14. In a high school, a committee has to be formed from a group of 6 boys M_1, M_2, M_3, M_4, M_5, M_6 and 5 girls G_1, G_2, G_3, G_4, G_5.

(i) Let α_1 be the total number of ways in which the committee can be formed such that the committee has 5 members, having exactly 3 boys and 2 girls.

(ii) Let α_2 be the total number of ways in which the committee can be formed such that the committee has at least 2 members, and having an equal number of boys and girls.

(iii) Let α_3 be the total number of ways in which the committee can be formed such that the committee has 5 members, at least 2 of them being girls.

(iv) Let α_4 be the total number of ways in which the committee can be formed such that the committee has 4 members, having at least 2 girls such that both M_1 and G_1 are NOT in the committee together.

(2018 Adv.)

	List-I		List-II
P.	The value of α_1 is	1.	136
Q.	The value of α_2 is	2.	189
R.	The value of α_3 is	3.	192
S.	The value of α_4 is	4.	200
		5.	381
		6.	461

The correct option is

(a) P $\to$ 4; Q $\to$ 6; R $\to$ 2; S $\to$ 1

(b) P $\to$ 1; Q $\to$ 4; R $\to$ 2; S $\to$ 3

(c) P $\to$ 4; Q $\to$ 6; R $\to$ 5; S $\to$ 2

(d) P $\to$ 4; Q $\to$ 2; R $\to$ 3; S $\to$ 1

8 Take r such that $1 \leq r \leq n$, and consider all subsets of r elements of the set $\{1, 2, \ldots, n\}$. Each subset has a smallest element. Let $F(n, r)$ be the arithmetic mean of these smallest elements. Prove that:

$$F(n, r) = \frac{n+1}{r+1}.$$

Example 38 A class contains 4 boys and g girls. Every sunday five students, including at least three boys go for a picnic to Appu Ghar, a different group being sent every week. During, the picnic, the class teacher gives each girl in the group a doll. If the total number of dolls distributed was 85, then value of g is

(a) 15 (b) 12 (c) 8 (d) 5

Ans. (d)

Q. 'n' lines, no. two of which are parallel and no '3' are concurrent. If their points of intersection are joined, then show that number of fresh lines thus formed is

$$\frac{n(n-1)(n-2)(n-3)}{8}$$

* Total no of sq. on a chess board

$= 8\times8 + 7\times7 + 6\times6 + 5\times5 + \cdots + 1\times1$

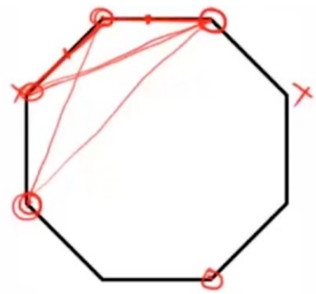
Total no of ways of selecting two squares so that they have exactly one common vertex.

(i) Total no of Δ's = ${}^8C_3 = \frac{8\cdot7\cdot6}{3\cdot2\cdot1} = 56$

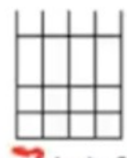
(ii) total no of Δs having exactly one side common $= 8\cdot{}^4C_1 = 32$

(iii) total no of Δs having exactly two sides common = 8 ways

(iv) No of Δs which do not have a side common with the octagon



A rectangle with sides $2m-1$ and $2n-1$ is divided into squares of unit length by drawing parallel lines as shown in the diagram, then the number of rectangles possible with odd side lengths is



(A) $(m+n+1)^2$ (B) 4^{m+n-1}

(C) m^2n^2 (D) $mn(m+1)(n+1)$

[JEE 2005 (Screening), 3]

Match the conditions/expressions in Column I with statement in Column II.

12. Consider all possible permutations of the letters of the word ENDEANOEL.

	Column I		Column II
A.	The number of permutations containing the word ENDEA, is	p.	5!
B.	The number of permutations in which the letter E occurs in the first and the last positions, is	q.	$2\times5!$
C.	The number of permutations in which none of the letters D, L, N occurs in the last five positions, is	r.	$7\times5!$
D.	The number of permutations in which the letters A, E, O occur only in odd positions, is	s.	$21\times5!$

3. Suppose Roger has 4 identical green tennis balls and 5 identical red tennis balls. In how many ways can Roger arrange these 9 balls in a line so that no two green balls are next to each other and no three red balls are together?

(A) 8 (B) 9 (C) 11 (D) 12

28. Two rows of n chairs, facing each other, are laid out. The number of different ways that n couples can sit on these chairs such that each person sits directly opposite to his/her partner is

(A) $n!$ (B) $n!/2$ (C) $2^n n!$ (D) $2n!$.

9. An up-right path is a sequence of points $\mathbf{a}_0 = (x_0, y_0), \mathbf{a}_1 = (x_1, y_1), \mathbf{a}_2 = (x_2, y_2), \ldots$ such that $\mathbf{a}_{i+1} - \mathbf{a}_i$ is either $(1,0)$ or $(0,1)$. The number of up-right paths from $(0,0)$ to $(100,100)$ which pass through $(1,2)$ is:

(A) $3 \cdot \binom{197}{99}$ (B) $3 \cdot \binom{100}{50}$ (C) $2 \cdot \binom{197}{98}$ (D) $3 \cdot \binom{197}{100}$.

*Q.14. In a hotel, four rooms are available. Six persons are to be accommodated in these four rooms in such a way that each of these rooms contains at least one person and at most two persons. Then the number of all possible ways in which this can be done is _____

Sol. **1080.00**

5 boys & 4 girls sit in a straight line. Find the number of ways in which they can be seated if 2 girls are together & the other 2 are also together but separate from the first 2.

In how many ways, we can paint 6 faces of a cube with 6 different colors.

The number of ways in which we can make a garland with 5 flowers of one kind and 3 flowers of another kind, is

20. If the word PERMUTE is permuted in all possible ways and the different resulting words are written down in alphabetical order (also known as dictionary order), irrespective of whether the word has meaning or not, then the 720^{th} word would be:

(A) EEMPRTU (B) EUTRPME (C) UTRPMEE (D) MEETPUR.

An eight digit number divisible by 9 is to be formed by using 8 digits out of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 without replacement. The number of ways in which this can be done is $\lambda^2 \lfloor 7$ then λ =

6. A number is called a palindrome if it reads the same backward or forward. For example, 112211 is a palindrome. How many 6-digit palindromes are divisible by 495?

(A) 10 (B) 11 (C) 30 (D) 45

Sum of all the numbers that can be formed using all the digits 2, 3, 3, 4, 4, 4 is :
(A*) 22222200 (B) 11111100 (C) 55555500 (D) 20333280

How many 4 digit numbers are there which contains not more than 2 different digits?

(a) How many divisors are there of the number $x = 21600$. Find also the sum of these divisors.
(b) In how many ways the number 7056 can be resolved as a product of 2 factors.
(c) Find the number of ways in which the number 300300 can be split into 2 factors which are relatively prime.

Let D_1, D_2,-----,D_{1000} are 1000 doors and P_1, P_2,-------P_{1000} are 1000 persons. Initially all the doors are closed .P_1 opens all the doors. Then P_2 closes D_2,D_4, D_6---- D_{998}, D_{1000}. Then P_3 changes the status of D_3, D_6, D_9, D_{12},-----etc.(doors having numbers which are multiples of 3).Changing the status of a door means closing it if it is open and opening if it is closed. Then P_4 changes the status of D_4,D_8, D_{12}, D_{16},-----etc (doors having numbers which are multiples of 4) And so on until lastly P_{1000} changes the status of D_{1000} , then which of the following is/are correct?

A) Number of doors are open is 31

B) Greatest number of consecutive doors that are closed finally 60

C) The door having the greatest number that is finally open is D_{961}

D) Greatest number of consecutive doors that are closed finally 61

If r, s, t are prime numbers and p, q are the positive integers such that their LCM of p, q is is $r^2t^4s^2$, then the numbers of ordered pair of (p, q) is
(A) 252 (B) 254 (C) 225 (D) 224

Paragraph-2

Let S_n be the set of all n-digit numbers.

59. The number of numbers in $\boxed{S_6}$ consisting of digits 0,3 and 5 only, with each of them appearing at least once, is

A) 360 **B)** 348 **C)** 342 **D)** 324

60. The number of numbers in S_6 having exactly three different digits is

A) 62328 **B)** 58320 **C)** 56328 **D)** 54320

A party of 10 consists of 2 Americans, 2 British men, 2 Chinese & 4 men of other nationalities (all different). Find the number of ways in which they can stand in a row so that no two men of the same nationality are next to one another. Find also the number of ways in which they can sit at a round table.

Derangement formula proof :

$$D(n) = n!\sum_{k=0}^{n}\frac{(-1)^k}{k!}$$

12. The number of ways in which 4 marries couples can be seated such that four persons are no left side and four are on right side of a long table and no wife is infront of her husband and same gender persons do not sit opposite to each other, is

a) $144\underline{|4}$ **b)** $9\underline{|4}$ **c)** $70\underline{|4}$ **d)** None of these

*43. Six cards and six envelopes are numbered 1, 2, 3, 4, 5, 6 and cards are to be placed in envelopes so that each envelope contains exactly one card and no card is placed in the envelope bearing the same number and moreover the card numbered $\boxed{1}$ is always placed in envelope numbered 2. Then the number of ways it can be done is

(A) 264 (B) 265

(C) 53 (D) 67

* Number of ONTO functions (SURJECTIVE function)

$f : A \to B$.

$y = f(x)$ wt $n(A) = n$ $n(B) = m$.

Number of Relations defined on AxA

1: Reflexive
2: Symmetric
3: Anti-symmetric
4: Transitive?
5: Equivalence

NO. of ONTO function =

f : A → B n(A) = 4 ; n(B) = 2

Number of functions defined from A to B

(i) n(AxB)
(ii) Relation on AxB
(iii) Functions on AxB
(iv) One-one / Many one functions
(v) Into/Onto functions
(vi) Bijective functions
(vii) Bijective functions with derangement condition
(viii) Strictly increasing & decreasing functions
(ix) Non-decreasing and Non-increasing functions

16. Let $A = \{a_1, a_2, \ldots, a_{10}\}$ and $B = \{1, 2\}$. The number of functions $f : A \to B$ for which the sum $f(a_1) + \cdots + f(a_{10})$ is an even number, is

(A) 768 (B) 512 (C) 256 (D) 128

14 Let $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{a, b, c, d, e\}$. How many functions $f : A \to B$ are there such that for every $x \in A$, there is one and exactly one $y \in A$ with $y \neq x$ and $f(x) = f(y)$?

(A) 450 (B) 540 (C) 900 (D) 5400.

If $x_1 + x_2 + x_3 ..... + x_r = n$

Then number of solutions of equation ${}^{n+r-1}C_n$ when x_i are $(i = 1, 2, 3 .... r)$ non-negative integers and ${}^{n-1}C_{r-1}$ when x_i are $(i = 1, 2, 3 .... r)$ positive integers

57. If a, b, c be three natural numbers in A.P. then number of solution of $a + b + c = 21$ is

A) 15 B) 14 C) 13 D) 16

58. Number of ways of distributing 22, identical toys among 4 children when each child must get odd number of toys is equal to

A) ${}^{8}c_3$ B) ${}^{12}c_9$ C) ${}^{21}c_3$ D) ${}^{25}c_{22}$

The no.of positive integer solutions of $x + y + z = 10$, where x, y, z are unequal is (20 + K) then K is

How many different ways can 15 Candy bars be distributed between Ram, Shyam, Ghanshyam and Balram, if Ram can not have more than 5 candy bars and Shyam must have at least two. Assume all Candy bars to be alike.

Find the number of distinct throws which can be thrown with 'n' six faced normal dice which are indistinguishable among themselves.

Q No. of positive integral soln of the inequality:

$3x + y + z \leq 30$. $x, y, z \geq 1$

24 What is the number of ordered triplets (a, b, c), where a, b, c are positive integers (not necessarily distinct), such that $abc = 1000$?

(A) 64 (B) 100 (C) 200 (D) 560

The number of combinations n together of 3n letters of which n are 'a' and n are 'b' and the rest unlike is $(n + 2).\ 2^{n-1}$.

An examination has 20 questions. For each question the marks that can be obtained are either -1 or 0 or 4. If S be the set of possible total marks that a student can score in an examination, then find the total number of elements in set S?

Non-Consecutive selection

(i) If 'n' things are arranged in circular order, then show that the number of ways of selecting four of the things no two of which are consecutive is

$$\frac{n(n-5)(n-6)(n-7)}{4!}$$

(ii) If the 'n' things are arranged in a row, then show that the number of such sets of four is

$$\frac{(n-3)(n-4)(n-5)(n-6)}{4!}$$

No. of Δ's formed from the vertices of regular octagon such that they do not form a common side with octagon $n = 8$ $r = 3$

Distribution of balls into boxes so that no box is empty

5 balls are to be placed in 3 boxes. Each box can hold all 5 balls. In how many different ways can we place the balls so that no box remains empty if,

(i) balls & boxes are different
(ii) balls are identical but boxes are different
(iii) balls are different but boxes are identical
(iv) balls as well as boxes are identical
(v) balls as well as boxes are identical but boxes are kept in a row.

5 balls are to be placed in 3 boxes. Each box can hold all 5 balls. In how many different ways can we place the balls if,

(i) balls & boxes are different
(ii) balls are identical but boxes are different
(iii) balls are different but boxes are identical
(iv) balls as well as boxes are identical

➤ **Example 11.** Find the number of combinations and the number of permutations of the letters of the word PARALLEL, taken four at a time.

There are counters available in 7 different colours. Counters are all alike except for the colour and they are atleast ten of each colour. Find the number of ways in which an arrangement of 10 counters can be made. How many of these will have counters of each colour.

Let f(n) denote the number of ways in which n letters go into n envelops so that no let is in the correct envelope, (where n>5), then $f(n)-nf(n-1)$ equals

A) $f(n-2)-(n-2)f(n-3)$

B) $(n-1)f(n-2)-f(n-1)$

C) $(n-3)f(n-4)-f(n-3)$

D) $(n-4)f(n-5)-f(n-4)$

Let a_n denotes the number of all n-digit positive integers formed by the digits 0, 1 or both such that no consecutive digits in them are 0. Let b_n = the number of such n-digit integers ending with digit 1 and C_n = the number of such n-digit integers ending with digit 0.

Q. The value of b_6 is (A) 7 (B) 8 (C) 9 (D) 11

Q. Which of the following is correct ? (A) $a_{17}=a_{16}+a_{15}$ (B) $c_{17}\neq c_{16}+c_{15}$ (C) $b_{17}\neq b_{16}+c_{16}$ (D) $a_{17}=c_{17}+b_{16}$

If p,q are randomly chosen from the set {1,2,3,4,5,6} with replacement, then the number of ways in which the expression $x^4+px^3+(q+1)x^2+px+q$ will have positive values for all real x is two digited number then its units place digit is

11. Let $n_1<n_2<n_3<n_4<n_5$ be positive integers such that $n_1+n_2+n_3+n_4+n_5=20$. The number of such distinct arrangements (n_1,n_2,n_3,n_4,n_5) is (2014 Adv.)

➤ **Problem 5.** Show that the number of ways in which three numbers in A.P. can be selected from 1, 2, 3,...., n is $\frac{1}{4}(n-1)^2$ or $\frac{1}{4}n(n-2)$ according as n is odd or even.