The locus of midpoint of chord of the parabola $y^2 = 4ax$

Column – I		Column – II	
(A)	which passes through focus, is	(p)	$y = 0$
(B)	which is normal, is	(q)	$a^2\ell^2 = (4ax - y^2)(y + 4a^2)$
(C)	for which line joining origin to the extremeties of chord are equally inclined to the axis of the parabola is	(r)	$y^2 = 2a(x - a)$
(D)	whose length is ℓ, is	(s)	$y^4 + 2a(2a - x)y^2 + 8a^4 = 0$

Ans. (A) → (r), (B) → (s), (C) → (p), (D) → (q)

A line $L : y = mx + 3$ meets y – axis at $E(0, 3)$ and the arc of the parabola $y^2 = 16x, 0 \le y \le 6$ at the point $F(x_0, y_0)$. The tangent to the parabola at $F(x_0, y_0)$ intersects the y-axis at $G(0, y_1)$. The slope m of the line L is chosen such that the area of the triangle EFG has a local maximum.

Match List I with List II and select the correct answer using the code given below the lists :

List I		List II	
P.	$m =$	1.	$\frac{1}{2}$
Q.	Maximum area of ΔEFG is	2.	4
R.	$y_0 =$	3.	2
S.	$y_1 =$	4.	1

Codes:

	P	Q	R	S
(a)	4	1	2	3
(b)	3	4	1	2
(c)	1	3	2	4
(d)	1	3	4	2

Let z be the complex number of maximum amplitude (argument) satisfying

$$|z - 3| = Re(z),$$

then find the value of $|z - 3|$?

If the tangent at the point $(t^2, 2t)$ on the parabola $y^2 = 4x$ is same as the normal drawn at point $(\sqrt{5}\cos\theta, 2\sin\theta)$ on the ellipse $4x^2 + 5y^2 = 20$, then

a) $\theta = \cos^{-1}\left(\frac{-1}{\sqrt{5}}\right)$ b) $\theta = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ c) $t = \frac{-2}{\sqrt{5}}$ d) $t = \frac{-1}{\sqrt{5}}$

Minimum distance between the curves $y^2 = x-1$ and $x^2 = y-1$ is equal d, then $2\sqrt{2}.d =$

$$\text{Find } \min[(x_1 - x_2)^2 + (12 - \sqrt{1 - x_1^{\,2}} - \sqrt{4x_2})^2]$$

AB is a chord of the parabola $y^2 = 4x$ such that the normals at A and B intersect at the point $C(9, 6)$

COLUMN – I	COLUMN – II
A) The length AB	p) 20
B) The area of ΔABC	q) $\frac{4}{\sqrt{13}}$
C) The distance of the origin from the line through AB	r) $\sqrt{13}$
D) The area bounded by the coordinate axes and the line through AB	s) 4/3

For the parabola y2 = 4ax, a normal chord AB is drawn at a point A suc AB is shortest then the circumcentre of ΔOAB is

If the normal at 'P' on $y^2 = 4ax$ cuts the axis of the parabola in G and if S is the focus then SG =

1) SP **2) 2SP** **3) $\frac{1}{2}$SP** **4) $\sqrt{SP}$**

The normal at 'P' cuts the axis of the parabola $y^2 = 4ax$ in G and S is the focus of the parabola If ΔSPG is equilateral then each side is of length

1) a **2) $2a$** **3) $3a$** **4) $4a$**

P is the point 't' on the parabola $y^2 = 4ax$ and PQ is a focal chord, PT is the tangent at P and QN is the normal at Q. If the angle betwen PT and QN, be α and the distance between PT and QN be d then.

a) $0 < \alpha < 90^0$, $d = a$

b) $\alpha = 0^0$, $d = \frac{a}{t^2}(t^2+1)^{3/2}$

c) $d = 0, \alpha = 15^0$

d) $d = a\left(\sqrt{1+t^2} + \frac{1}{\sqrt{1+t^2}}\right)$, $\alpha = 35^0$

If the locus of centres of a family of circles passing through the vertex of the parabola $y^2 = 4ax$ and cutting the parabola orthogonally at the other point of intersection is $2y^2(2y^2+x^2-12ax) = ax(kx-4a)^2$, then find the value of k

The common tangents to the circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ touch the circle at the points P, Q and the parabola at the points R, S. Then the area of the quadrilateral $PQRS$ is

(a) 3 (b) 6 (c) 9 (d) 15

Length of chord of parabola $y^2 = 4ax$ whose equation is $y - \sqrt{2}x + 4\sqrt{2}a = 0$

1) $2\sqrt{11}a$ 2) $4\sqrt{2}a$ 3) $8\sqrt{2}a$ 4) $6\sqrt{3}a$

$y = mx$ bisects two distinct chords drawn from (4, 4) on $y^2 = 4x$ if

(a) $m = -1/2$ (b) $m = 0$

(c) $m = 1/2$ (d) $m = 1$

Chords of the parabola $y^2 + 4y = \frac{4}{3}x - \frac{16}{3}$ which subtend right angle at the vertex pass through

(a) (7/3, −2) (b) (1/3, 0)

(c) (4/3, 0) (d) (0, 4/3)

The straight line joining any point P on the parabola $y^2 = 4ax$ to the vertex and perpendicular from the focus to the tangent at P, intersect at R, then the equation of the locus of R is

a) $x^2 + 2y^2 - ax = 0$ b) $2x^2 + y^2 - 2ax = 0$

c) $2x^2 + 2y^2 - ay = 0$ d) $2x^2 + y^2 - 2ay = 0$

The parabola $x^2 = 12y$ rolls without slipping around the parabola $x^2 = -12y$ then find the locus of focus of rolling parabola and also find the locus of vertex of rolling parabola

Two straight lines are perpendicular to each other. One of them touches the parabola $y^2 = 4a(x+a)$ and the other touches $y^2 = 4b(x+b)$. Then the locus of point of intersection of two lines is

1) $x + a = 0$ 2) $x + b = 0$ 3) $x + a + b = 0$ 4) $x - a - b = 0$

Tangent is drawn to parabola $y^2 - 2y - 4x + 5 = 0$ at a point P which cuts the directrix at the point Q. A point R is such that it divides QP externally in the ratio 1/2 : 1. Find the locus of point R.

Passages - I :

A tangent is drawn at any point $P(t)$ on the parabola $y^2 = 8x$ and on it is taken a point $Q(\alpha, \beta)$ from which pair of tangents QA and QB are drawn to the circle $x^2 + y^2 = 8$. Using this information answer the following questions.

The locus of the point of concurrency of the chord of contact AB of the circle $x^2 + y^2 = 8$ is ______

a) $y^2 - 2x = 0$ b) $y^2 - x^2 = 4$ c) $y^2 + 4x = 0$ d) $y^2 - 2x^2 = 4$

The point from which perpendicular tangents can be drawn both to the given circle and the parabola is

a) $(4, \pm\sqrt{3})$ b) $(-1, \sqrt{2})$ c) $(-\sqrt{2}, -\sqrt{2})$ d) $(-2, \pm 2\sqrt{3})$

The locus of circumcentre of ΔAQB if $t = 2$ is

a) $x - 2y + 2 = 0$ b) $x + 2y - 4 = 0$ c) $x - 2y + 4 = 0$ d) $x + 2y + 4 = 0$

Let M be the foot of perpendicular from a point P on parabola $y^2 = 8(x-3)$ to its directrix and S is the focus of parabola and SPM is an equilateral Δ. find length of sides of a Δ?

Let S be the focus of the parabola $y^2 = 8x$ and let PQ the common chord of the circle $x^2+y^2-2x-4y=0$ and the given parabola. find area of ΔPQS?

Consider the parabola $(y-2)^2 = 4(x-2)$ & $(x-2)^2 = 4(y-2)$. Let S be the largest circle touching the two parabolas internally. Find center and radius of the circle.

If eqn of parabola is $25[(x-2)^2+(y+5)^2] = (3x+4y-1)^2$. Find focus, directrix, axis, tangent at vertex.

A ray of light is coming along the line $y = 2$ from the positive direction of x-axis and strikes a concave mirror whose intersection with the xy-plane is a parabola $y^2 = 8x$, then the equation of the reflected ray is

a) $2x+5y=4$ b) $3x+2y=6$ c) $4x+3y=8$ d) $5x+4y=10$

A ray of light moving parallel to the x-axis gets reflected from a parabolic mirror whose equation is $y^2 + 10y - 4x + 17 = 0$. After reflection, the ray must pass through the point

a) $(-2,-5)$ b) $(-1,-5)$ c) $(-3,-5)$ d) $(-4,-5)$.

A ray of light is coming along the line $x = \lambda, (\lambda < 0)$ from the positive direction of y-axis and strikes a concave mirror $x^2 = 4(y-1)$, the ray is reflected from it is $x + y = 2$, if this reflected ray again strikes the same mirror, then the equation of new reflected ray is

a) $y+1=2\sqrt{2}$ b) $x+1=2\sqrt{2}$ c) $y+2=2\sqrt{2}$ d) $x+2=2\sqrt{2}$

3. Let $\Omega = \{z = x + iy \in \mathbb{C} : |y| \leq 1\}$. If $f(z) = z^2 + 2$, then draw a sketch of

$$f(\Omega) = \{f(z) : z \in \Omega\}.$$

Justify your answer.

If $y = x + 1$ is axis of parabola, $y + x = 4$ is tangent of same parabola at its vertex and $y = 2x + 3$ is one of its tangent, then

COLUMN - I	COLUMN - II
A) If equation of directrix of parabola is $ax + by - 29 = 0$, then $a + b = 0$	p) 9
B) If length of latus rectum of parabola is $\frac{a\sqrt{2}}{b}$ where a and b are relatively prime natural numbers, then $a + b =$	q) 18
C) Let extremities of latus rectum are (a_1, b_1) and (a_2, b_2), then $[a_1 + b_1 + a_2 + b_2] =$ (where [.] denote greatest integer function)	r) 23
D) If equation of parabola is $a(x - y + 1)^2 = b(x + y - 4)$ where a and b are relatively prime natural numbers then $a + b =$	s) 37

Let z be a complex number satisfying |z-2|=Re(z).

Find the minimum value of |z-4|?

A movable parabola touches the x-axis and the y-axis at (1,0) and (0,1). Then the locus of the focus of the parabola is

a) $2x^2 - 2x + 2y^2 - 2y + 1 = 0$

b) $x^2 - 2x + 2y^2 - 2y + 1 = 0$

c) $2x^2 - 2x + 2y^2 + 2y + 2 = 0$

d) $2x^2 + 2x - 2y^2 - 2y - 2 = 0$

Normals are drawn from point (4, 1) to the parabola $y^2 = 4x$. The tangents at the feet of normals to the parabola $y^2 = 4x$ from a triangle ABC

A) The distance of focus of parabola $y^2 = 4x$ from centroid of ΔABC is	p) 5/3
B) The distance of focus of parabola $y^2 = 4x$ from orthocentre of ΔABC is	q) $\frac{\sqrt{10}}{2}$
C) The distance of focus of parabola $y^2 = 4x$ from circumcenter of ΔABC is	r) $\frac{\sqrt{7}}{2}$
D) Area of ΔABC is	s) $\frac{\sqrt{5}}{2}$
	t) $\sqrt{5}$