}$

Ans. (c)

Example 10 Let $a = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$, $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ and $\mathbf{c} = c_1\mathbf{i} + c_2\mathbf{j} + c_3\mathbf{k}$ be three non-zero vectors such that **c** is a unit vector perpendicular to both **a** and **b**. If the angle between **a** and **b** is $\pi/6$, then

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$$

is equal to

(a) 0 (b) 1

(c) $\frac{1}{4}(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)$

(d) $\frac{3}{4}(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)(c_1^2 + c_2^2 + c_3^2)$

Ans. (c)

Example 6 If **p, q, r** are three mutually perpendicular vectors of the same magnitude and if a vector **x** satisfies the equation $\mathbf{p} \times ((\mathbf{x} - \mathbf{q}) \times \mathbf{p}) + \mathbf{q} \times ((\mathbf{x} - \mathbf{r}) \times \mathbf{q}) + \mathbf{r} \times ((\mathbf{x} - \mathbf{p}) \times \mathbf{r}) = \mathbf{0}$, then vector the **x** is

(a) $(1/2)(\mathbf{p} + \mathbf{q} - 2\mathbf{r})$ (b) $(1/2)(\mathbf{p} + \mathbf{q} + \mathbf{r})$

(c) $(1/3)(\mathbf{p} + \mathbf{q} + \mathbf{r})$ (d) $(1/3)(2\mathbf{p} + \mathbf{q} - \mathbf{r})$.

Ans. (b)

Consider the set of eight vectors

$V = \{a\hat{i} + b\hat{j} + c\hat{k} : a, b, c \in \{-1, 1\}\}$. Three non-coplanar vectors can be chosen from V in 2^p ways. Then p is

If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors satisfying

$|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 = 9$, then $|2\vec{a} + 5\vec{b} + 5\vec{c}|$ is

Let $\vec{a} = -\hat{i} - \hat{k}, \vec{b} = -\hat{i} + \hat{j}$ and $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r}.\vec{a} = 0$, then the value of $\vec{r}.\vec{b}$ is

If $\vec{a}$ and $\vec{b}$ are vectors in space given by $\vec{a} = \dfrac{\hat{i} - 2\hat{j}}{\sqrt{5}}$ and $\vec{b} = \dfrac{2\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{14}}$, then find the value of $\left(2\vec{a} + \vec{b}\right)$. $\left[\left(\vec{a} \times \vec{b}\right) \times \left(\vec{a} - 2\vec{b}\right)\right]$.

7. Match List I with List II and select the correct answer using the code given below

List I

P. Volume of parallelepiped determined by vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is 2. Then the volume of the parallelepiped determined by vectors $2(\vec{a} \times \vec{b}), 3(\vec{b} \times \vec{c})$ and $2(\vec{c} \times \vec{a})$ is

Q. Volume of parallelepiped determined by vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is 5. Then the volume of the parallelepiped determined by vectors $3(\vec{a} + \vec{b}), 3(\vec{b} + \vec{c})$ and $2(\vec{c} + \vec{a})$ is

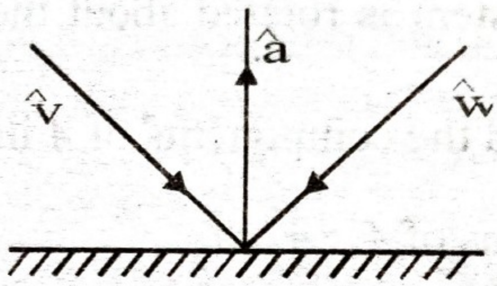
R. Area of a triangle with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 20. Then the area of the triangle with adjacent sides determined by vectors $(2\vec{a} + 3\vec{b})$ and $(\vec{a} - \vec{b})$ is

S. Area of a parallelogram with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 30. Then the area of the parallelogram with adjacent sides determined by vectors $(\vec{a} + \vec{b})$ and $\vec{a}$ is

Codes:

	P	Q	R	S
(a)	4	2	3	1
(b)	2	3	1	4
(c)	3	4	1	2
(d)	1	4	3	2

If the incident ray on a surface is along the unit vector $\hat{v}$, the reflected ray is along the unit vector $\hat{w}$ and the normal is along unit vector $\hat{a}$ outwards. Express $\hat{w}$ in terms of $\hat{a}$ and $\hat{v}$.



Let V be the volume of the parallelopiped formed by the vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$. If a_r, b_r, c_r, where $r = 1, 2, 3$, are non-negative real numbers and $\sum_{r=1}^{3}(a_r + b_r + c_r) = 3L$, show that $V \le L^3$.

Let $\vec{A}(t) = f_1(t)\hat{i} + f_2(t)\hat{j}$ and

$\vec{B}(t) = g_1(t)\hat{i} + g_2(t)\hat{j}, t \in [0, 1]$,

where f_1, f_2, g_1, g_2 are continuous functions. If $\vec{A}(t)$ and $\vec{B}(t)$ are nonzero vectors for all t and $\vec{A}(0) = 2\hat{i} + 3\hat{j}$, $\vec{A}(1) = 6\hat{i} + 2\hat{j}$, $\vec{B}(0) = 3\hat{i} + 2\hat{j}$ and $\vec{B}(1) = 2\hat{i} + 6\hat{j}$. Then show that $\vec{A}(t)$ and $\vec{B}(t)$ are parallel for some t.

Let u and v be unit vectors. If w is a vector such that $w + (w \times u) = v$, then prove that $|(u \times v) \cdot w| \le 1/2$ and that the equality holds if and only if u is perpendicular to v.

The position vectors of the vertices A, B and C of a tetrahedron $ABCD$ are $\hat{i} + \hat{j} + \hat{k}$, $\hat{i}$ and $3\hat{i}$, respectively. The altitude from vertex D to the opposite face ABC meets the median line through A of the triangle ABC at a point E. If the length of the side AD is 4 and the volume of the tetrahedron is $\frac{2\sqrt{2}}{3}$, find the position vector of the point E for all its possible positions.

If $a = i + j + k, \vec{b} = 4i + 3j + 4k$ and $c = i + \alpha j + \beta k$ are linearly dependent vectors and $|c| = \sqrt{3}$, then

(a) $\alpha = 1, \beta = -1$ (b) $\alpha = 1, \beta = \pm 1$

(c) $\alpha = -1, \beta = \pm 1$ (d) $\alpha = \pm 1, \beta = 1$

If $\vec{a}$ and $\vec{b}$ are vectors such that $\left|\vec{a} + \vec{b}\right| = \sqrt{29}$ and $\vec{a} \times \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) = \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) \times \vec{b}$, then a possible value of $\left(\vec{a} + \vec{b}\right).\left(-7\hat{i} + 2\hat{j} + 3\hat{k}\right)$ is

(a) 0 (b) 3 (c) 4 (d) 8

Let $a = 2i + j - 2k$ and $b = i + j$. If c is a vector such that a $c = |c|, |c - a| = 2\sqrt{2}$ and the angle between $(a \times b)$ and c i 30°, then $|(a \times b) \times c| =$

(a) 2/3 (b) 3/2 (c) 2 (d) 3

In ΔABC, K and L are points on AB and BC such that AK:KB= 1 : 2 and BL : LC =1: lines AL and CK interest in Q. If the area of ΔBQC is 1, then find the area of ΔABC.

Q.11. Let a and b be positive real numbers. Suppose $\overrightarrow{PQ} = a\hat{i} + b\hat{j}$ and $\overrightarrow{PS} = a\hat{i} - b\hat{j}$ are adjacent sides of a parallelogram PQRS. Let $\vec{u}$ and $\vec{v}$ be the projection vectors of $\vec{w} = \hat{i} + \hat{j}$ along $\overrightarrow{PQ}$ and $\overrightarrow{PS}$, respectively. If $|\vec{u}| + |\vec{v}| = |\vec{w}|$ and if the area of the parallelogram PQRS is 8, then which of the following statements is/are TRUE?

(A) $a + b = 4$
(B) $a - b = 2$
(C) The length of the diagonal PR of the parallelogram PQRS is 4
(D) $\vec{w}$ is an angle bisector of the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PS}$

Sol. **A, C**

16. In a triangle PQR, let $\vec{a} = \overrightarrow{QR}, \vec{b} = \overrightarrow{RP}$ and $\vec{c} = \overrightarrow{PQ}$. If

$$|\vec{a}| = 3,\ |\vec{b}| = 4 \text{ and } \frac{\vec{a}\cdot(\vec{c} - \vec{b})}{\vec{c}\cdot(\vec{a} - \vec{b})} = \frac{|\vec{a}|}{|\vec{a}| + |\vec{b}|},$$

then the value of $|\vec{a} \times \vec{b}|^2$ is _____

Sol. **108**

Q.2 Let $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ be two vectors. Consider a vector $\vec{c} = \alpha\vec{a} + \beta\vec{b}$, $\alpha, \beta \in \mathbb{R}$. If the projection of $\vec{c}$ on the vector $(\vec{a} + \vec{b})$ is $3\sqrt{2}$, then the minimum value of $(\vec{c} - (\vec{a} \times \vec{b}))\cdot\vec{c}$ equals ____

Sol. **18.00**

Q. 6 Three lines are given by $\vec{r} = \lambda\hat{i}, \lambda \in R$, $\vec{r} = \mu(\hat{i} + \hat{j}), \mu \in R$ and $\vec{r} = v(\hat{i} + \hat{j} + \hat{k}), v \in R$. Let the lines cut the plane $x + y + z = 1$ at the points A, B and C respectively. If the area of the triangle ABC is Δ then the value of $(6\Delta)^2$ equals

Q.13 Consider the cube in the first octant with sides OP, OQ and OR of length 1, along the x-axis, y-axis and z-axis, respectively, where O(0, 0, 0) is the origin. Let $S\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$ be the centre of the cube and T be the vertex of the cube opposite to the origin O such that S lies on the diagonal OT. If $\vec{p}=\overrightarrow{SP}$, $\vec{q}=\overrightarrow{SQ}$, $\vec{r}=\overrightarrow{SR}$ and $\vec{t}=\overrightarrow{ST}$, then the value of $\left|(\vec{p}\times\vec{q})\times(\vec{r}\times\vec{t})\right|$ is ______ .

Sol. **0.5**

Q.12 Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $\vec{a}\cdot\vec{b}=0$. For some x, y ∈ R, let $\vec{c}=x\vec{a}+y\vec{b}+(\vec{a}\times\vec{b})$. If $|\vec{c}|=2$ and the vector $\vec{c}$ is inclined at the same angle α to both $\vec{a}$ and $\vec{b}$, then the value of $8\cos^2\alpha$ is _____ .

Sol. **3**

Q.52 If the triangle PQR varies, then the minimum value of

$$\cos(P+Q)+\cos(Q+R)+\cos(R+P)$$

is

[A] $-\frac{5}{3}$ [B] $-\frac{3}{2}$

[C] $\frac{3}{2}$ [D] $\frac{5}{3}$

Sol. **B**

PARAGRAPH 1

Let O be the origin, and $\overrightarrow{OX}$, $\overrightarrow{OY}$, $\overrightarrow{OZ}$ be three unit vectors in the directions of the sides $\overrightarrow{QR}$, $\overrightarrow{RP}$, $\overrightarrow{PQ}$, respectively, of a triangle PQR.

Q.51 $|\overrightarrow{OX}\times\overrightarrow{OY}|=$

[A] $\sin(P+Q)$ [B] $\sin 2R$

[C] $\sin(P+R)$ [D] $\sin(Q+R)$

Sol. **A**

Q.38 Let O be the origin and let PQR be an arbitrary triangle. The point S is such that

$$\overrightarrow{OP}\cdot\overrightarrow{OQ}+\overrightarrow{OR}\cdot\overrightarrow{OS}=\overrightarrow{OR}\cdot\overrightarrow{OP}+\overrightarrow{OQ}\cdot\overrightarrow{OS}=\overrightarrow{OQ}\cdot\overrightarrow{OR}+\overrightarrow{OP}\cdot\overrightarrow{OS}$$

Then the triangle PQR has S as its

[A] centroid [B] circumcentre

[C] incentre [D] orthocenter

Sol. **D**

46. Let $\hat{u}=u_1\hat{i}+u_2\hat{j}+u_3\hat{k}$ be a unit vector in $\mathbb{R}^3$ and $\hat{w}=\frac{1}{\sqrt{6}}(\hat{i}+\hat{j}+2\hat{k})$. Given that there exists a vector $\vec{v}$ in $\mathbb{R}^3$ such that $|\hat{u}\times\vec{v}|=1$ and $\hat{w}\cdot(\hat{u}\times\vec{v})=1$. Which of the following statement(s) is(are) correct?

(A) There is exactly one choice for such $\vec{v}$ (B) There are infinitely many choices for such $\vec{v}$

(C) If $\hat{u}$ lies in the xy-plane then $|u_1|=|u_2|$ (D) If $\hat{u}$ lies in the xz-plane then $2|u_1|=|u_3|$

Sol. **(B, C)**

In $\mathbb{R}^2$, let $\sqrt{3}\hat{i}+\hat{j}$, $\hat{i}+\sqrt{3}\hat{j}$ and $\beta\hat{i}+(1-\beta)\hat{j}$ be the position vectors of *X*, *Y* and *Z* with respect of the origin O, respectively. If the distance of *Z* from the bisector of the acute angle of $\overrightarrow{OX}$ with $\overrightarrow{OY}$ is $\frac{3}{\sqrt{2}}$, then possible value(s) of $|\beta|$ is (are)

49. Let ΔPQR be a triangle. Let $\vec{a}=\overrightarrow{QR}$, $\vec{b}=\overrightarrow{RP}$ and $\vec{c}=\overrightarrow{PQ}$. If $|\vec{a}|=12$, $|\vec{b}|=4\sqrt{3}$ and $\vec{b}.\vec{c}=24$, then which of the following is (are) true ?

(A) $\frac{|\vec{c}|^2}{2}-|\vec{a}|=12$ (B) $\frac{|\vec{c}|^2}{2}+|\vec{a}|=30$

(C) $|\vec{a}\times\vec{b}+\vec{c}\times\vec{a}|=48\sqrt{3}$ (D) $\vec{a}.\vec{b}=-72$

Let $A_1, A_2, \ldots, A_n (n > 2)$ be the vertices of a regular polygon of n sides with its centre at the origin. Let $\overrightarrow{a_k}$ be the position vector of the point A_k, $k = 1, 2, \ldots n$. If $\left|\sum_{k=1}^{n-1}\left(\overrightarrow{a_k} \times \overrightarrow{a_{k+1}}\right)\right| = \left|\sum_{k=1}^{n-1}\left(\overrightarrow{a_k} \cdot \overrightarrow{a_{k+1}}\right)\right|$, then the minimum value of n is

55. Let $\vec{a}, \vec{b}$, and $\vec{c}$ be three non-coplanar unit vectors such that the angle between every pair of them is $\frac{\pi}{3}$. If $\vec{a}\times\vec{b}+\vec{b}\times\vec{c} = p\vec{a}+q\vec{b}+r\vec{c}$, where p, q and r are scalars, then the value of $\frac{p^2+2q^2+r^2}{q^2}$ is ______

45. Let $\vec{x}, \vec{y}$ and $\vec{z}$ be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\frac{\pi}{3}$. If $\vec{a}$ is a non-zero vector perpendicular to $\vec{x}$ and $\vec{y}\times\vec{z}$ and $\vec{b}$ is a non-zero vector perpendicular to $\vec{y}$ and $\vec{z}\times\vec{x}$, then

(A) $\vec{b} = (\vec{b}\cdot\vec{z})(\vec{z}-\vec{x})$
(B) $\vec{a} = (\vec{a}\cdot\vec{y})(\vec{y}-\vec{z})$
(C) $\vec{a}\cdot\vec{b} = -(\vec{a}\cdot\vec{y})(\vec{b}\cdot\vec{z})$
(D) $\vec{a} = (\vec{a}\cdot\vec{y})(\vec{z}-\vec{y})$

48. Let $\overline{PR} = 3\hat{i}+\hat{j}-2\hat{k}$ and $\overline{SQ} = \hat{i}-3\hat{j}-4\hat{k}$ determine diagonals of a parallelogram PQRS and $\overline{PT} = \hat{i}+2\hat{j}+3\hat{k}$ be another vector. Then the volume of the parallelepiped determined by the vectors $\overline{PT}, \overline{PQ}$ and $\overline{PS}$ is

(A) 5
(B) 20
(C) 10
(D) 30

The vectors $(x^2-1)\bar{i}+2(x^2-1)\bar{j}-3(x^2-1)\bar{k}, (2x^2-1)\bar{i}+(2x^2+1)\bar{j}+x^2\bar{k}$, and $(3x^2+2)\bar{i}+(x^2+4)\bar{j}+(x^2+1)\bar{k}$ are non-coplanar. The number of real values that x cannot take is

(A) 0
(B) 1
(C) 2
(D) 4

If $|\vec{a}| = 2$, $|\vec{b}| = 3$, and $|\vec{c}| = 4$ and maximum of $|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2$ = 10 p + q where p, q $\in$ {0, 1, 2, 3, ….. 9} then |p – q| is ________

Let $\vec{a}, \vec{b}, \vec{c}$ be three coplanar unit vectors such that $\vec{a}+\vec{b}+\vec{c} = \vec{0}$. If three vectors $\vec{p}, \vec{q}, \vec{r}$ parallel to $\vec{a}, \vec{b}, \vec{c}$ respectively and having integral but different magnitudes, then among the following options $|\vec{p}+\vec{q}+\vec{r}|$ can take a value equal to

(A) 1
(B) 0
(C) $\sqrt{3}$
(D) 2

Solve the simultaneous vector equations for the vectors $\vec{x}$ and $\vec{y}$.

$\vec{x}+\vec{c}\times\vec{y}=\vec{a}$ and $\vec{y}+\vec{c}\times\vec{x}=\vec{b}$ where $\vec{c}$ is a non zero vector.

Let $\vec{a}, \vec{b}$ & $\vec{c}$ be non coplanar unit vectors, equally inclined to one another at an angle θ. If $\vec{a}\times\vec{b}+\vec{b}\times\vec{c} = p\vec{a}+q\vec{b}+r\vec{c}$. Find scalars p, q & r in terms of θ.

Q. If $A(\vec{a})$; $B(\vec{b})$ & $C(\vec{c})$ are three non collinear points, then for any point $P(\vec{p})$ in the plane of the ΔABC, prove that;

(i) $[\vec{a}\,\vec{b}\,\vec{c}] = \vec{p}\cdot(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a})$

(ii) The vector $\vec{v}$ perpendicular to the plane of the triangle ABC drawn from the origin 'O' is given by $\vec{v} = \pm\frac{[\vec{a}\vec{b}\vec{c}](\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a})}{4\Delta^2}$ where $\vec{\Delta}$ is the vector area of the triangle ABC.

If $p\vec{x}+(\vec{x}\times\vec{a})=\vec{b}; (p\neq 0)$ prove that $\vec{x} = \frac{p^2\vec{b}+(\vec{b}.\vec{a})\vec{a}-p(\vec{b}\times\vec{a})}{p(p^2+\vec{a}^2)}$.

Two forces $\overrightarrow{AB}$ and $\overrightarrow{AD}$ are acting at the vertex A of a quadrilateral $ABCD$ and two forces $\overrightarrow{CB}$ and $\overrightarrow{CD}$ at C. Prove that their resultant is given by $4\overrightarrow{EF}$, where E and F are the midpoints of AC and BD, respectively.

Prove the result (Lagrange's identity) $(\vec{p}\times\vec{q})\cdot(\vec{r}\times\vec{s}) = \begin{vmatrix} \vec{p}.\vec{r} & \vec{p}.\vec{s} \\ \vec{q}.\vec{r} & \vec{q}.\vec{s} \end{vmatrix}$ & use it to prove the following. I (ab)denote the plane formed by the lines a,b. If (ab) is perpendicular to (cd) and (ac) is perpendicular (bd) prove that (ad) is perpendicular to (bc).

Let $\vec{a} = \alpha\hat{i} + 2\hat{j} - 3\hat{k}$, $\vec{b} = \hat{i} + 2\alpha\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} - \alpha\hat{j} + \hat{k}$. Find the value(s) of α, if any, such that $\{(\vec{a}\times\vec{b})\times(\vec{b}\times\vec{c})\}\times(\vec{c}\times\vec{a}) = 0$. Find the vector product when $\alpha = 0$.

$\hat{a}$, $\hat{b}$, $\hat{c}$ are non–coplanar unit vectors . The angle between $\hat{b}$ & $\hat{c}$ is α, between $\hat{c}$ & $\hat{a}$ is β and between $\hat{a}$ & $\hat{b}$ is γ . If A $(\hat{a}\cos\alpha)$, B $(\hat{b}\cos\beta)$, C $(\hat{c}\cos\gamma)$, then show that in Δ ABC,

$$\frac{|\hat{a}\times(\hat{b}\times\hat{c})|}{\sin A} = \frac{|\hat{b}\times(\hat{c}\times\hat{a})|}{\sin B} = \frac{|\hat{c}\times(\hat{a}\times\hat{b})|}{\sin C} = \frac{\prod|\hat{a}\times(\hat{b}\times\hat{c})|}{|\sum \sin\alpha\cos\beta\cos\gamma\,\hat{n}_1|} \quad \text{where}$$

$$\hat{n}_1 = \frac{\hat{b}\times\hat{c}}{|\hat{b}\times\hat{c}|},\ \hat{n}_2 = \frac{\hat{c}\times\hat{a}}{|\hat{c}\times\hat{a}|}\ \&\ \hat{n}_3 = \frac{\hat{a}\times\hat{b}}{|\hat{a}\times\hat{b}|}.$$

Find the scalars α & β if $\vec{a}\times(\vec{b}\times\vec{c}) + (\vec{a}.\vec{b})\vec{b} = (4-2\beta-\sin\alpha)\vec{b} + (\beta^2-1)\vec{c}$ & $(\vec{c}.\vec{c})\vec{a} = \vec{c}$ while $\vec{b}$ & $\vec{c}$ are non zero non collinear vectors.

The vector $\overrightarrow{OP} = \hat{i} + 2\hat{j} + 2\hat{k}$ turns through a right angle, passing through the positive x-axis on the way. Find the vector in its new position.

Let $\vec{a} = \sqrt{3}\hat{i} - \hat{j}$ and $\vec{b} = \frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}$ and $\vec{x} = \vec{a} + (q^2-3)\vec{b}$, $\vec{y} = -p\vec{a} + q\vec{b}$. If $\vec{x}\perp\vec{y}$, then express p as a function of q, say $p = f(q)$, ($p \neq 0$ & $q \neq 0$) and find the intervals of monotonicity of $f(q)$.

If $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are position vectors of the vertices of a cyclic quadrilateral ABCD prove that :

$$\frac{|\vec{a}\times\vec{b} + \vec{b}\times\vec{d} + \vec{d}\times\vec{a}|}{(\vec{b}-\vec{a}).(\vec{d}-\vec{a})} + \frac{|\vec{b}\times\vec{c} + \vec{c}\times\vec{d} + \vec{d}\times\vec{b}|}{(\vec{b}-\vec{c}).(\vec{d}-\vec{c})} = 0$$

The internal bisectors of the angles of a triangle ABC meet the opposite sides in D, E, F ; use vectors to prove that the area of the triangle DEF is given by

$$\frac{(2abc)\,\Delta}{(a+b)(b+c)(c+a)}$$ where Δ is the area of the triangle.

$\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are the position vectors of the points A $\equiv$ (x, y, z) ; B $\equiv$ (y, – 2z, 3x) ; C $\equiv$ (2z, 3x, – y) and D$\equiv$(1,–1, 2) respectively. If $|\vec{a}| = 2\sqrt{3}$; $(\widehat{\vec{a}\ \vec{b}}) = (\widehat{\vec{a}\ \vec{c}})$; $(\widehat{\vec{a}\ \vec{d}}) = \frac{\pi}{2}$ and $(\widehat{\vec{a}\ \hat{j}})$ is obtuse, then find x, y, z.

Let OACB be paralelogram with O at the origin & OC a diagonal. Let D be the mid point of OA. Using vector method prove that BD & CO intersect in the same ratio. Determine this ratio.

Points X & Y are taken on the sides QR & RS , respectively of a parallelogram PQRS, so that $\overrightarrow{QX} = 4\overrightarrow{XR}$ & $\overrightarrow{RY} = 4\overrightarrow{YS}$. The line XY cuts the line PR at Z . Prove that $\overrightarrow{PZ} = \left(\frac{21}{25}\right)\overrightarrow{PR}$.

If $O(\vec{0})$ is the circumcentre and O' the orthocentre of a triangle ABC, then prove that

i. $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}=\overrightarrow{OO'}$

ii. $\overrightarrow{O'A}+\overrightarrow{O'B}+\overrightarrow{O'C}=2\overrightarrow{O'O}$

iii. $\overrightarrow{AO'}+\overrightarrow{O'B}+\overrightarrow{O'C}=2\overrightarrow{AO}=\overrightarrow{AP}$

where AP is the diameter through A of the circumcircle.

Consider the non zero vectors $\vec{a}, \vec{b}, \vec{c} \,\&\, \vec{d}$ such that no three of which are coplanar then prove tha

$\vec{a}[\vec{b}\,\vec{c}\,\vec{d}]+\vec{c}[\vec{a}\,\vec{b}\,\vec{d}]=\vec{b}[\vec{a}\,\vec{c}\,\vec{d}]+\vec{d}[\vec{a}\,\vec{b}\,\vec{c}]$. Hence prove that $\vec{a}, \vec{b}, \vec{c} \,\&\, \vec{d}$ represent the position vectors o

the vertices of a plane quadrilateral if and only if $\dfrac{[\vec{b}\,\vec{c}\,\vec{d}]+[\vec{a}\,\vec{b}\,\vec{d}]}{[\vec{a}\,\vec{c}\,\vec{d}]+[\vec{a}\,\vec{b}\,\vec{c}]}=1$.

Solve the following equation for the vector $\vec{p}$; $\vec{p}\times\vec{a}+(\vec{p}\cdot\vec{b})\vec{c}=\vec{b}\times\vec{c}$, where $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-zero non-coplanar vectors and $\vec{a}$ is neither perpendicular to $\vec{b}$ nor to $\vec{c}$, hence show that $\left(\vec{p}\times\vec{a}+\dfrac{[\vec{a}\,\vec{b}\,\vec{c}]}{\vec{a}\cdot\vec{c}}\vec{c}\right)$ is perpendicular to $(\vec{b}-\vec{c})$.

Show that the circumcentre of the tetrahedron OABC is given by $\dfrac{\vec{a}^2(\vec{b}\times\vec{c})+\vec{b}^2(\vec{c}\times\vec{a})+\vec{c}^2(\vec{a}\times\vec{b})}{2[\vec{a}\,\vec{b}\,\vec{c}]}$,

where $\vec{a}, \vec{b}$ & $\vec{c}$ are the position vectors of the points A, B, C respectively relative to the origin 'O'.

$\hat{a}$ and $\hat{b}$ are two given unit vectors at right angle. The unit vector equally inclined with $\hat{a}$, $\hat{b}$ and $\hat{a}\times\hat{b}$ will be:

(A*) $-\dfrac{1}{\sqrt{3}}(\hat{a}+\hat{b}+\hat{a}\times\hat{b})$ (B*) $\dfrac{1}{\sqrt{3}}(\hat{a}+\hat{b}+\hat{a}\times\hat{b})$

(C) $\dfrac{1}{\sqrt{3}}(\hat{a}+\hat{b}-\hat{a}\times\hat{b})$ (D) $-\dfrac{1}{\sqrt{3}}(\hat{a}+\hat{b}-\hat{a}\times\hat{b})$

Which of the following statement(s) is/are true?

(A*) If $\vec{n}\cdot\vec{a}=0, \vec{n}\cdot\vec{b}=0$ and $\vec{n}\cdot\vec{c}=0$ for some non-zero vector $\vec{n}$, then $[\vec{a}\,\vec{b}\,\vec{c}]=0$.

(B) there exist a vector having direction angles $\alpha = 30°$ and $\beta = 45°$

(C*) locus of point for which $x = 3$ and $y = 4$ is a line parallel to the Z-axis whose distance from the Z-axis is 5

(D*) the vertices of a regular tetrahedron are OABC where 'O' is the origin. Then vector $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}$ is perpendicular to the plane ABC.

If $\vec{a}$, $\vec{b}$ are two non-collinear unit vectors and $\vec{a}, \vec{b}, x\vec{a}-y\vec{b}$ form a triangle, then:

(A*) $x=-1; y=1$ and $|\vec{a}+\vec{b}|=2\cos\left(\dfrac{\vec{a}\wedge\vec{b}}{2}\right)$

(B*) $x=-1; y=1$ and $\cos(\vec{a}\wedge\vec{b})+|\vec{a}+\vec{b}|\cos(\vec{a}\wedge-(\vec{a}+\vec{b}))=-1$

(C) $|\vec{a}+\vec{b}|=-2\cot\left(\dfrac{\vec{a}\wedge\vec{b}}{2}\right)\cos\left(\dfrac{\vec{a}\wedge\vec{b}}{2}\right)$ and $x=-1, y=1$

(D) none of these

Consider the vectors $\hat{i}+\cos(\beta-\alpha)\hat{j}+\cos(\gamma-\alpha)\hat{k}$, $\cos(\alpha-\beta)\hat{i}+\hat{j}+\cos(\gamma-\beta)\hat{k}$ and $\cos(\alpha-\gamma)\hat{i}+\cos(\beta-\gamma)\hat{j}+a\hat{k}$, where α, β and γ are different angles. If these vectors are coplanar, show that a is independent of α, β and γ.

Given four points P_1, P_2, P_3 and P_4 on the coordinate plane with origin O which satisfy the condition $\overrightarrow{OP}_{n-1}+\overrightarrow{OP}_{n+1}=\frac{3}{2}\overrightarrow{OP}_n$.

i. If P_1 and P_2 lie on the curve $xy = 1$, then prove that P_3 does not lie on the curve.

ii. If P_1, P_2 and P_3 lie on the circle $x^2 + y^2 = 1$, then prove that P_4 also lies on this circle.