

42. The number of ways of distributing 3 identical physics books and 3 identical mathematics books among three students such that each student gets at least one book is  $50 + K$ , where  $K$  is single digit number, then  $K$  is \_\_\_\_\_.
43. The no. of positive integer solutions of  $x + y + z = 10$ , where  $x, y, z$  are unequal is  $(20 + K)$  then  $K$  is \_\_\_\_\_.
44. The total number of ways of selecting 5 letters of word INDEPENDENT is  $x$  then sum of digits of  $x$  is \_\_\_\_\_.
48. Let  $f(n) = (\sqrt{2} + 1)^n$ , 'n' being an odd positive integer [.] the greatest integer function then the value of 'n' for which  $[f(n)] = 82$  is \_\_\_\_\_.

49. Which of the following statement(s) is(are) true?

- A) In a 12 storeyed house, 10 people enter the lift cabin at ground floor. It is known that they will leave lift in groups of particular 2, 3 and 5 people at different storey. The number of ways this can be done if the lift does not stop at first and second floors is 720.
- B) Each of three ladies have brought their one child for admission to a school. The principal wants to interview the six persons one by one, subject to the condition that no mother is interviewed before her child. The number of ways in which interviews can be arranged is 90.
- C) The number of ways in which one can put three balls numbered 1, 2, 3 in three boxes labelled a, b, c such that at most one box is empty is equal to 18.
- D) A box contains 5 different red balls and 6 different white balls. The total number of ways in which 4 balls can be selected taking atleast 1 ball of each colour is 310.
50. A fair coin is tossed  $n$  times. Let  $a_n$  denotes the no. of cases in which no two heads occur consecutively, then
- A)  $a_1 = 2$                       B)  $a_2 = 3$                       C)  $a_5 = 14$                       D)  $a_8 = 55$

51. Four balls numbered 1, 2, 3, 4 are to be placed into five boxes numbered 1, 2, 3, 4, 5, such that exactly one box remains empty and no ball goes to its own numbered box. The no. of ways is

- A)  $5! \sum_{r=0}^5 \frac{(-1)^r}{r!}$                       B)  $4! \sum_{r=0}^4 \frac{(-1)^r}{r!}$
- C)  $4! \sum_{r=0}^4 \frac{(-1)^r}{r!} + 5! \sum_{r=0}^5 \frac{(-1)^r}{r!}$                       D) 54

52. Let  $f(n)$  denote the number of ways in which  $n$  letters go into  $n$  envelopes so that no letter is in the correct envelope, (where  $n > 5$ ), then  $f(n) - nf(n-1)$  equals
- A)  $f(n-2) - (n-2)f(n-3)$   
 B)  $(n-1)f(n-2) - f(n-1)$   
 C)  $(n-3)f(n-4) - f(n-3)$   
 D)  $(n-4)f(n-5) - f(n-4)$
53. The letters of the word "ARRANGE" are arranged in all possible ways. Let  $m$  be the number of arrangements in which the two  $A$ 's are together and the two  $R$ 's are not together and  $n$  be the number of arrangements in which neither the two  $A$ 's nor the two  $R$ 's are together. Then
- A)  $m + n = 900$       B)  $m + n = 1260$       C)  $n - m = 780$       D)  $n - m = 420$
54. Thirteen persons are sitting in a row. The number of ways in which four persons can be selected so that no two of them are consecutive is equal to \_\_\_\_\_
- A) The number of ways in which all the letters of the word "MARRIAGE" can be permuted if no two vowels are together.  
 B) The number of numbers lying between 100 and 1000 using only the digits 1,2,3,4,5,6,7 without repetition.  
 C) The number of ways in which 4 alike chocolates can be distributed among 10 children so that each child get at most one chocolate.  
 D) The number of triangles can be formed by joining 12 points in a plane, of which exactly 5 are collinear
55. The number of ways in which we can choose 2 distinct integers (the order does not matter) from 1 to 200, so that the difference between them is at most 20, is
- A) 3790      B)  ${}^{200}C_2 - {}^{180}C_2$   
 C)  ${}^{180}C_1 \times 20 + \frac{19 \times 20}{2}$       D)  ${}^{180}C_2$
56. The number of ways in which five different books to be distributed among 3 persons so that each person gets at least one book, is equal to the number of ways in which
- A) 5 persons are allotted 3 different residential flats so that each person is allotted at most one flat and no two persons are allotted the same flat  
 B) number of parallelograms (some of which may be overlapping) formed by one set of 6 parallel lines and other set of 5 parallel lines that goes in other direction  
 C) 5 different toys are to be distributed among 3 children, so that each child gets at least one toy  
 D) 3 professors of mathematics are assigned five different lectures to be delivered, so that each professor gets at least one lecture

**Paragraph-1**

If  $x_1 + x_2 + x_3 + \dots + x_r = n$

Then number of solutions of equation  ${}^{n+r-1}C_n$  when  $x_i$  are ( $i = 1, 2, 3 \dots r$ ) non-negative integers and  ${}^{n-1}C_{r-1}$  when  $x_i$  are ( $i = 1, 2, 3 \dots r$ ) positive integers

57. If  $a, b, c$  be three natural numbers in A.P. then number of solution of  $a + b + c = 21$  is  
A) 15                      B) 14                      C) 13                      D) 16
58. Number of ways of distributing 22, identical toys among 4 children when each child must get odd number of toys is equal to  
A)  ${}^8C_3$                       B)  ${}^{12}C_9$                       C)  ${}^{21}C_3$                       D)  ${}^{25}C_{22}$

**Paragraph-2**

Let  $S_n$  be the set of all  $n$ -digit numbers.

59. The number of numbers in  $S_6$ , consisting of digits 0,3 and 5 only, with each of them appearing at least once, is  
A) 360                      B) 348                      C) 342                      D) 324
60. The number of numbers in  $S_6$  having exactly three different digits is  
A) 62328                      B) 58320                      C) 56328                      D) 54320

Let  $n$  be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that all the girls stand consecutively in the queue. Let  $m$  be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that exactly four girls stand consecutively in the queue. Then which of the following option(s) is/are correct?

- ~~A)  $m=5n$~~                       B)  $n=5m$                       C)  $n=6(5!)^2$                       D)  $m=6!4!$   
~~Y)  $n=0(5!)$~~                       ~~D)  $m=0!4!$~~

47. If  $N$  is the number of eight digit numbers formed using all the digits 1,3,4,5,6,7,8,9 which are divisible by 275 then

- ✓ A)  $\sqrt{N} \in Z$   
✓ B)  $N$  can be expressed as a sum of twin prime  
C)  $\sqrt{N} \notin Z$   
D)  $N$  can be expressed as sum of cubes of three consecutive positive integers

[Note: A twin prime is a pair of consecutive prime numbers that differs by two]

48. Let  $\omega \neq 1$  be the cube root of unity and  $S$  be the set of all non-singular matrices of the

form  $\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$  where each of  $a, b$  and  $c$  is either  $\omega$  or  $\omega^2$ . If the number of distinct

matrices in the set  $S$  is  $n$  then

- A)  $n=2$       B)  $n=4$       C)  $a=\omega$       D)  $c \neq \omega^2$

63. Let  $f: A \rightarrow A$  be an invertible function where  $A = \{1, 2, 3, 4, 5, 6\}$ . The number of these functions in which at least three have self image is

- 1) 40      2) 56      3) 16      4) 3

The number of functions  $f$  from the set  $A = \{0, 1, 2\}$  into the set  $B = \{0, 1, 2, 3, 4, 5, 6, 7\}$  such that  $f(i) \leq f(j)$  for  $i < j$  and  $i, j \in A$  is

- 1)  ${}^8C_3$       2)  ${}^8C_3 + 2({}^8C_2)$       3)  ${}^{10}C_3$       4)  ${}^8C_3 + {}^{10}C_3$

47. Let  $S = \{1, 2, 3, \dots, n\}$ , If  $X$  denote the set of all subsets of  $S$  containing exactly two elements, then the value of  $\sum_{A \in X} (\min A)$  is  ${}^{n+1}C_\lambda$  then  $\lambda = \underline{\hspace{2cm}}$  (min  $A$  means minimum element in  $A$ )

48. Consider  $n \times n$  graph paper where  $n$  is a natural number. Consider the right angled isosceles triangles whose vertices are integer points of this graph and whose sides forming right angle are parallel to  $x$  and  $y$  axes (integral point means it is a point whose both the coordinates are integers). If the number of such triangles is  $\frac{2}{K}n(n+1)(2n+1)$ , then  $K$  is  $\underline{\hspace{2cm}}$ .

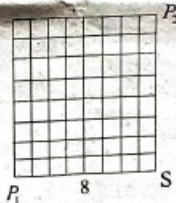
D. Two numbers 'a' & 'b' are chosen from the set of  $\{1, 2, 3, \dots, 3n\}$ . In how many ways can these integers be selected such that  $a^2 - b^2$  is divisible by 3

- A)  $\frac{3}{2}n(n+1) + n^2$       B)  $\frac{3}{2}n(n-1) + n^2$       C) 70 for  $n=5$       D) 55 for  $n=5$

51. Consider the following statements.

- (i) The number of a cube can be paint with the 6 different colour is equal to 30
  - (ii) Each of three ladies have brought their one child for admission to a school. The principal wants to interview the six persons one by one, subject to the condition that no mother is interviewed before her child. The number of ways in which interviews can be arranged is 90.
  - (iii) The number of ways in which one can put three balls numbered 1, 2, 3 in three boxes labelled a, b, c such that at most one box is empty is equal to 18.
  - (iv) A box contains 5 different red balls and 6 different white balls. The total number of ways in which 4 balls can be selected, taking atleast 1 ball of each colour is 310.
- A) statements (i), (ii) are correct.  
B) statements (ii) and (iv) are correct.  
C) statements (i) and (iii) are correct.  
D) All statements are correct.

52. On a chess board as shown  $P_1$  and  $P_2$  are two persons which start moving towards each other Each person moving with same constant speed.  $P_1$  can move only to the left or downward along the lines ; while  $P_2$  can move only to the right or upward along the lines of chess board.



- A) Total number of ways the two person meet at some point is K then sum of digit of K is 18  
B) Total number of ways the two person meet at some point is K then sum of digits of K is 16  
C) Total number of ways the two person meet at some point is K then middle digit of K is 8  
D) Total number of ways the two person meet at some point is K then middle digit of K is 6

55. Let A, B, C, D, E be the smallest positive integers having 10, 12, 15, 16, 20 positive integral divisors respectively. Then

- A)  $A + B = 108$     B)  $C + D = 354$     C)  $A + E = 288$     D)  $A + E = 300$

56. Let  $D_1, D_2, \dots, D_{1000}$  are 1000 doors and  $P_1, P_2, \dots, P_{1000}$  are 1000 persons. Initially all the doors are closed.  $P_1$  opens all the doors. Then  $P_2$  closes  $D_2, D_4, D_6, \dots, D_{998}, D_{1000}$ . Then  $P_3$  changes the status of  $D_3, D_6, D_9, D_{12}, \dots$  etc. (doors having numbers which are multiples of 3). Changing the status of a door means closing it if it is open and opening it if it is closed. Then  $P_4$  changes the status of  $D_4, D_8, D_{12}, D_{16}, \dots$  etc (doors having numbers which are multiples of 4) And so on until lastly  $P_{1000}$  changes the status of  $D_{1000}$ , then which of the following is/are correct?

- A) Number of doors are open is 31
- B) Greatest number of consecutive doors that are closed finally 60
- C) The door having the greatest number that is finally open is  $D_{961}$
- D) Greatest number of consecutive doors that are closed finally 61

59. 5 digit numbers are formed using the digits 0, 1, 2, 3, 4, 5. Answer the following

Column - I	Column-II
A) How many of them are divisible by 3 if repetition is not allowed	P) 216
B) How many of them are divisible by 3 if repetition of digits is allowed	Q) 108
C) How many of them are divisible by 3 but not by 2 if repetition is not allowed	R) 1000
D) Number of 4 digit numbers divisible by 5 (without repetition)	S) 42

60. Let  $n(P)$  represents the number of points  $P(\alpha, \beta)$  lying on the rectangular hyperbola  $xy = 15!$ , under the conditions given in column I, match the value of  $n(P)$  given in column II.

Column -I	Column -II
A) $\alpha, \beta \in I$	P) 32
B) $\alpha, \beta \in I^+$ and $\text{HCF}(\alpha, \beta) = 1$	Q) 64
C) $\alpha, \beta \in I^+$ and $\alpha$ divides $\beta$	R) 96
D) $\alpha, \beta \in I^+$ and $\text{HCF}(\alpha, \beta) = 35$	S) 4032
	T) 8064

68. A father with 8 children takes them 3 at a time to the zoological garden, as often as he can without taking the same 3 children together more than once. Then

1) number of times he will go is 56

✓ 2) number of times each child will go is 25

3) number of times a particular child will not go is 53

4) number of times a particular child will not go is 35

(22)

14  
14

43. If  $x, y, z$  are natural numbers such that  $\text{LCM}(x, y) = 2744$ ,  $\text{LCM}(y, z) = 1372$  and  $\text{LCM}(z, x) = 2744$ , then the number of triplets of  $(x, y, z)$  is 'n' where sum of the digits of n is

46. If  $\alpha = x_1x_2x_3$  and  $\beta = y_1y_2y_3$  be two three digit numbers, the number of pairs  $\alpha$  and  $\beta$  can be formed so that  $\alpha$  can be subtracted from  $\beta$  without borrowing is  $3^p5^q11^r$  then the value of  $p+q+r$  is

41. The number of onto functions which are non decreasing from  $A = \{1, 2, 3, 4, 5\}$  to  $B = \{7, 8, 9\}$  is

42. A is a set containing 8 elements. A subset  $P_1$  of A is selected. The set A is reconstructed by replacing the elements of  $P_1$ . Next, a subset  $P_2$  of A is chosen and again the set is reconstructed by replacing elements of  $P_2$ . In this way subsets  $P_1, P_2, \dots, P_{10}$  of A are chosen. The number of ways of choosing  $P_1, P_2, \dots, P_{10}$  so that,  $P_i \cap P_j = \phi$  for  $i \neq j$  is given by  $\lambda$ . then numbers of factors of  $\lambda$  is

43. If  $x, y, z$  are natural numbers such that  $\text{LCM}(x, y) = 2744$ ,  $\text{LCM}(y, z) = 1372$  and  $\text{LCM}(z, x) = 2744$ , then the number of triplets of  $(x, y, z)$  is 'n' where sum of the digits of n is

57. There are 'n' intermediate stations on a railway line from one terminus to another. In how many ways can the train stop at 3 of these intermediate stations,

- A) If All the three stations are consecutive is  $(n - 1)$
- B) If At least two of the stations are consecutive  $(n - 2)(n - 1)$
- C) If At least two of the stations are consecutive is  $(n - 2)^2$
- D) If No two of these stations are consecutive  ${}^n C_3 - (n - 2)^2$

58. Which of following is/ are correct

- A) Tom has 15 ping-pong balls each uniquely numbered from 1 to 15. He also has a red box, a blue box, and a green box. Then Number of ways Tom place the 15 distinct balls into the three boxes so that no box is empty is  $3^{15} - 3 \cdot 2^{15} + 3$
- B) Jerry has 15 ping-pong balls each uniquely numbered from 1 to 15. He also has a red box, a blue box, and a green box. Suppose now that Jerry has placed 5 ping-pong balls in each box. Number of ways he choose 5 balls from the three boxes so that he chooses at least one from each box 2250
- C) If  $A = \{1, 2, 3\}$ ,  $B = \{1, 2, 3, 4, 5\}$ ,  $f: A \rightarrow B$  is a mapping, then number of functions satisfying  $f(i) \leq f(j), i < j$  is 35
- D) If  $A = \{1, 2, 3\}$ ,  $B = \{1, 2, 3, 4, 5\}$ ,  $f: A \rightarrow B$  is a mapping, then number of functions satisfying  $f(i) \neq i$ , is 64