All the 7 digit numbers containing each of the digits 1, 2, 3, 4, 5, 6, 7 exactly once, and not divisible by 5 are arranged in the increasing order. Find the $(2004)^{th}$ number in this list.

5 boys & 4 girls sit in a straight line. Find the number of ways in which they can be seated if 2 girls are together & the other 2 are also together but separate from the first 2.

A firm of Chartered Accountants in Bombay has to send 10 clerks to 5 different companies, two clerks in each. Two of the companies are in Bombay and the others are outside. Two of the clerks prefer to work in Bombay while three others prefer to work outside. In how many ways can the assignment be made if the preferences are to be satisfied.

How many arrangements each consisting of 2 vowels & 2 consonants can be made out of the letters of the word **'DEVASTATION'**?

There are 5 white , 4 yellow , 3 green , 2 blue & 1 red ball. The balls are all identical except for colour. These are to be arranged in a line in 5 places. Find the number of distinct arrangements.

How many 4 digit numbers are there which contains not more than 2 different digits?

(a) How many divisors are there of the number x = 21600. Find also the sum of these divisors.
(b) In how many ways the number 7056 can be resolved as a product of 2 factors.
(c) Find the number of ways in which the number 300300 can be split into 2 factors which are relatively prime.

How many ten digits whole number satisfy the following property they have 2 and 5 as digits, and there are no consecutive 2's in the number (i.e. any two 2's are separated by at least one 5).

How many different ways can 15 Candy bars be distributed between Ram, Shyam, Ghanshyam and Balram, if Ram can not have more than 5 candy bars and Shyam must have at least two. Assume all Candy bars to be alike.

Find the number of distinct throws which can be thrown with 'n' six faced normal dice which are indistinguishable among themselves.

How many integers between 1000 and 9999 have exactly one pair of equal digit such as 4049 or 9902 but not 4449 or 4040?

Find the number of ways in which 3 distinct numbers can be selected from the set $\{3^1, 3^2, 3^3, ... 3^{100}, 3^{101}\}$ so that they form a G.P.

There are counters available in 7 different colours. Counters are all alike except for the colour and they are atleast ten of each colour. Find the number of ways in which an arrangement of 10 counters can be made. How many of these will have counters of each colour.

For each positive integer k, let S_k denote the increasing arithmetic sequence of integers whose first term is 1 and whose common difference is k. For example, S_3 is the sequence 1, 4, 7, 10...... Find the number of values of k for which S_k contain the term 361.

A shop sells 6 different flavours of ice-cream. In how many ways can a customer choose 4 ice-cream cones if

(i) they are all of different flavours

(ii) they are non necessarily of different flavours

(iii) they contain only 3 different flavours

(iv) they contain only 2 or 3 different flavours?

6 white & 6 black balls of the same size are distributed among 10 different urns. Balls are alike except for the colour & each urn can hold any number of balls. Find the number of different distribution of the balls so that there is atleast 1 ball in each urn.

There are 2n guests at a dinner party. Supposing that the master and mistress of the house have fixed seats opposite one another, and that there are two specified guests who must not be placed next to one another. Show that the number of ways in which the company can be placed is $(2n-2)!.(4n^2-6n+4)$.

How many 15 letter arrangements of 5 A's, 5 B's and 5 C's have no A's in the first 5 letters, no B's in the next 5 letters, and no C's in the last 5 letters.

Find the number of ways in which the number 30 can be partitioned into three unequal parts, each part being a natural number. What this number would be if equal parts are also included.

Find the number of three digits numbers from 100 to 999 inclusive which have any one digit that is the average of the other two.

25 passengers arrive at a railway station & proceed to the neighbouring village. At the station there are 2 coaches accommodating 4 each & 3 carts accommodating 3 each. Find the number of ways in which they can proceed to the village assuming that the conveyances are always fully occupied & that the conveyances are all distinguishable from each other.

The members of a chess club took part in a round robin competition in which each plays every one else once. All members scored the same number of points, except four juniors whose total score were 17.5. How many members were there in the club? Assume that for each win a player scores 1 point, for draw 1/2 point and zero for losing.

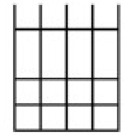
Six faces of an ordinary cubical die marked with alphabets A, B, C, D, E and F is thrown n times and the list of n alphabets showing up are noted. Find the total number of ways in which among the alphabets A, B, C, D, E and F only three of them appear in the list.

Find the number of integer betwen 1 and 10000 with at least one 8 and at least one 9 as digits.

The number of combinations n together of 3n letters of which n are 'a' and n are 'b' and the rest unlike is $(n+2).\ 2^{n-1}$.

A man goes in for an examination in which there are 4 papers with a maximum of m marks for each paper; show that the number of ways of getting 2m marks on the whole is $\frac{1}{3}(m+1)(2m^2+4m+3)$.

A rectangle with sides $2m-1$ and $2n-1$ is divided into squares of unit length by drawing parallel lines as shown in the diagram, then the number of rectangles possible with odd side lengths is



(A) $(m+n+1)^2$ (B) 4^{m+n-1}

(C) m^2n^2 (D) $mn(m+1)(n+1)$

[JEE 2005 (Screening), 3]

If r, s, t are prime numbers and p, q are the positive integers such that their LCM of p, q is is $r^2t^4s^2$, then the numbers of ordered pair of (p, q) is

(A) 252 (B) 254 (C) 225 (D) 224

[JEE 2006, 3]

A bag contains 6n tickets numbered from 0, 1, 2, ... 6n–1. In how many ways 3 tickets can be selected so that the sum of the numbers shown on them is equal to 6n.

(vii) Find the number of integers between 1 to 100000 if the sum of their digits is 15.

[Sol. (X X X X X)

coefficient of x^{15} in $(1 + x + x^2 + x^3 + ... + x^9)^5 = ({}^{19}C_4 - 5.{}^{9}C_4) = 735$ Ans]

Prove that the number of combinations of n letters out of 3n letters of which n are 'a' and n are 'b' and the rest unlike is $(n+2)\,2^{n-1}$ [T/S]

[Sol : Coeff of x^n in $(1 + x + x^2 + ... + x^n)(1 + x + x^2 + ... + x^n)(1 + x)^n$

(d) Sum of all the numbers that can be formed using all the digits 2, 3, 3, 4, 4, 4 is :

(A*) 22222200 (B) 11111100 (C) 55555500 (D) 20333280

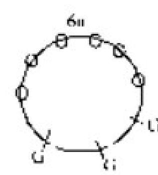
[Hint : 4 x 30 [x] + 3 x 20 [x] + 2 x 10 [x] where $[x] = 1 + 10 + 10^2 + 10^3 + 10^4 + 10^5$]

(viii) There are 6n flowers of one kind and 3 flowers of another kind. How many differen garlands can be made using all the flowers.

when '0' is always taken	'1' is always taken	'2' is always taken		(2n–1) is always taken	'2n' is always taken
0 0 6n 0 1 6n–1 0 2 6n–2 ⋮ ⋮ ⋮ 0 3n 3n	1 1 6n–2 1 2 6n–3 1 3 6n–4 ⋮ ⋮ ⋮ 1 3n–1 3n	2 2 6n–4 2 3 6n–5 ⋮ ⋮ ⋮ 2 3n–2 3n 2 3n–1 3n+1		2n–1 2n–1 2n+2 2n–1 2n 2n+1	2n 2n 2n
(3n+1) ways	(3n–1) ways	(3n–2) ways		2 way	1 way

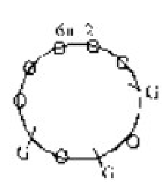
Total = 1 +(2+4)+(5+7)+(8+10) .... ((3n–1) + (3n+1))

$$= 1 + 6\,[\,1+2+3+ ....+n) = 1 + 6 + \frac{n(n+1)}{2} = 3n^2 + 3n + 1$$



Note: 'O' is always taken means all 6n flowers on the same side of the extreme G

|||ly 1 is always taken means at least 1 different flower between two successive G's.



(f) Number of 7 digit numbers if the sum of their digits is 59, how many of these are divisible by 11 (Digits to be used can be 5, 6, 7, 8, 9) [Ans: 210 ; 40]

[Sol. Possible digits are 5, 6, 7, 8, 9

Category	Number of 7 digits number	Divisible by 11
9 9 9 9 9 9 5	$\frac{7!}{6!} = 7$	×
9 9 9 9 9 7 7	$\frac{7!}{5! \cdot 6!} = 21$	×
9 9 9 9 9 6 8	$\frac{7!}{5!} = 42$	× □ × □ × □ × $\frac{4!}{3!} \frac{3!}{2!} = 12$
9 9 9 9 8 8 7	$\frac{7!}{4! \cdot 2!} = 105$	$\frac{4!}{3!} \cdot 3! = 24$
9 9 9 8 8 8 8	$\frac{7!}{3! \cdot 4!} = 35$	$\frac{4!}{3!} \cdot 1! = 4$
	210	40]

If $N = 2^{p-1} . (2^p - 1)$, where $2^p - 1$ is a prime, then the sum of the divisors of N expressed in terms of N is equal to ______ .

5. Number of 4 digit number of the form $N = x_1\, x_2\, x_3\, x_4$ such that $2000 \le N \le 5000$, $4 \le x_2 < x_3 \le 8$ and N is multiple of 5 is/are

(A) 80 (B) 60

(C) 40 (D) 30

7. The product of all the even divisor of N = 9000 is

(A) 721×10^5 (B) 726×10^6

(C) 729×10^6 (D) none of these

Match the conditions/expressions in Column I with statement in Column II.

12. Consider all possible permutations of the letters of the word ENDEANOEL.

	Column I		Column II
A.	The number of permutations containing the word ENDEA, is	p.	5!
B.	The number of permutations in which the letter E occurs in the first and the last positions, is	q.	$2 \times 5!$
C.	The number of permutations in which none of the letters D, L, N occurs in the last five positions, is	r.	$7 \times 5!$
D.	The number of permutations in which the letters A, E, O occur only in odd positions, is	s.	$21 \times 5!$

14. In a high school, a committee has to be formed from a group of 6 boys M_1, M_2, M_3, M_4, M_5, M_6 and 5 girls G_1, G_2, G_3, G_4, G_5.

(i) Let α_1 be the total number of ways in which the committee can be formed such that the committee has 5 members, having exactly 3 boys and 2 girls.

(ii) Let α_2 be the total number of ways in which the committee can be formed such that the committee has at least 2 members, and having an equal number of boys and girls.

(iii) Let α_3 be the total number of ways in which the committee can be formed such that the committee has 5 members, at least 2 of them being girls.

(iv) Let α_4 be the total number of ways in which the committee can be formed such that the committee has 4 members, having at least 2 girls such that both M_1 and G_1 are NOT in the committee together.

(2018 Adv.)

	List-I		List-II
P.	The value of α_1 is	1.	136
Q.	The value of α_2 is	2.	189
R.	The value of α_3 is	3.	192
S.	The value of α_4 is	4.	200
		5.	381
		6.	461

The correct option is

(a) $P \to 4; Q \to 6; R \to 2; S \to 1$

(b) $P \to 1; Q \to 4; R \to 2; S \to 3$

(c) $P \to 4; Q \to 6; R \to 5; S \to 2$

(d) $P \to 4; Q \to 2; R \to 3; S \to 1$

9. Words of length 10 are formed using the letters A, B, C, D, E, F, G, H, I, J. Let x be the number of such words where no letter is repeated; and let y be the number of such words where exactly one letter is repeated twice and no other letter is repeated. Then, $\frac{y}{9x} =$ **(2017 Adv.)**

10. Let n be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that all the girls stand consecutively in the queue. Let m be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that exactly four girls stand consecutively in the queue. Then, the value of $\frac{m}{n}$ is **(2015 Adv.)**

11. Let $n_1 < n_2 < n_3 < n_4 < n_5$ be positive integers such that $n_1 + n_2 + n_3 + n_4 + n_5 = 20$. The number of such distinct arrangements $(n_1, n_2, n_3, n_4, n_5)$ is **(2014 Adv.)**

8. The number of 5 digit numbers which are divisible by 4, with digits from the set $\{1, 2, 3, 4, 5\}$ and the repetition of digits is allowed, is .... **(2018 Adv.)**

56. Consider the set of eight vectors $V = \left\{a\hat{i} + b\hat{j} + c\hat{k} \,;\, a, b, c \in \{-1, 1\}\right\}$. Three non-coplanar vectors can be chosen from V in 2^p ways. Then p is ________

Sol. **(5)**

41. For a $\in$ R (the set of all real numbers), a $\neq -1$, $\lim\limits_{n\to\infty} \dfrac{\left(1^a + 2^a + \ldots + n^a\right)}{(n+1)^{a-1}\left[(na+1) + (na+2) + \ldots + (na+n)\right]} = \dfrac{1}{60}$.

Then a =

(A) 5 (B) 7

(C) $\dfrac{-15}{2}$ (D) $\dfrac{-17}{2}$

Sol. **(B, D)**

*51. Let $n_1 < n_2 < n_3 < n_4 < n_5$ be positive integers such that $n_1 + n_2 + n_3 + n_4 + n_5 = 20$. Then the number of such distinct arrangements $(n_1, n_2, n_3, n_4, n_5)$ is _________

*52. Let $n \geq 2$ be an integer. Take n distinct points on a circle and join each pair of points by a line segment. Colour the line segment joining every pair of adjacent points by blue and the rest by red. If the number of red and blue line segments are equal, then the value of n is _________

41. Three boys and two girls stand in a queue. The probability, that the number of boys ahead of every girl is at least one more than the number of girls ahead of her, is

(A) $\dfrac{1}{2}$ (B) $\dfrac{1}{3}$

(C) $\dfrac{2}{3}$ (D) $\dfrac{3}{4}$

*43. Six cards and six envelopes are numbered 1, 2, 3, 4, 5, 6 and cards are to be placed in envelopes so that each envelope contains exactly one card and no card is placed in the envelope bearing the same number and moreover the card numbered 1 is always placed in envelope numbered 2. Then the number of ways it can be done is

(A) 264 (B) 265

(C) 53 (D) 67

*45. Let n be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that all the girls stand consecutively in the queue. Let m be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that exactly four girls stand consecutively in the queue. Then the value of $\dfrac{m}{n}$ is

* Q.47 Words of length 10 are formed using the letters, A, B, C, D, E, F, G, H, I, J. Let x be the number of such words where no letter is repeated ; and let y be the number of such words where exactly one letter is repeated twice and no other letter is repeated. Then, $\dfrac{y}{9x} =$

Sol. **5**

Q.41 How many 3×3 matrices M with entries from $\{0, 1, 2\}$ are there, for which the sum of the diagonal entries of M^TM is 5 ?

[A] 126 [B] 198

[C] 162 [D] 135

Sol. **B**

Q.42 Let $S = \{1, 2, 3, \ldots, 9\}$. For $k = 1, 2, \ldots, 5$, let N_K be the number of subsets of S, each containing five elements out of which exactly k are odd. Then $N_1 + N_2 + N_3 + N_4 + N_5 =$

[A] 210 [B] 252

[C] 125 [D] 126

Sol. **D**

*Q.8 The number of 5 digit numbers which are divisible by 4, with digits from the set $\{1, 2, 3, 4, 5\}$ and the repetition of digits is allowed, is _______ .

Sol. **625**

Q.8 Let P be a matrix of order 3×3 such that all the entries in P are from the set $\{-1, 0, 1\}$. Then, the maximum possible value of the determinant of P is _____ .

Sol. **4**

*Q.9 Let X be a set with exactly 5 elements and Y be a set with exactly 7 elements. If α is the number of one-one functions from X to Y and β is the number of onto functions from Y to X, then the value of $\frac{1}{5!}(\beta - \alpha)$ is _____ .

Sol. **119**

*Q.16 In a high school, a committee has to be formed from a group of 6 boys $M_1, M_2, M_3, M_4, M_5, M_6$ and 5 girls G_1, G_2, G_3, G_4, G_5.

(i) Let α_1 be the total number of ways in which the committee can be formed such that the committee has 5 members, having exactly 3 boys and 2 girls.

(ii) Let α_2 be the total number of ways in which the committee can be formed such that the committee has at least 2 members, and having an equal number of boys and girls.

(iii) Let α_3 be the total number of ways in which the committee can be formed such that the committee has 5 members, at least 2 of them being girls.

(iv) Let α_4 be the total number of ways in which the committee can be formed such that the committee has 4 members, having atleast 2 girls and such that both M_1 and G_1 are **NOT** in the committee together.

LIST–I		LIST–II	
P.	The value of α_1 is	1.	136
Q.	The value of α_2 is	2.	189
R.	The value of α_3 is	3.	192
S.	The value of α_4 is	4.	200
		5.	381
		6.	461

The correct option is :

(A) **P → 4; Q → 6; R → 2; S → 1**

(B) **P → 1; Q → 4; R → 2; S → 3**

(C) **P → 4; Q → 6; R → 5; S → 2**

(D) **P → 4; Q → 2; R → 3; S → 1**

Sol. **C**

*Q.3 Five persons A, B, C, D and E are seated in a circular arrangement. If each of them is given a hat of one of the three colours red, blue and green, then the number of ways of distributing the hats such that the persons seated in adjacent seats get different coloured hats is ______

Sol. **30.00**

*Q.5 Let |x| denote the number of elements in a set X. Let S = {1, 2, 3, 4, 5, 6} be a sample space, where each element is equally likely to occur. If A and B are independent events associated with S, then the number of ordered pairs (A, B) such that $1 \le |B| < |A|$, equals _____

Sol. **422.00**

*14. Let $a_1, a_2, a_3, \ldots$ be a sequence of positive integers in arithmetic progression with common difference 2. Also, let $b_1, b_2, b_3, \ldots$ be a sequence of positive integers in geometric progression with common ratio 2. If $a_1 = b_1 = c$, then the number of all possible values of c, for which the equality

$$2(a_1 + a_2 + \ldots + a_n) = b_1 + b_2 + \ldots + b_n$$

holds for some positive integer n, is _____

Sol. **1**

*Q.13. An engineer is required to visit a factory for exactly four days during the first 15 days of every month and it is mandatory that no two visits take place on consecutive days. Then the number of all possible ways in which such visits to the factory can be made by the engineer during 1-15 June 2021 is _____

Sol. **495.00**

*Q.14. In a hotel, four rooms are available. Six persons are to be accommodated in these four rooms in such a way that each of these rooms contains at least one person and at most two persons. Then the number of all possible ways in which this can be done is _____

Sol. **1080.00**

6. A batsman can score 0, 2, 3 or 4 runs for each ball he receives the no. of ways of scoring a total of 20 runs in one over of 6 balls is

a) 90 **b) 95** **c) 45** **d) 96**

The number of 3 × 3 symmetric matrices using −1, −1, −1, 1, 1, 1, 2, 2, 2 is

a) 24 **b) 36** **c) 48** **d) 52**

10. The number of three digit numbers with three distinct digits such that one of the digits is the arithmetic mean of the other two is

a) 120 **b) 180** **c) 112** **d) 104**

12. The number of ways in which 4 marries couples can be seated such that four persons are no left side and four are on right side of a long table and no wife is infront of her husband and same gender persons do not sit opposite to each other, is

a) $144\underline{|4}$ **b)** $9\underline{|4}$ **c)** $70\underline{|4}$ **d) None of these**

16. If $\alpha = x_1x_2x_3$ and $\beta = y_1y_2y_3$ be two three digits numbers, the number of pairs of α and β can be formed so that α can be subtracted form β without borrowing is

a) $2!10!10!$ **b)** $(45)(55)^2$ **c)** $3^2.5^3.11^2$ **d) 136125**

17. If a seven digit number made up of all distinct digits 8,7,6,4,2, x and y is divisible by 3 then

a) Maximum value of x–y is 9 **b) Maximum value of x+y is 12**

c) Minimum value xy is 0 **d) Minimum value of x+y is 3**

Passage - I :

There are 12 seats in the first row of a theater of which 4 are to be occupied.

18. Find the number of ways of arranging 4 persons so that, no two persons sit side by side

a) 3023 **b) 3024** **c) 3025** **d) 326**

19. Find the number of ways of arranging 4 persons so that, there should be atleast 2 empty seats between any two persons

a) 360 **b) 260** **c) 560** **d) 230**

20. Find the number of ways of arranging 4 presons so that, each person has exactly one neighbour

a) 860 **b) 862** **c) 867** **d) 864**

28. Let $A=\{x|x$ is a prime number and $x < 30\}$. The number of different rational numbers, whose numarator and denominator belong to A, is $13x$ then $x =$

31. The number of three digit numbers having only two consecutive digits identical is abc then $a+b+c=$

32. The number of triplets (x, y, z) of positive integers satisfying $2^x + 2^y + 2^z = 2336$ is

26. If the words formed by using the letters of the word HARFET are arranged in the form of dictiona then 261th word

1) FATHER 2) FETHAR 3) FATHRE 4) FATERH

9. The number of ordered pairs (m, n) where $m, n, \in \{1, 2, 3, ...100\}$ such that $7^m + 7^n$ is divisible by 5 is

a) 250 b) 2500 c) 1500 d) 25200

12. There are unlimited number of identical balls of four different colours. The number of arrangem of atmost 8 balls in a row that can be made by using them is

a) 87380 **b) 65625** **c) 84614** **d) 70042**

31. **The number of ways in which we can make a garland with 5 flowers of one kind and 3 flowers of another kind, is**

28. **An eight digit number divisible by 9 is to be formed by using 8 digits out of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 without replacement. The number of ways in which this can be done is $\lambda^2 \lfloor 7$ then $\lambda =$**

Passage - I

There are five different boxes and seven different balls. All the seven balls are to be distributed in the five boxes placed in a row so that any box can receive any number of balls.

21. In how many ways can these balls be distributed so that no box is empty?

a) 71 b) 16800 c) 1775 d) 2160

22. Suppose, all the balls are identical, then in how many ways can all these balls be distributed into these boxes?

a) 110 b) 220 c) 330 d) 1440

23. In how many ways can these balls be distributed so that box 2 and box 4 contain only 1 and 2 balls, respectively?

a) 5522 b) 8505 c) 2305 d) 2160

29. If N is the number of ways in which a person can walk up a stair-way which has 7 steps if he can take 1 or 2 steps up the stairs at a time, then the value of $\frac{N}{3}$ is

31. If N is the number of ways in which 3 distinct numbers can be selected from the set $\{3^1, 3^2, 3^3, ..., 3^{10}\}$ so that they form a G.P then the value of $\frac{N}{5}$ is

32. Consider a set of 8 vectors $V = \{a\hat{i} + b\hat{j} + c\hat{k}; a, b, c \in \{-1, 1\}\}$ the number of ways of choosing 3 non coplanar vectors from V is 2^p then the value of p is

33. Let $f: A \to B$ be any function where A is the set containing the positive integral solutions of the in equality $Sin^{-1}(Sin2) > x^2 - 3x$ and B is the set of all divisors of 30. If $f(i) \le f(j) \forall i < j$ then the number of mapping from A to B is 15K then the value of K is

Passage - II :

Let *f(n)* denotes the numebr of different ways the positive integer *n* can be expressed as the sum of 1's and 2's. For example *f*(4) = 5

i.e, 4=1+1+1+1=1+1+2 =1+2+1=2+1+1=2+2

28. The value of *f*(6) is

a) 10 b) 13 c) 16 d) 19

29. The value of *f*(*f*(6)) is

a) 356 b) 377 c) 389 d) 427

30. The number of solutions of the equation $f(n) = n$, where $n \in N$ is

a) 1 b) 2 c) 3 d) 4

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32. If N is the number of rational numbers between 0 and 1 whose digits after the decimal point are non zero and are in the decreasing order then the sum of digits of the number N is

33. Let n = 2005. The least positive integer k for which $k(n^2)(n^2-1^2)(n^2-2^2)(n^2-3^2)..........(n^2-(n-1)^2) = r!$ for some positive integer r is

Passage - II :

10 digit numbers are formed by using all the digits 0,1,2,3,4,5,6,7,8 and 9 such that they divisible by 11111.

20. The digit in the tens place, in the smallest of such numbers, is

a) 9 b) 8 c) 7 d) 6

21. The digit in the units place, in the greatest of such numbers, is

a) 4 b) 3 c) 2 d) 1

23. Consider $S=\{1,2,3,4,.......10\}$. Then sum of all products of numbers by taking two or more fro is $(11!-k)$ than $\left[\frac{k}{11}\right]$ where [ ] is G.I.F is

25. If a, b, c are positive two digit numbers 2 and 0 are digits in the units place of b and c

$D=\begin{vmatrix} a & 5 & 2 \\ b & 0 & 1 \\ c & 1 & 0 \end{vmatrix}$ such that $D+2$ is divided by 10. Then the digit in the units place of a is

26. Let $f(n)=\sum_{r=0}^{n}\sum_{k=r}^{n}\binom{k}{r}$ The total number of divisors of $f(9)$ is.

0. The number of positive integer pairs (x, y) such that $\frac{1}{x}+\frac{1}{y}=\frac{1}{2007}, x<y$ is

1. Let $x=\{1,2,3,4\}$ the no.of one – one functions $f:x\to x$ satisfying $f(f(i))=i,\ \forall 1\le i\le 4$ is $2K$ then $K=$

3. If p,q are randomly chosen from the set {1,2,3,4,5,6} with replacement, then the number of ways in which the expression $x^4+px^3+(q+1)x^2+px+q$ will have positive values for all real x is two digited number then its units place digit is

25. The number of monotonically increasing functions from the set $\{1,2,3,...6\}$ to itself with the property that $f(x)\ge x\,\forall x\in\{1,2,3,...6\}$ is equal to $\frac{2}{k}\cdot{}^{11}c_5$ where k is equal to ______

26. 'A' & 'B' are two students among 100 students appearing for a competitive exam. If all 100 students are given ranks the number of ways in which 'A' will be better ranked than B is $50\underline{|k}$ where $\frac{k}{11}$ is equal to ______

PASSAGE - 3

Let p be an odd prime number and T_p be the following set of 2×2 matrices :

$$T_P = \left\{ A = \begin{bmatrix} a & b \\ c & a \end{bmatrix} : a, b, c \in \{0, 1, 2, \ldots, p-1\} \right\}$$ **(2010)**

7. The number of A in T_p such that A is either symmetric or skew-symmetric or both, and det(A) divisible by p is

(a) $(p-1)^2$ (b) $2(p-1)$

(c) $(p-1)^2 + 1$ (d) $2p - 1$

8. The number of A in T_p such that the trace of A is not divisible by p but det (A) is divisible by p is

[**Note:** The trace of a matrix is the sum of its diagonal entries.]

(a) $(p-1)(p^2 - p + 1)$ (b) $p^3 - (p-1)^2$

(c) $(p-1)^2$ (d) $(p-1)(p^2 - 2)$

9. The number of A in T_p such that det (A) is not divisible by p is

(a) $2p^2$ (b) $p^3 - 5p$

(c) $p^3 - 3p$ (d) $p^3 - p^2$

5. Let $z = \dfrac{-1 + \sqrt{3}i}{2}$, where $i = \sqrt{-1}$, and r, s $\in \{1, 2, 3\}$. Let

$P = \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}$ and I be the identity matrix of order 2.

Then the total number of ordered pairs (r, s) for which $P^2 = -I$ is ***(JEE Adv. 2016)***

Example 38 A class contains 4 boys and g girls. Every sunday five students, including at least three boys go for a picnic to Appu Ghar, a different group being sent every week. During, the picnic, the class teacher gives each girl in the group a doll. If the total number of dolls distributed was 85, then value of g is

(a) 15 (b) 12 (c) 8 (d) 5

Ans. (d)

Example 39 The sum of the factors of 9! which are odd and are of the form $3m + 2$, where m is a natural number is

(a) 40 (b) 45

(c) 51 (d) none of these

Example 52 The number of ways of choosing triplets (x, y, z) such that $z \geq \max\{x, y\}$ and $x, y, z \in \{1, 2, \ldots, n, n + 1\}$ is

(a) ${}^{n+1}C_3 + {}^{n+2}C_3$ (b) $\frac{1}{6} n(n + 1)(2n + 1)$

(c) $1^2 + 2^2 + \ldots + n^2$ (d) $2({}^{n+2}C_3) - {}^{n+1}C_2$

Ans. (a), (b), (c), (d)

Example 62 Let N denote the number of ways in which $3n$ letters can be selected from $2n$ A's, $2n$ B's and $2n$ C's. Then

(a) $3 \mid (N - 1)$ (b) $n \mid (N - 1)$

(c) $(n + 1) \mid (N - 1)$ (d) $3n(n + 1) \mid (N - 1)$

Ans. (a), (b), (c), (d)

20. Let $p, q \in \{1, 2, 3, 4\}$. The number of equations of the form $px^2 + qx + 1 = 0$ having real roots is

(a) 15 (b) 9

(c) 7 (d) 8

➤ **Example 11.** Find the number of combinations and the number of permutations of the letters of the word PARALLEL, taken four at a time.

➤ **Example 13.** Find the number of combinations and permutations of 4 letters taken from the word EXAMINATION.

➤ **Example 18.** 12 seats are to be occupied by 4 people. Find the number of possible arrangements if

(i) no two persons sit side by side.

(ii) there should be atleast two empty seats between any two persons.

(iii) each person has exactly one neighbour.

➤ **Example 6.** In how many ways we can choose two squares of same dimension from a square grid, formed by (2m + 1) horizontal and (2m + 1) vertical equi-spaced lines, such that they have either a edge or corner edge common.

➤ **Problem 5.** Show that the number of ways in which three numbers in A.P. can be selected from 1, 2, 3,...., n is $\frac{1}{4}(n-1)^2$ or $\frac{1}{4}n(n-2)$ according as n is odd or even.

➤ **Problem 11.** Find the number of positive unequal integral solutions of the equation $x_1 + x_2 + x_3 = 14$.

➤ **Problem 13.** Show that the number of positive integral solutions of $x + y + z + w = 26$ such $x > y$ is 1078.

➤ **Problem 19.** Find the number of quadratic polynomials, $ax^2 + bx + c$, which satisfy the following conditions :

(a) a, b, c are distinct ; (b) $a, b, c \in \{1, 2, 3, \ldots, 1999\}$ and (c) $x + 1$ divides $ax^2 + bx + c$.

➤ **Problem 16.** A shop sells 6 different flavours of ice-cream. In how many ways can a customer choose 4 ice cream cones if

(i) they are all of different flavours :

(ii) they are not necessarily of different flavours

(iii) they contain only 3 different flavours

(iv) they contain only 2 or 3 different flavours ?

PASSAGE - XV

A triangle is called an integer triangle if all the sides are integers. If a, b, c are sides of a triangle then we can assume $a \le b \le c$ (any other permutation will yield same triangle). Since sum of two sides is greater than the third side therefore if c is fixed a + b will vary from c + 1 to 2c. The number of such integer triangles can be found by finding integer solutions of $a+b=c+1$, $a+b=c+2$, ... , $a+b=2c$

122. The number of integer isosceles or equilateral triangle none of whose sides exceed 4 must be

A)9 B)10 C)11 D)12

123. The number of integer isosceles or equilateral triangles none of whose sides exceed 2c must be

A) c^2 B) $2c^2$ C) $3c^2$ D) $\frac{3c^2}{2}$

124. If "c" is fixed and odd, the number of integer isosceles or equilateral triangle whose sides are a,b,c $(a \le b \le c)$ must be

A) $\frac{2c-1}{2}$ B) $\frac{2c+1}{2}$ C) $\frac{3c+1}{2}$ D) $\frac{3c-1}{2}$

PASSAGE - XVII

10-digit numbers are formed by using all the digits 0,1,2,3,4,5,6,7,8 and 9 such that they are divisible by 11111.

127. The digit in the ten's place, in the smallest of such numbers, is

A) 9 B) 8
C) 6 D) 7

128. The digit in the unit's place, in the greatest of such numbers, is

A) 2 B) 3
C) 4 D) 1

129. The total number of such numbers is

A) 3456 B) 5634
C) 6543 D) 4365

130. The unit digit of such smallest number is

(A) 3 (B) 5
(C) 2 (D) 4

131. The unit digit of such a greatest number is

(A) 3 (B) 5
(C) 2 (D) 4

PASSAGE - XVIII

The sides of a triangle a, b, c be positive integers and given $a \leq b \leq c$. If c is given, then

132. The number of triangle that can be formed when c is odd are

A) $\frac{(c+1)^2}{4}$ B) $\frac{3c-1}{2}$

C) $\frac{1}{4}c(c+2)$ D) $\frac{1}{2}(3c-2)$

133. The number of triangle that can be formed when c is even are

A) $\frac{(c+1)^2}{4}$ B) $\frac{3c-1}{2}$

C) $\frac{1}{4}c(c+2)$ D) $\frac{1}{2}(3c-2)$

134. The no.of isosceles or equletent triangle that can be formed when c is odd is

A) $\frac{(c+1)^2}{4}$ B) $\frac{3c-1}{2}$

C) $\frac{1}{4}c(c+2)$ D) $\frac{1}{2}(3c-2)$

). The number of ways in which five different books to be distributed among 3 persons so that each person gets at least one book, is equal to the number of ways in which

(A) 5 persons are allotted 3 different residential flats so that each person is allotted at most one flat and no two persons are allotted the same flat.

B) number of parallelograms (some of which may be overlapping) formed by one set of 6 parallel lines and other set of 5 parallel lines that goes in other direction.

C) 5 different toys are to be distributed among 3 children, so that each child gets at least one toy

D) 3 professors of mathematics are assigned five different lectures to be delivered, so that each professor gets at least one lecture

0. Thirteen persons are sitting in a row. Number of ways in which four persons can be selected so that no two of them are consecutive is equal to

A) number of ways in which all the letters of the word "M A R R I A G E" are permutated if no two vowels are never together.

B) number of numbers lying between 100 and 1000 using only the digits 1,2,3,4,5,6,7 without repetition.

C) number of ways in which 4 alike chocolates can be distributed among 10 children so that each child getting at most one chocolate.

D) number of triangles that can be formed by joining 12 points in a plane, of which 5 are collinear.

1. **The number of different ordered triplets (a, b, c), a, b, c∈I, $a+b+c=21$ number of triangles of**

(A) Non equilateral isossceles triangles $=4$

(B) Isossceles right angled trianlges $=0$

(C) Scalene triangles $=7$

(D) Equailateral triangles $=1$

72. **12 small sticks of length 1 cm each are distributed into three children A,B and C. These children join the sticks in the form of line segments individually. If 'n' is the number of ways in which the sticks can be distributed to the children so that the line segments joined by them form a triangle**

A) Number of distributions so that they form a triangle $=10$

B) Number of distributions so that they form right angle triangle $=3$

C) Number of distributions so that they form isossceles $=0$

D) Number of distributions so that they form isossceles right angle triangle $=0$

73. **The number of ways of choosing two naturals such that the least common multiple (LCM) of two numbers is $2^3 5^7 11^{13}$ is M**

A) $M=2835$

B) Two choosen are even then $M=2025$

C) two choosen are odd then $M=0$

D) Exactly one of them is even $=810$

210. Find all triplets of natural numbers (a, b, c) such that a, b and c are in geometric progression and a + b + c = 111.

13. The number of ways in which $2n$ objects of one type, $2n$ of another type and $2n$ of a third type can be divided between 2 persons so that each may have $3n$ objects is $\alpha n^2 + \beta n + \gamma$. Find the value of $(\alpha + \beta + \gamma)$.

Paragraph for Question Nos. 1 to 2

Consider all the six digit numbers that can be formed using the digits 1, 2, 3, 4, 5 and 6, each digit being used exactly once. Each of such six digit numbers have the property that for each digit, not more than two digits smaller than that digit appear to the right of that digit.

1. A six digit number which does not satisfy the property mentioned above, is :
   (a) 315426 (b) 135462 (c) 234651 (d) None of these
2. Number of such six digit numbers having the desired property is :
   (a) 120 (b) 144 (c) 162 (d) 210