Point "O" is the centre of an ellipse with major axis AB and minor axis CD. Point F is one focus of the ellipse. If $OF = 6$ and the diameter of the inscribed circle of the triangle OCF is 2 and the product of $(AB)(CD)$ is k. Then the value of $[\sqrt{k}]$ = ______. Where [.] denotes greatest integer function

An ellipse is drawn with major and minor axis of lengths 10 and 8 respectively. Using one focus as centre, a circle is drawn that is tangent to the ellipse, with no part of the circle being outside the ellipse then radius of the circle is

a) 4 b) 5 c) 2 d) 1

Consider an ellipse and a concentric circle. The circle passes through the foci of the ellipse and intersects the ellipse in four distinct points. The length of major axis of the ellipse is 15 units. If S_1 and S_2 are the foci of the ellipse and area of triangle PS_1S_2 is 26 sq. units, then ecenricity of ellipse is equal to (where P is one of points of intersection of ellipse and circle)

a) $\frac{2}{3}$ b) $\frac{7}{15}$ c) $\frac{13}{15}$ d) $\frac{11}{15}$

Equation of the largest circle with centre (1, 0) that can be inscribed in the ellipse $x^2 + 4y^2 = 16$, is

a) $2x^2 + 2y^2 - 4x + 7 = 0$ b) $x^2 + y^2 - 2x + 5 = 0$

c) $3x^2 + 3y^2 - 6x - 8 = 0$ d) $x^2 + y^2 - 2x = 0$

4. Let a, b and λ be positive real numbers. Suppose P is an end point of the latus rectum of the parabola $y^2 = 4\lambda x$, and suppose the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through the point P. If the tangents to the parabola and the ellipse at the point P are perpendicular to each other, then the eccentricity of the ellipse is

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{2}{5}$

Example 35 In a ΔABC with fixed base BC, the vertex A moves such than

$$\cos B + \cos C = 4\sin^2(A/2)$$

If a, b and c denote the long this of the sides of the triangle opposite to the angles A, B and C respectively, then

(a) $b + c = 4a$

(b) $b + c = 2a$

(c) locus of point A is an ellipse

(d) locus of point A is a pair of straight lines.

A variable point P on the ellipse of eccentricity e is joined to the foci S and S'. The eccentricity of the locus of incentre the triangle PSS' is

a) $\sqrt{\frac{2e}{1+e}}$ b) $\sqrt{\frac{e}{1+e}}$ c) $\sqrt{\frac{1-e}{1+e}}$ d) $\frac{e}{2(1+e)}$

If maximum distance of any point on the ellipse $x^2+2y^2+2xy=1$ from its centre be r, then r is equal to

a) $3+\sqrt{3}$ b) $2+\sqrt{2}$ c) $\frac{\sqrt{2}}{\sqrt{3-\sqrt{5}}}$ d) $\sqrt{2-\sqrt{2}}$

COLUMN - I	COLUMN - II
A) S and S' are foci and B is end of minor axis of an ellipse. If $\Delta SS'B$ is equilateral, then eccentricity of ellipse is	p) 10
B) If $P(x, y)$ be a point on the ellipse $16x^2+25y^2=400$ and $F_1=(3, 0)$, $F_2=(-3, 0)$ then $PF_1+PF_2=$	q) $\frac{(x-2)^2}{4}+\frac{(y+3)^2}{3}=1$
C) A circle with centre at (0, 3) passes through the foci of ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$. Its radius is of length	r) $\frac{1}{2}$
D) In an ellipse $C=(2,-3)$, $S=(3,-3)$ an A is $(4,-3)$, then the equation of ellipse is	s) 4

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$, $y_1<0$, $y_2<0$, be the end points of the latus rectum of the ellipse $x^2+4y^2=4$. The equations of parabolas with latus rectum PQ are *(2008)*

(a) $x^2+2\sqrt{3}\,y=3+\sqrt{3}$ (b) $x^2-2\sqrt{3}\,y=3+\sqrt{3}$

(c) $x^2+2\sqrt{3}\,y=3-\sqrt{3}$ (d) $x^2-2\sqrt{3}\,y=3-\sqrt{3}$

The equation of the common tangent in 1st quadrant to the circle $x^2+y^2=16$ and the ellip $\frac{x^2}{25}+\frac{y^2}{4}=1$. Makes intercept between the co-ordinate axes. Then find the value of k for whi length of this intercept is $\frac{14}{\sqrt{k}}$

If the ellipse $\frac{x^2}{a^2-7}+\frac{y^2}{13-5a}=1$ is inscribed in a square of side length $\sqrt{2}a$ then a belongs to

a) $\left(\frac{6}{5},\infty\right)$ b) $(-\infty,-\sqrt{7})\cup(\sqrt{7},13/5)$

c) $(-\infty,-\sqrt{7})\cup(13/5,\sqrt{7})$ d) no such a exists

A vertical line passing through the point (h, 0) intersects the ellipse $\frac{x^2}{4}+\frac{y^2}{3}=1$ at the points P and Q. Let the tangents to the ellipse at P and Q meet at the point R. If $\Delta(h)$ = area of the triangle PQR, $\Delta_1 = \max\limits_{1/2 \le h \le 1} \Delta(h)$ and $\Delta_2 = \min\limits_{1/2 \le h \le 1} \Delta(h)$,

then $\frac{8}{\sqrt{5}}\Delta_1 - 8\Delta_2 =$

Let $F_1(x_1, 0)$ and $F_2(x_2, 0)$ for $x_1 < 0$ and $x_2 > 0$, be the foci of the ellipse $\frac{x^2}{9}+\frac{y^2}{8}=1$. Suppose a parabola having vertex at the origin and focus at F_2 intersects the ellipse at point M in the first quadrant and at point N in the fourth quadrant.

13. The orthocentre of the triangle F_1MN is

(a) $\left(-\frac{9}{10}, 0\right)$ (b) $\left(\frac{2}{3}, 0\right)$

(c) $\left(\frac{9}{10}, 0\right)$ (d) $\left(\frac{2}{3}, \sqrt{6}\right)$

14. If the tangents to the ellipse at M and N meet at R and the normal to the parabola at M meets the x-axis at Q, then the ratio of area of the triangle MQR to area of the quadrilateral MF_1NF_2 is

(a) 3:4 (b) 4:5

(c) 5:8 (d) 2:3

Tangents are drawn from the point P(3, 4) to the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ touching the ellipse at points A and B. *(2010)*

6. The coordinates of A and B are

(a) (3, 0) and (0, 2)

(b) $\left(-\frac{8}{5},\frac{2\sqrt{161}}{15}\right)$ and $\left(-\frac{9}{5},\frac{8}{5}\right)$

(c) $\left(-\frac{8}{5},\frac{2\sqrt{161}}{15}\right)$ and (0, 2)

(d) (3,0) and $\left(-\frac{9}{5},\frac{8}{5}\right)$

7. The orthocenter of the triangle PAB is

(a) $\left(5,\frac{8}{7}\right)$ (b) $\left(\frac{7}{5},\frac{25}{8}\right)$ (c) $\left(\frac{11}{5},\frac{8}{5}\right)$ (d) $\left(\frac{8}{25},\frac{7}{5}\right)$

If equation of the ellipse is $3x^2 + 2y^2 + 6x - 8y + 5 = 0$, then which of the following are true :

a) $e=\frac{1}{\sqrt{3}}$

b) centre is (–1, 2)

c) foci are (–1, 1) and (–1, 3)

d) directrices are $y=2\pm\sqrt{3}$

Let E_1 and E_2 be two ellipses whose centers are at the origin. The major axes of E_1 and E_2 lie along the x-axis and the y-axis, respectively. Let S be the circle $x^2 + (y-1)^2 = 2$. The straight line $x + y = 3$ touches the curves S, E_1 and E_2 at P, Q and R respectively. Suppose that $PQ = PR = \frac{2\sqrt{2}}{3}$. If e_1 and e_2 are the eccentricities of E_1 and E_2, respectively, then the correct expression(s) is (are)

(a) $e_1^2 + e_2^2 = \frac{43}{40}$ (b) $e_1 e_2 = \frac{\sqrt{7}}{2\sqrt{10}}$

(c) $\left|e_1^2 - e_2^2\right| = \frac{5}{8}$ (d) $e_1 e_2 = \frac{\sqrt{3}}{4}$

(2, 6) and (12, k) are the foci of an ellipse which touches both the coordinate axes. Then its

a) Point of contact with x-axis is (8, 0) **b) Point of contact with y – axis is $\left(0, \frac{40}{7}\right)$**

c) Eccentricity of the ellipse is $\frac{\sqrt{13}}{5}$ **d) Length of latus rectum of the ellipse is $\frac{24\sqrt{2}}{5}$**

A coplanar beam of light emerging from a point source has the equation $\lambda x - y + 2(1+\lambda) = 0, \lambda \in R$, the rays of the beam strike an elliptical surface and get reflected. The reflected rays form another convergent beam having equation $\mu x - y + 2(1-\mu) \in R$. Further it is found that the foot of the perpendicular from the point (2, 2) upon any tangent to the ellipse lies on the circle $x^2 + y^2 - 4y - 5 = 0$

37. The eccentricity of the ellipse is equal to

a) $\frac{1}{3}$ b) $\frac{1}{\sqrt{3}}$ c) $\frac{2}{3}$ d) $\frac{1}{2}$

38. The area of the largest triangle that an incident ray and the corresponding reflected ray can enclose with axis of the ellipse is equal to

a) $4\sqrt{5}$ b) $2\sqrt{5}$ c) $\sqrt{5}$ d) None of these

39. Length of the latus rectum of the ellipse

a) $\frac{15}{3}$ b) $\frac{5}{3}$ c) $\frac{10}{3}$ d) None of these

If tangent at point P on the ellipse intersect major axis at T. If N is foot of perpendicular from P on major axis. Then circle drawn on NT as diameter intersect the auxillary circle orthogonally.

If any tangent to an ellipse intersect the tangents at the vertices at T and T'. Then the circle considering TT' as diameter passess through both foci.

Match the following loci for the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \, (a > b)$:

COLUMN - I	COLUMN - II
A) Locus of point of intersection of two perpendicular tangents	p) $(x^2+y^2)^2 = a^2x^2 + b^2y^2$
B) Locus of foot of perpendicular from any focus upon any tangent	q) $4(x^2+y^2)^2 = a^2x^2 + b^2y^2$
C) Locus of foot of the perpendicular from centre on any tangent	r) $x^2 + y^2 = a^2$
D) Locus of mid point of segment OM where M is the foot of the perpendicular from centre O to any tangent.	s) $x^2 + y^2 = a^2 + b^2$

If two concentric ellipses be such that the foci of one be on the other and their major axes are equal. If e_1 and e_2 be their eccentricities, then the angle θ between their axes is given by :

a) $\cos\theta = \sqrt{\frac{1}{e_1^2} + \frac{1}{e_2^2} - \frac{1}{e_1^2 e_2^2}}$ b) $\cos\theta = \sqrt{\frac{1}{e_1^2} + \frac{1}{e_2^2} + \frac{1}{e_1^2 e_2^2}}$

c) $\cos^2\theta = \sqrt{\frac{1}{e_1^2} + \frac{1}{e_2^2} + \left(\frac{1}{e_1^2 e_2^2}\right)^2}$ d) $\sin\theta = \sqrt{\left(1 - \frac{1}{e_1^2}\right)\left(1 - \frac{1}{e_2^2}\right)}$

The ellipse $E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point (0, 4) circumscribes the rectangle R. The eccentricity of the ellipse E_2 is *(2012)*

(a) $\frac{\sqrt{2}}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

COLUMN - I	COLUMN - II
A) The number of rational points on the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ is	P) infinite
B) The number of integral points on the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ is	Q) 4
C) The number of rational points on the ellipse $\frac{x^2}{3} + y^2 = 1$	R) 0
D) The number of integral points on the ellipse $\frac{x^2}{3} + y^2 = 1$	S) 2

$C: x^2 + y^2 = 9$, $E: \dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$, $L: y = 2x$

Example 44 P is a point on the circle C, the perpendicular PQ to the major axis of the ellipse E meets the ellipse at M, then $\dfrac{MQ}{PQ}$ is equal to

(a) 1/3 (b) 2/3

(c) 1/2 (d) none of these

Example 45 If L represents the line joining the point P on C to its centre O, then equation of the tangent at M to the ellipse E is

(a) $x + 3y = 3\sqrt{5}$ (b) $4x + 3y = \sqrt{5}$

(c) $x + 3y + 3\sqrt{5} = 0$ (d) $4x + 3 + \sqrt{5} = 0$

Example 46 Equation of the diameter of the ellipse E conjugate to the diameter represented by L is

(a) $9x + 2y = 0$ (b) $2x + 9y = 0$

(c) $4x + 9y = 0$ (d) $4x - 9y = 0$

The locus of the foot of perpendicular drawn from the centre of the ellipse $x^2 + 3y^2 = 6$ on any tangent to it is

(a) $\left(x^2 + y^2\right)^2 = 6x^2 + 2y^2$ **(b)** $\left(x^2 + y^2\right)^2 = 6x^2 - 2y^2$

(c) $\left(x^2 - y^2\right)^2 = 6x^2 + 2y^2$ **(d)** $\left(x^2 - y^2\right)^2 = 6x^2 - 2y^2$

If $(5, 12)$ and $(24, 7)$ are the foci of an ellipse passing through the origin, then the eccentricity of the conic is

A. $\frac{\sqrt{386}}{12}$

B. $\frac{\sqrt{386}}{13}$

C. $\frac{\sqrt{386}}{25}$

D. $\frac{\sqrt{386}}{38}$

Let $S = 0$ be the equation of reflection of $\frac{(x-4)^2}{16} + \frac{(y-3)^2}{9} = 1$ about the line $x - y - 2 = 0$. Then the locus of point of intersection of perpendicular tangents of S is

a) $x^2 + y^2 + 10x - y - 4 = 0$ b) $x^2 + y^2 = 25$

c) $x^2 + y^2 - 10x - 4y + 4 = 0$ d) $x^2 + y^2 - 10x - 4y = 0$

Passage - II :

An ellipse (E) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, centered at point O have AB and CD as its major and minor axe respectively, let S_1 be one of the foci of the ellipse, radius of incircle of triangle OCS_1 be 1 un and $OS_1 = 6$ units.

27. Perimeter of ΔOCS_1 is

a) 20 units b) 10 units c) 15 units d) 25 units

28. Equation of the director circle of ellipse (E) is

a) $x^2 + y^2 = (48.5)$ b) $x^2 + y^2 = \sqrt{97}$ c) $x^2 + y^2 = 97$ d) $x^2 + y^2 = \sqrt{48.5}$

29. Area of ellipse (E) is

a) $\frac{65\pi}{4}$ b) $\frac{64\pi}{5}$ c) 64π d) 65π

If the normals at $P(\theta)$ and $Q\left(\frac{\pi}{2} + \theta\right)$ to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meet the major axis at G and g, respectively, then $PG^2 + Qg^2 =$

a) $b^2(1 + e^2)(2 - e^2)$ b) $a^2(e^4 - e^2)(2 - e^2)$ c) $a^2(1 + e^2)(2 + e^2)$ d) $b^2(1 + e^2)(2 + e^2)$

Passage II :

Length of the normal drawn at $P\left(\frac{\pi}{4}\right)$ on $\frac{x^2}{a^2}+\frac{y^2}{16}=1$ is $2\sqrt{3}$ units Then.

26. Length of major axis is

a) 8 b) $4\sqrt{2}$ c) $8\sqrt{2}$ d) 16

27. Eccentricity $e =$

a) $\frac{1}{\sqrt{2}}$ b) $\frac{1}{2}$ c) $\frac{\sqrt{3}}{2}$ d) $\cos 18^0$

28. Equns of directrices are

a) $x=\pm 8$ b) $x=\pm 16$ c) $x=\pm 4\sqrt{2}$ d) $x=\pm 8\sqrt{2}$

$P_1, P_2, \ldots, P_r, \ldots, P_n$ are the points on the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ and $Q_1, Q_2, \ldots, Q_r, \ldots Q_n$ are the corresponding points on the auxiliary circle of the ellipse. If the line joining centre C to Q_i meets the normal at P_i w.r.t. the given ellipse at K_i and $\sum_{i=1}^{n}(CK_i) = 175$, then the value of $n/5$ is

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A normal is drawn to the ellipse $\frac{x^2}{(a^2+2a+2)^2}+\frac{y^2}{(a^2+1)^2}=1$ whose centre is at O. If maximum radius of the circle, centered at the origin and touching the normal, is 5 then the positive value of 'a' is

If the normal at any given point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets its auxiliary circle at Q and R such that $\angle QOR = 90°$, where O is the centre of ellipse, then

a) $a^4 + 2b^4 \geq 3a^2b^2$

b) $a^4 + 2b^4 \geq 5a^2b^2 + 2a^2b$

c) $a^4 + 2b^4 \geq 3a^2b^2 + ab$

d) none of these

The coordinates (2, 3) and (1, 5) are the foci of an ellipse which passes through the origin, then the equation of

a) tangent at the origin is $(3\sqrt{2}-5)x+(1-2\sqrt{2})y=0$

b) tangent at the origin is $(3\sqrt{2}+5)x+(1+2\sqrt{2y})=0$

c) normal at the origin is $(3\sqrt{2}+5)x-(2\sqrt{2}+1)y=0$

d) normal at the origin is $x(3\sqrt{2}-5)-y(1-2\sqrt{2})=0$

PQ is variable chord of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. If PQ subtends a right angle at the center of the ellipse then $\frac{1}{OP^2}+\frac{1}{OQ^2}$, ('$O$' being the origin) is equal to

a) $\frac{1}{a^2}+\frac{1}{b^2}$　　b) $\frac{2}{a^2}+\frac{1}{b^2}$　　c) $\frac{1}{a^2}+\frac{2}{b^2}$　　d) $2\left(\frac{1}{a^2}+\frac{1}{b^2}\right)$

If the line $lx + my + n = 0$ cuts the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in points whose eccentric angles differ by $\frac{\pi}{2}$, then $\frac{a^2l^2+b^2m^2}{n^2}=$

a) 1　　b) 2　　c) 4　　d) $\frac{3}{2}$