Let the foci of the hyperbola $\frac{x^2}{A^2}-\frac{y^2}{B^2}=1$ be the vertices of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the fo... the ellipse be the vertices of the hyperbola. Let the eccentricities of the ellipse and hyperbola b... and e_H, respectively. Then the match the following.

COLUMN - I	COLUMN - II
A) $\frac{b}{B}$ is equal to	p) 1
B) $e_H + e_E$ is always greater than	q) 2
C) If angle between the asymptotes of hyperbola is $\frac{2\pi}{3}$, then $4e_E$ is equal to	r) 3
D) If $e_E^2=\frac{1}{2}$ and (x, y) is point of intersection of ellipse and the hyperbola then $\frac{x^2}{y^2}$ is	s) 4

The equation of a hyperbola, conjugate to the hyperbola $2x^2+3xy-2y^2+3x+y+2=0$ is $2x^2+3xy-2y^2+3x+y+k=0$ then $k=$

1) 0 2) 1 3) –4 4) 4

Example 37 An ellipse intersects the hyperbola $2x^2-2y^2=1$ orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. if the axes of the ellipse are along the coordinates axes, then

(a) Equation of ellipse is $x^2+2y^2=2$
(b) The foci of ellipse are $(\pm 1, 0)$
(c) Equation of ellipse is $x^2+2y^2=4$
(d) The foci of ellipse are $(\pm\sqrt{2}, 0)$

An ellipse and a hyperbola are confocal (have the same focus) and the conjugate axis of the hyperbola is equal to the minor axis of the ellipse. If e_1, e_2 are the eccentricites of the ellipse and hyperbola then the value $(1/e_1^2)+(1/e_2^2)$ is

The equation of the hyperbola which passes throug the point(2, 3)and has the asymptotes $4x+3y-7=0$ and $x-2y-1=0$ is

1) $4x^2+5xy-6y^2-11x+11y+50=0$
2) $4x^2+5xy-6y^2-11x+11y-43=0$
3) $4x^2-5xy-6y^2-11x+11y+57=0$
4) $x^2-5xy-y^2-11x+11y-43=0$

Example 28 Consider a branch of the hyperbola $x^2 - 2y^2 - 2\sqrt{2}x - 4\sqrt{2}y - 6 = 0$ with vertex at the point A. Let B be one of the end points of its latus rectum. If C is the focus of the hyperbola nearest to the point A, the area of the triangle ABC is

(a) $1-\sqrt{2/3}$ (b) $\sqrt{3/2}-1$

(c) $1+\sqrt{2/3}$ (d) $\sqrt{3/2}+1$

Example 28 Consider a branch of the hyperbola $x^2 - 2y^2 - 2\sqrt{2}x - 4\sqrt{2}y - 6 = 0$ with vertex at the point A. Let B be one of the end points of its latus rectum. If C is the focus of the hyperbola nearest to the point A, the area of the triangle ABC is

(a) $1-\sqrt{2/3}$ (b) $\sqrt{3/2}-1$

(c) $1+\sqrt{2/3}$ (d) $\sqrt{3/2}+1$

If a directrix of a hyperbola centred at the origin and passing through the point $(4, -2\sqrt{3})$ is $5x = 4\sqrt{5}$ and its eccentricity is e, then

(a) $4e^4 - 12e^2 - 27 = 0$

(b) $4e^4 - 24e^2 + 27 = 0$

(c) $4e^4 + 8e^2 - 35 = 0$

(d) $4e^4 - 24e^2 + 35 = 0$

For the hyperbola $9x^2 - 16y^2 - 18x + 32y - 151 = 0$

a) one of the directrices is $x = \frac{21}{5}$

b) length of latus rectum $= \frac{9}{2}$

c) foci are (6, 1) and (−4, 1)

d) eccentricity is $\frac{5}{[illegible]}$

If a hyperbola has one focus at the origin and its eccentricity is $\sqrt{2}$. One of the directrices is $x+y+1$ Then the centre of the hyperbola is

a) $(-1, -1)$ b) $(1, -1)$ c) $(-2, -1)$ d) $(2, 2)$

Example 23 If PQ is a double ordinate of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ such that OPQ is an equilateral triangle, O being the centre of the hyperbola. Then the eccentricity e of the hyperbola, satisfies

(a) $1 < e < 2/\sqrt{3}$ (b) $e = 2/\sqrt{3}$

(c) $e = \sqrt{3}/2$ (d) $e > 2/\sqrt{3}$.

Example 36 If a variable straight line $x\cos\alpha + y\sin\alpha = p$ which is a chord of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $(b > 0)$ subtend a right angle at the centre of the hyperbola, then it always touches a fixed circle whose

(a) radius is $\frac{ab}{\sqrt{b-2a}}$

(b) radius is $\frac{ab}{\sqrt{b^2-a^2}}$

(c) centre is $(0, 0)$

(d) centre is at the centre of the hyperbola

Passage - I :

$H : x^2 - y^2 = 9, P : y^2 = 4(x-5), L : x = 9$

25. If L is the chord of contact of the hyperbola H, then the equation of the corresponding pair of tangents is

a) $9x^2 - 8y^2 + 18x - 9 = 0$
b) $9x^2 - 8y^2 - 18x + 9 = 0$
c) $9x^2 - 8y^2 - 18x - 9 = 0$
d) $9x^2 - 8y^2 + 18x + 9 = 0$

26. If R is the point of intersection of the tangents to H at the extremities of the chord L, then equation of the chord of contact of R with respect to the parabola P is

a) $x = 7$ b) $x = 9$ c) $y = 7$ d) $y = 9$

27. If the chord of contact of R with respect to the parabola P meets the parabola at T and T', S is the focus of the parabola, then area of the triangle STT' is equal to

a) 8 sq. units b) 9 sq. units c) 12 sq. units d) 16 sq. units

Tangent are drawn from any point on the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ to the circle $x^2 + y^2 = 1$ to the chord of contact is $a(x^2 + y^2)^2 = bx^2 - cy^2$, then the value of $a^2 + b^2 + c^2 = k$ then the last digit in k is ____

An ellipse and a hyperbola have their principal axes along the coordinate axes and have a common foci separated by a distance $2\sqrt{13}$, the difference of their focal semi axes is equal to 4. If the ratio of their eccentricities is 3/7. Find the equation of these curves.

Let a hyperbola passes through the focus of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$. The transverse and conjugate axes of this hyperbola coincide with the major and minor axes of the given ellipse, also the product of eccentricities of given ellipse and hyperbola is 1, then *(2006 - 5M, –1)*

(a) the equation of hyperbola is $\frac{x^2}{9} - \frac{y^2}{16} = 1$

(b) the equation of hyperbola is $\frac{x^2}{9} - \frac{y^2}{25} = 1$

(c) focus of hyperbola is (5, 0)

(d) vertex of hyperbola is $(5\sqrt{3}, 0)$

Let a hyperbola passes through the focus of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$. The transverse and conjugate axes of this hyperbola coincide with the major and minor axes of the given ellipse, also the product of eccentricities of given ellipse and hyperbola is 1, then *(2006 - 5M, –1)*

(a) the equation of hyperbola is $\frac{x^2}{9}-\frac{y^2}{16}=1$

(b) the equation of hyperbola is $\frac{x^2}{9}-\frac{y^2}{25}=1$

(c) focus of hyperbola is (5, 0)

(d) vertex of hyperbola is $(5\sqrt{3}, 0)$

The tangent at any point P on a standard hyperbola with centre C, meets the asymptotes in Q and R.

i) Area of triangle ΔCQR constant equal to ab

ii) Portion of the tangent intercepted between the asymptote is bisected at the point of contact.

iii) Locus of the centre of the circle circumscribing the ΔCQR in case of a rectangular hyperbola is the hyperbola itself and for a standard hyperbola the locus would be the curve, $4(a^2x^2-b^2y^2)=(a^2+b^2)^2$

The locus of middle points of normal chords of the rectangular hyperbola $x^2-y^2=a^2$ is

a) $(x^2+y^2)^3+4a^2x^2y^2=0$
b) $(x^2-y^2)^3+4a^2x^2y^2=0$
c) $(x^2+y^2)^3-4a^2x^2y^2=0$
d) $(x^2-y^2)^2-4a^2x^2y^2=0$

If a chord joining the points P $(a\sec\theta, a\tan\theta)$ & Q $(a\sec\phi, a\tan\phi)$ on the hyperbola $x^2-y^2=a^2$ is a normal to it at P, then show that $\tan\phi=\tan\theta\,(4\sec^2\theta-1)$.

8. Let a and b be positive real numbers such that $a > 1$ and $b < a$. Let P be a point in the first quadrant that lies on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Suppose the tangent to the hyperbola at P passes through the point (1,0), and suppose the normal to the hyperbola at P cuts off equal intercepts on the coordinate axes. Let Δ denote the area of the triangle formed by the tangent at P, the normal at P and the x-axis. If e denotes the eccentricity of the hyperbola, then which of the following statements is/are TRUE?

(A) $1 < e < \sqrt{2}$ (B) $\sqrt{2} < e < 2$

(C) $\Delta = a^4$ (D) $\Delta = b^4$

Ans. A,D

The exhaustive set of values of α^2 such that there exist a tangent to the ellipse $x^2 + \alpha^2 y^2 = \alpha^2$ such that the portion of the tangent intercepted by the hyperbola $\alpha^2 x^2 - y^2 = 1$ subtends a right angle at the centre of the curve is

a) $\left[\frac{\sqrt{5}+1}{2}, 2\right]$ b) (1, 2] c) $\left[\frac{\sqrt{5}-1}{2}, 1\right)$ d) $\left[\frac{\sqrt{5}-1}{2}, 1\right) \cup \left(1, \frac{\sqrt{5}+1}{2}\right]$

If the tangent at the point (h, k) to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ cuts the auxiliary circle in points whose ordinates are y_1 and y_2 then prove that $\frac{1}{y_1} + \frac{1}{y_2} = \frac{2}{k}$.

An ellipse has eccentricity 1/2 and one focus at the point P (1/2, 1). Its one directrix is the common tangent, nearer to the point P, to the circle $x^2 + y^2 = 1$ and the hyperbola $x^2 - y^2 = 1$. Find the equation of the ellipse in the standard form.

Passage - I :

The locus of foot of perpendicular from any focus of a hyperbola upon any tangent to the hyperbola is the auxiliary circle of the hyperbola. Consider the foci of a hyperbola as (–3, –2) and (5, 6) and the foot of perpendicular from the focus (5, 6) upon a tangent to the hyperbola as (2, 5)

18. The conjugate axis of the hyperbola is

a) $4\sqrt{11}$ b) $2\sqrt{11}$ c) $4\sqrt{22}$ d) $2\sqrt{22}$

19. The directrix of the hyperbola corresponding to the focus (5, 6) is

a) $2x + 2y - 1 = 0$ b) $2x + 2y - 11 = 0$ c) $2x + 2y - 7 = 0$ d) $2x + 2y - 9 = 0$

20. Eccentricity of the hyperbola

a) $\frac{\sqrt{5}}{4}$ b) $\frac{4}{\sqrt{5}}$ c) $\frac{5}{2}$ d) $\sqrt{10}$

Consider the hyperbola $H: x^2 - y^2 = 1$ and a circle S with centre $N(x_2, 0)$. Suppose that H and S touch each other at a point $P(x_1, y_1)$ with $x_1 > 1$ and $y_1 > 0$. The common tangent to H and S at P intersects the X-axis at point M. If (l, m) is the centroid of ΔPMN, then the correct expression(s) is/are (2015 Adv.)

(a) $\frac{dl}{dx_1} = 1 - \frac{1}{3x_1^2}$ for $x_1 > 1$ (b) $\frac{dm}{dx_1} = \frac{x_1}{3(\sqrt{x_1^2 - 1})}$ for $x_1 > 1$

(c) $\frac{dl}{dx_1} = 1 + \frac{1}{3x_1^2}$ for $x_1 > 1$ (d) $\frac{dm}{dy_1} = \frac{1}{3}$ for $y_1 > 0$

If $\alpha, \beta, \gamma, \delta$ are eccentric angles of four co-normal points to the hyperbola, then $\alpha + \beta + \gamma + \delta$ is an odd multiple of π.

If α, β, γ are eccentric angles of three points on hyperbola, the normals at which are concurrent then $\sin(\alpha + \beta) + \sin(\beta + \gamma) + \sin(\gamma + \alpha) = 0$

The circle $x^2 + y^2 - 8x = 0$ and hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ intersect at the points A and B.

4. Equation of a common tangent with positive slope to the circle as well as to the hyperbola is

(a) $2x - \sqrt{5}y - 20 = 0$ (b) $2x - \sqrt{5}y + 4 = 0$

(c) $3x - 4y + 8 = 0$ (d) $4x - 3y + 4 = 0$

5. Equation of the circle with AB as its diameter is

(a) $x^2 + y^2 - 12x + 24 = 0$ (b) $x^2 + y^2 + 12x + 24 = 0$

(c) $x^2 + y^2 + 24x - 12 = 0$ (d) $x^2 + y^2 - 24x - 12 = 0$

Passage - I :

The difference between the second degree curve and pair of asymptotes is constant. If second degree curve represented by a hyperbola S = 0, then equation of its asymptotes is $S+\lambda=0$ where λ is constant. which will be a pair of straight lines, then we get λ. Then equation of asymptotes is $A\equiv S+\lambda=0$ and if equation of conjugate hyperbola of S represented by S_1, then A is the arithmetic mean of S and S_1

15. Pair of asymptotes of the hyperbola $xy - 3y - 2x = 0$ is

a) $xy - 3y - 2x + 2 = 0$ b) $xy - 3y - 2x + 4 = 0$

c) $xy - 3y - 2x + 6 = 0$ d) $xy - 3y - 2x + 12 = 0$

16. The asymptotes of a hyperbola having centre at the point (1, 2) are parallel to the lines $2x + 3y = 0$ and $3x + 2y = 0$. If the hyperbola passes through the point (5, 3), then its equation is

a) $(2x + 3y - 3)(3x + 2y - 5) = 256$ b) $(2x + 3y - 7)(3x + 2y - 8) = 156$

c) $(2x + 3y - 5)(3x + 2y - 3) = 252$ d) $(2x + 3y - 8)(3x + 2y - 7) = 154$

17. If angle between the asymptotes of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\pi/3$ then the eccentricity of conjugate hyperbola is

a) $\sqrt{2}$ b) 2 c) $2/\sqrt{3}$ d) $4/\sqrt{3}$

If (h, k) is the point of intersection of the normals at P and Q, then k is equal to *(1999 - 2 Marks)*

(a) $\frac{a^2+b^2}{a}$ (b) $-\left(\frac{a^2+b^2}{a}\right)$

(c) $\frac{a^2+b^2}{b}$ (d) $-\left(\frac{a^2+b^2}{b}\right)$

The normal at P to a hyperbola of eccentricity e, intersects its transverse and conjugate axes at L and M respectively. If locus of the mid point of LM is hyperbola, then eccentricity of the hyperbola is

1) $\left(\frac{e+1}{e-1}\right)$ 2) $\frac{e}{\sqrt{(e^2-1)}}$ 3) e 4) $\sqrt{\frac{e-1}{e+1}}$

Q.1 For a complex number z, let $\mathrm{Re}(z)$ denote the real part of z. Let S be the set of all complex numbers z satisfying $z^4 - |z|^4 = 4iz^2$, where $i = \sqrt{-1}$. Then the minimum possible value of $|z_1 - z_2|^2$, where $z_1, z_2 \in S$ with $\mathrm{Re}(z_1) > 0$ and $\mathrm{Re}(z_2) < 0$, is ____

(a)	Circle	(p)	The locus of the point (h, k) for which the line $hx + ky = 1$ touches the circle $x^2 + y^2 = 4$.
(b)	Parabola	(q)	Points Z in the complex plane satisfying $\lvert Z + 2\rvert - \lvert Z - 2\rvert = \pm 3$
(c)	Ellipse	(r)	Points of the conic have parametric representation $x = \sqrt{3}\left(\frac{1-t^2}{1+t^2}\right)$, $y = \frac{2t}{1+t^2}$
(d)	Hyperbola	(s)	The eccentricity of the conic lies in the interval $1 \le x \le \infty$
		(t)	Points Z in the complex plane satisfying $Re(Z + 1)^2 = \lvert Z\rvert^2 + 1$.

In any hyperbola tangent and normal at point P bisects the internal and external angle bisector respectively between the focal distance of P.

Passage - II :

A point P moves such that sum of the slopes of the normal drawn from it to the hyperbola $xy = 16$ is equal to the sum of ordinates of feet of normal. The locus of P is a curve C.

21. The equation of the curve C is

a) $x^2 = 4y$ b) $x^2 = 16y$ c) $x^2 = 12y$ d) $y^2 = 8x$

22. If the tangent to the curve C cuts the coordinate axes at A and B, then the locus of the middle point of AB is

a) $x^2 = 4y$ b) $x^2 = 2y$ c) $x^2 + 2y = 0$ d) $x^2 + 4y = 0$

23. Area of the equilateral triangle, inscribed in the curve C, having one ventex as the vertex of curve C is

a) $772\sqrt{3}$ sq.units b) $776\sqrt{3}$ sq units c) $760\sqrt{3}$ sq.units d) $768\sqrt{3}$ sq.units

The sides of a triangle ABC, inscribed in a hyperbola $xy = c^2$, makes angles α, β, γ with an asymptote. Prove that the nomals at A, B, C will meet in a point if $\cot 2\alpha + \cot 2\beta + \cot 2\gamma = 0$

Passage - I :

A conic passes through the point (2, 4) and is such that the segment of any of its tangents at any point contained between the coordinate axes is bisected at the point of tangency.

18. The eccentricity of the conic is

a) 2 b) $\sqrt{2}$ c) $\sqrt{3}$ d) $\sqrt{\frac{3}{2}}$

19. The foci of the conic are

a) $(-2\sqrt{2},0)$ and $(-2\sqrt{2},0)$ b) $(2\sqrt{2},2\sqrt{2})$ and $(-2\sqrt{2},-2\sqrt{2})$

c) (4, 4) and (–4, –4) d) $(4\sqrt{2},4\sqrt{2})$ and $(-4\sqrt{2},-4\sqrt{2})$

20. The equations of directrices are

a) $x+y=\pm 8$ b) $x+y=\pm 4$ c) $x+y=\pm 4\sqrt{2}$ d) $x+y=\pm 1$

Passage - II :

Consider a hyperbola $xy = 4$ and a line $y + 2x = 4$. O is centre of hyperbola. Tangent at any point P of hyperbola intersect the co-ordinate axis *at* A and B.

28. Locus of circum centre of ΔOAB is

a) an ellipse with eccentricity $\frac{1}{\sqrt{2}}$ b) an ellipse with eccentricity $\frac{1}{\sqrt{3}}$

c) a hyperbola with eccentricity $\sqrt{2}$ d) a circle

29. Shortest distance between the line and hyperbola is

a) $8\sqrt{\frac{2}{3}}$ b) $\frac{4(\sqrt{2}-1)}{5}$ c) $\frac{2\sqrt{2}}{\sqrt{5}}$ d) $\frac{4(\sqrt{2}+1)}{\sqrt{5}}$

30. Let the given line intersect the x-axis at R. If a line through R intersect the hyperbola at S and T, then the minimum value of $RS \times RT$ is

a) 2 b) 4 c) 6 d) 8

Consider an ellipse $\frac{x^2}{36}+\frac{y^2}{18}=1$. There is a hyperbola whose one asymptote is major axis of given ellipse. If eccentricity of given ellipse and hyperbola are reciprocal to each other, both have same centre and both touch each other in first and third quadrant

18. Focus of hyperbola 1st equal to

a) $\left(\frac{3}{2},\frac{3}{2}\right)$ b) $\left(\frac{3}{\sqrt{2}},\frac{3}{\sqrt{2}}\right)$ c) $(3\sqrt{2},3\sqrt{2})$ d) $\left[3\left(3^{\frac{1}{4}}\right),3\left(2^{\frac{1}{4}}\right)\right]$

19. Number of points in $x-y$ plane from where perpendicular tangents can be drawn to hyperb

a) 0 b) 1 c) infinite d) none of these

20. The equation of common tangent to given ellipise and hyperbola in first quadrant is

a) $\frac{x}{\sqrt{2}}+y=3$ b) $\frac{x}{\sqrt{2}}+y=6$ c) $x+y\sqrt{2}=3\sqrt{2}$ d) $x+y\sqrt{2}=6$

If we rotate the axes of the rectangular hyperbola $x^2 - y^2 = a^2$ through an angle $\pi/4$ clockwise direction, then the equation $x^2 - y^2 = a^2$ reduces to $xy = \frac{a^2}{2} = \left(\frac{a}{\sqrt{2}}\right)^2 = c^2$ (say $x = ct,\ y = \frac{c}{t}$ satisfies $xy = c^2$. $\therefore (x, y) = \left(ct, \frac{c}{t}\right) (t \neq 0)$ is called a 't' point on the rect hyperbola.

16. If t_1 and t_2 are the roots of the equation $x^2 - 4x + 2 = 0$, then the point of intersection of tange and 't_2' on $xy = c^2$ is

a) $\left(\frac{c}{2}, 2c\right)$ b) $\left(2c, \frac{c}{2}\right)$ c) $\left(\frac{c}{2}, c\right)$ d) $\left(c, \frac{c}{2}\right)$

17. Let α, γ be the roots of the equation $t_1x^2 - 4x + 1 = 0$ and β, δ be the roots of $t_2x^2 - 6x + 1$ = $\alpha, \beta, \gamma, \delta$ are in HP then the point of intersection of normals at 't_1' and 't_2' on $xy = c^2$ is

a) $\left(\frac{327c}{264}, \frac{921c}{264}\right)$ b) $\left(\frac{237c}{264}, \frac{291c}{264}\right)$ c) $\left(\frac{723c}{264}, \frac{129c}{264}\right)$ d) none of these

18. If e_1 and e_2 are the eccentricities of the hyperbolas $xy = 9$ and $x^2 - y^2 = 25$, then (e_1, e_2) lie on

Passage - II :

The vertices of ΔABC lie on a rectangular hyperbola such that the orthocenter of the triangle is (3, 2) and the asymptotes of the rectangular hyperbola are parallel to the coordinate axes. The two perpendicular tangents of the hyperbola intersect at the point (1, 1).

21 The equation of the asymptotes is

a) $xy - 1 = x - y$ b) $xy + 1 = x + y$ c) $2xy = x + y$ d) $2xy = x - y$

22. Equation of the rectangular hyperbola is

a) $xy = 2x + y - 2$ b) $2xy = x + 2y + 5$ c) $xy = x + y + 1$ d) $xy = x + y$

23. Number of real tangents that can be drawn from the point (1, 1) to the rectangular hyperbola is

a) 4 b) 0 c) 3 d) 2

The tangent to the hyperbola $xy = c^2$ at the point P intersects the x-axis at T and the y-axis at T'. The normal to the hyperbola at P intersecs the x-axis at N and the y-axis at N'. The areas of the triangles PNT and $PN'T'$ are Δ adn Δ' respectively, then $\frac{1}{\Delta} + \frac{1}{\Delta'}$ is

a) equal to 1 b) depends on t c) depennds on c d) equal to 2